THE GROVND OF ARTS.

TEACHING THE PERFECT worke and practise of Arithmeticke, both in whole Numbers and Fractions, after a more easie and exact forme than in former time hath been set forth: Made by M. *Robert Recorde*, D. in Physick.

Afterward, augmented by M^r. John Dee.

And since enlarged with a Third part of Rules of Practise, abridged into a briefer method than hitherto hath been published, with divers necessary Rules incident to the Trade of Merchandise: with Tables of the valuation of all Coynes, as they are currant at this present time.

By *Iohn Mellis.*

And now diligently perused, corrected, illustrated and enlarged; with an *Appendix* of figurate Numbers, and the Extraction of their Roots, according to the Method of *Christian Urstitius*: with Tables of Board and Timber measure, and new Tables of Interest upon Interest, after 10 and 8 per 100; with the true value of Annuities to be bought or sold present, Respited or in Reversion: the first calculated by R. C. but corrected, and the latter diligently calculated by Rob: Hartwell, *Philomathemat.*

Scientia non habet inimicum nisi ignorantem.

Fide——Sed——Vide.

LONDON,
Printed by J. RAWORTH, for JOHN HARISON, and are to be sold at his shop in Pater noster Row, at the signe of the Unicorne. 1640.

THat which my friend hath well begun
For very love to Common-weale,
Need not all whole to be new done,
But now encrease I do reveale.

Something herein I once redrest,
And now again for thy behoofe,
Of zeale I do, and at request,
Both mend and adde, fit for all prroofe.

Of numbers use, the endlesse might,
No wit nor language can expresse,
Apply and try both day and night,
And then this truth thou wilt confesse.

J. De

The Books Verdict.

TO please or displease sure I am,
But not of one sort to every man:
To please the best sort would I faine,
The froward displease shall I certaine,
Yet wish I will, though not with hope,
All eares or mouths to please or stoppe.

TO THE MOST mighty Prince *Edward* the sixth, by the grace of God, King of *England*, *France*, and *Ireland*, *&c.*

THe Excellency of mans nature being such, as it is by Gods divine favour (most mighty Prince) not only created in highnesse of degree far above all other corporall things, but by perfection, reason and search of wit, much approaching toward the Image of God as not only the holy Scriptures do testifie, but also those naturall Philosophers, which exactly did consider the nature of man, and namely the far reach and infinite compasse of the words of the mind, were inforced to confesse, that man scarcely was able to know himselfe. And if he would duely ponder the nature of himselfe, he would find it so strange, that it might seeme unto him a very miracle. And thereof sprang that saying: *Magnum miraculum est homo, maximum miraculum sapiens homo.* For undoubtedly, as man is one of the greatest miracles that ever God wrought, so a wise man is plainly the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not only of the strange workmanship that is in it, but much more of the efficacy of reason, wit, memory, imagination, and such other powers, and works of the mind, which can more easily conceive any thing in a manner, then understand it selfe. Amongst all the creatures of God, it findeth none more difficult to be perceived than these same powers of it selfe; whereby it doth conceive and judge: as it may be well conjectured by the diversity of opinions, that the wisest Philosophers did utter touching the spirit of man, and the substance of it: whereof, I now intend to make no rehearsall; but whoso listeth to read thereof, may

find it largely ſet forth, not only in *Ariſtotle* his books, De *Anima*, but alſo in *Galen*, his book called *Hiſtoria Philoſophica*: and again in *Plutarch* his worke, De *Philoſophorum placitis*, whoſe words are alſo repeated of *Euſebius* in the xv. booke, Τῆς δ' ἀγγελτικῆς προπαρασκευῆς, unto whom I remit them that have deſired to underſtand intricate difficulty of knowing our owne ſelves, as touching our beſt part, and that part whereby we deſerve to beare the name of men.

This matter ſeemed ſo obſcure and difficult in knowledge; that *Galen*, who for his excellent wiſdome and judgment in naturall works, is called of many men a Miracle in Nature, yet in ſearching the nature and ſubſtance of the ſpirit of man, hee not onely confeſſeth himſelfe ignorant, but counteth it plaine temerity to attempt to find it. So farre above the hope of mans knowledge is that part, whereby man doth know and judge of things. And although the ignorant ſort (which hate all things that they know not) doe little eſteeme the profoundneſſe of mans ſpirit and reaſon, the chiefe power and faculty of it: yet as there is a kind of feare and obedience of unreaſonable beaſts unto man, by the working power of God, ſo is there in thoſe ſmall reaſoned perſons a certaine kind of reverence toward wiſdome and reaſon, which they do ſhew oftentimes, and by power of perſwaſion, are inforced to obey reaſon, will they nill they.

And hereby came it to paſſe, that the rudeneſſe of the firſt age of Man was brought unto ſome more civill trade, as it is well declared by *Cicero*, in the beginning of his firſt book, De *inventione Rhetorica*, where hee ſaith thus: *Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant, nec ratione animi quicquam, ſed pleraque viribus corporis adminiſtrabant. Nondum divinæ religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos; non jus æquabile quid utilitatis haberet, acceperat: ita propter errorem atque inſcitiam cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam viribus corporis abutebatur, pernicioſiſſimis ſatellitibus. Quo tempore quidam, magnus vi-*

delicet

delicet vir & ſapiens, cognovit quæ materia eſſet, & quanta ad maximas res opportunitas in animis ineſſet hominum, ſi quis eam poſſit elicere, & præcipiendo meliorem reddere. Qui diſperſos homines in Agris,& in tectis Sylveſtribus abditos,ratione quadam compulit in unum locum, & congregavit: & eos in unamquamque rem inducens utilem atque honeſtam, primo propter inſolentiam reclamantes, deinde propter rationem atque orationem ſtudioſius audientes, ex feris & immanibus, mittes reddidit, & manſuetos.

This long repetition of *Tullyes* words will ſeeme tedious to them that love but little, and care much leſſe for the knowledge of reaſon, but unto your Majeſty (I dare ſay) it is a delectable remembrance, and unto me it ſeemed ſo pleaſant, that I could ſcarce ſtay my pen from writing all that mine eyes did ſo greedily reade.

This ſentence of *Cicero* am I loath to tranſlate into Engliſh,partly for that unto your Majeſty it needeth no tranſlation,but eſpecially knowing how far the grace of *Tullyes* eloquence doth excell any Engliſhmans tongue,and much more exceedeth the baſſeneſſe of my barbarous ſtile: yet for the fruit of the ſentence, I had rather unto my meere Engliſh Countrimen utter the rudeneſſe of my tranſlation, then to defraud them the benefit of ſo good a leſſon; truſting they will ſo learne to love reaſon, that they will alſo gladly and greedily embrace all good Sciences, that may helpe to the juſt furniture of the ſame, when they conſider that informed reaſon was the onely inſtrument, or at leaſt the chiefeſt meanes to bring men into civill regiment, from barbarous manners, and beaſtly conditions.

“ For the time was (ſaith *Tully*) that men wandred abroad in the fields up and down like beaſt, and uſed no better order in feeding than they; ſo that by reaſons rule they wrought nothing, but moſt of their doings did they atchieve by force of ſtrength. At this time there was no juſt regard of religion towards God, nor of duty toward man. No man had ſeen right uſe of marriage, neither did any man know their own children from other; nor no man had felt the commodity of juſt Laws: ſo that through error and ignorance, wilfull luſt, like a blind and heady ruler, abuſed bodi-

" ly ſtrength as a moſt mortall miniſter for the ſatisfying of
" his deſire. At that time was there one which not only
" in power, but alſo in wiſdome was great, and he conſide-
" red how that in the minds of men was both apt inſtru-
" ments, and great occaſion to the due accompliſhment
" of moſt weighty affaires, if a man could apply them to
" uſe, and by teaching of rules frame them to better
" trade. This man with perſwaſion of reaſon gathered
" into one place the people that were wandring about the
" fields, and lay lurking in wild cottages, and woods,
" and bringing them into one common ſociety, did
" trade them to all ſuch things, as either were profitable
" or honeſt, although not without repining at the firſt,
" by reaſon that they had not beene ſo accuſtomed be-
" fore. Yet at length through reaſon and perſwaſion of
" words they obeyed him more diligently, and ſo of a
" wild and cruell people, hee made them curteous and
" gentle.

Thus hath *Tully* ſet forth the efficacy of reaſon and perſwaſion, how it was able to convert wild people to a mildneſſe, and to change their furious cruelneſſe into gentle curteſie: were it not now a great reproach in this our time (when knowledge reigneth ſo large) that men ſhould ſhew themſelves leſſe obſequious to reaſon? Unleſſe it may be thought, that now every man having ſufficient knowledge of himſelfe, needeth not to hearken to the perſwaſion of others.

Indeed he that thinketh himſelfe wiſe, will not eſteeme the reaſon of any other, be he never ſo wiſe; ſo that of ſuch a one it may well be ſaid: He that thinketh himſelfe wiſer than he is, may juſtly be counted a double foole. Wherefore ſuch men are not to be permitted in open audience to to talke, but muſt be put to ſilence, and bee made to give eare to reaſon; which reaſon conſiſteth not in a multitude of words heaped raſhly together, and applied for one purpoſe, but reaſon is the expreſſing of a juſt matter with witty perſwaſions furniſhed with learned knowledge: ſuch knowledge had *Moſes*, being expert in all learning of the Egyptians, as the Scriptures declare, and therefore was able to perſwade the ſtubborne people of the Jews, although

not without paine. Such knowledge and such reasons did *Druys* shew, which was the first Law maker of all the West part of Europe. Like reason and wisdome did *Xamolxis* use amongst the Goths. *Lycurgus* unto the Lacedemonians. *Zeleucus* to the Locrians. *Solon* to the Athenienses, and *Dunwallo Molmutius* two thousand yeares past amongst the old Britains of this Realme. And hereby came it to passe, that their Lawes continued long, till more perfect reason altered many of them, and wilfull power oppressed most of them.

Druys was son to King Sarron, and succeeded him in his Kingdome.

At the beginning when these wise men perceived how hard it was to bring the rude people to understand reason, they judged the best meanes to attaine this honest purpose to depend of learning in every kind: for by learning (as *Ovid* saith:) *Pectora mollescunt asperitasque fugit*, Stout Stomackes doe waxe milde, and sharpe fiercenesse is exilde. Therefore as *Berosus* doth testifie, *Sarron* that was the third King over all this West part of Europe for to bring the people from beastly rage to manly reason, did erect Schooles of liberall Arts, which tooke so good successe, that his name continued in that sort famous above two thousand yeares after: for *Diodorus Siculus* which was in the time of *Iulius Cæsar*, maketh mention of the learned men or Gothes of Celtes, and nameth them *Sarronides*, that is to say, *Sarron his Schollers and followers.*

Among these Arts that then were taught, some did informe the tongue, and make them able both to utter aptly their mind, and also to perswade; as Grammar, Logicke, and Rhetoricke, although not so curiously as in this time; some other did appertaine to the just order of partition of Lands, the true using of Weights, Measures and reckonings in all sorts of bargaines, and for order of building and sundry other uses; those were Arithmeticke and Geometry. Againe, to incourage men to the honor of God, they taught Astronomy, whereby the wonderfull workes of God were so manifestly set forth, that no mans tongue, nor penne can in like sort expresse his infinite power, his unspeakable wisedome, and his exceeding goodnesse toward man, whereby hee doth bountifully provide for man all

necessaries, not only to live, but also to live pleasantly. And so was their confidence in Gods providence strongly stayed, knowing his goodnesse to be such, that he would help man as he could, and his power to be so great, that he would do nothing, but that that was best. Beside these Sciences they taught also Musicke, which most commonly they did apply partly to religious Services, to draw men to delight therein, and partly to songs made of the manners of men, in praise of Vertue, and discommandation of Vice, whereby it came to passe, that no man would displease them, nor do any thing evill that might come to their hearing: for their songs did make evill men more abhorred in that time than any excommunication doth in this time. The posterity of these Musitians continue yet both in Wales and Ireland, called *Bardes* unto this day, by the ancient name of *Bardus*, their first founder.

This Bardus Druydius the 5 King of the Celtes, reigned 60 yeares, and died 1832 yeares before Christ.

And as these Sciences did encrease, so did Vertue encrease thereby. Againe, as those Sciences did decay, so Vertue lost her estimation, and consequently was little in use: whereof to make a full declaration were a thing meet for a Prince to heare, but it would require a peculiar Treatise. Wherefore at this present I count it sufficient lightly to have touched this matter in generall words, and to say no more of the particularity thereof, but onely touching one of those Sciences, that is *Arithmeticke*, by which not onely just partition of lands was made, but also touching buying and selling, all Assises, Weights, and Measures were devised, and all reckonings and accounts driven; yea by proportion of it were the true orders of Iustice limited, as *Aristotle* in his Ethicks doth declare, and the degrees of estates in the Common wealth established: Although that proportion be called Geometricall, and not Arithmeticall, yet doth that proportion appertaine to the Art of *Arithmeticke*, and in *Arithmeticke* is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seeing *Arithmeticke* is so many wayes needfull unto the first planting of a Commonwealth, it must needs

needs be as much required to the preservation of it also, for by the same meanes is any Common-wealth continued, by which it was erected and established. And if I shall in small matters in appearance, but indeed very waighty, put one example or two. What shall wee say for the Statutes of this Realme, which be the onely stay of good order in manner now? As touching the measuring of ground by length and breadth, there is a good and an ancient Statute made by Art of *Arithmeticke*; and now it shall be to little use, if by the same Art it be not practised and tried. For the assise of Bread and Drink, the two most common and most necessary things for sustentation of man, there was a goodly ordinance in the Law made, which by ignorance hath so growne out of knowledge, and use, that few man do understand it, and therefore the Statutes bookes wonderfully corrupted, and the Commons cruelly oppressed: notwithstanding some men have written, that it is too doubtfull a matter to execute those Assises by those Statutes, by reason they depend of the standerd of the coyne, which is much changed from the State of that time, when those Statutes were made. Thus shall every man read (that listeth) in the Abridgement of the Statutes, in the title of Weights and Measures, in the seventh number of the English Book, where hee should have translated a good ordinance which is set forth in the French Booke: but no marvell if the Abridgement doth omit it, seeing the great Booke of Statutes doth omit the same Statute as it hath done divers other very good Lawes. And this is the fruit of ignorance, to reject and condemne all that it understandeth not, although they use some cloakes for it: but such cloakes as being allowed, might serve to repell all good Laws; which God forbid.

Again, there is an ancient order for Assise of fire, Wood and Coales, which was renued not many yeares past; and now how Avarice and Ignorance doth canuase that Statute, it is too pittifull to talke of, and more miserable to feele.

Furthermore, for the Statute of Coynage, and the standred thereof, if the people understood rightly the Statute,

Statute, they should not, nor would not (as they often do) gather an excuse for their folly thereby: but as I said, these Statuts by wisdome and good knowledge of Arithmeticke were made, and by the same must they be continued. And let ignorance no more meddle with the use of them, than it did with the making of them. Oh in how miserable case is that Realme, where the ministers and interpreters of the Law are destitute of all good Sciences, which be the Keyes of the Lawes. How can they either make good Lawes, or maintaine them that lack that true knowledge whereby to judge them? And happy may that Realme bee accounted, where the Prince himselfe is studious of learning, and desireth to understand equity in all Lawes. Therefore most happy are wee the loving subjects of your Majesty, which may see in your Highnesse not only such towardnesse, but also such knowledge of divers Arts, as seldome hath beene seene in any Prince of such yeares, whereby we are enforced to conceive this hope certainly, that he which in those yeares seeketh knowledge, when knowledge is least esteemed, and of such an age can discerne them to bee enemies both to his Royall person, and to his Realmes, which labour to with-draw him from knowledge, to excessive pastime, and from reasonable study, to idle or noysome pleasures, hee must needs, when hee commeth to more mature yeares, be a most prudent Prince, a most just Governor, and a right judge, not only of his Subjects commonly, but also of the ministers of his Lawes, yea, and of the Lawes themselves: and to bee able to conceive the true equity and exact understanding of all his Lawes and Statutes, to the comfort of his good Subjects, and the confusion and reproach of them which labor to obscure or pervert the equity of the same Lawes and Statutes. How some of these Statutes may be applyed to use, as well in this our time, as in any other time, I have peculiarly declared in this Book, and some other I have omitted for just considerations, till I may offer them first unto your Majesty, to weigh them, as to your Highnesse shall seeme good: for many things in them are not to bee published without your Highnesse knowledge and approbation, namely, because in them is declared all the rates of alloyes for all standards from one

ounce

ounce upward, with other mysteries of mint matters, and also most part of the varieties of coynes that have beene currant in this your Majesties Realme by the space almost of six hundred yeares last past, and many of them that were currant in the time that the Romanes ruled here.

All which with the ancient description of England and Ireland, and my simple censure of the same, I have almost completed to be exhibited to your Highnesse. In the meane season most humbly beseeching your Majesty to accept this simple Treatise, not worthy to be presented to so high a Prince, but that my lowly request to your Majesty is, that this amongst other of my bookes may passe under the protection of your Highnesse, whom I beseech God most earnestly and daily, according to my duty, to advance in all honour, and Princely Regalty, and to increase in all knowledge, justice, and godly policy. Amen.

Your Majesties most

obedient subject

and servant,

ROBERT RECORD.

TO

To the loving Readers: The Preface of Maister *Robert Record.*

Sore oft times have I lamented with my selfe the unfortunate condition of England, seeing so many great Clerks to arise in sundry other parts of the world, and so few to appeare in this our Nation: whereas for pregnancy of naturall wit (I thinke) few Nations do excell Englishmen: But I cannot impute the cause to any other thing, then to be contempt or misregard of learning. For as Englishmen are inferiour to no men in mother wit, so they passe all men in vain pleasures, to which they may attaine with great paine and labour: and are as slack to any never so great commodity, if there hang of it any painfull study or travelsome labour.

Howbeit, yet all men are not of that sort, though the most part be, the more pity it is: but of them that are so glad, not only with painfull study, and studious paine to attain learning, but also with as great study and paine to communicate their learning to other, and make all England (if it might be) partakers of the same; the most part are such, that unneth they can support their own necessary charges, so that they are not able to beare any charges in doing of that good, that else they desire to do.

But a greater cause of lamentation is this, that when learned men have taken pains to do things for the

the aid of the unlearned, scarce they shall be allowed for their wel-doing, but derided and scorned, and so utterly discouraged to take in hand any like enterprise againe. So that if any be found (as there are some) that do favour learning, and learne wits, and can be content to further knowledge, yea only with their word; such persons though they be rare, yet shall they encourage learned men to enterprise some thing at the least that England may rejoyce of. And I have good hope that England will (after she hath take some sure tast of learning) not only bring forth more favourers of it, but also such learned men that she shall be able to compare with any Realme in the world. But in the meane season, where so few regarders of learning are, how greatly they are to be esteemed that do favour and further it, my pen will not suffice at full to declare.

Therefore, gentle Reader, whereas I do upon most just occasion judge, yea and know assuredly, that there be some men in this Realme, which both love and also much desire to further good learning, and yet am not well able to write their condigne praise for the same, I think it better with silence to overpasse it, than either say too little of it, or to provoke against them the malice of such other, which do nothing themselves that is praise-worthy, and therefore cannot abide to heare the praise of any other mans good deed.

And considering their great favour unto learning, though I my selfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my selfe in Presse, with such ability as God hath

hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man, to help my Country-men, and will not ceaſe daily, (as much as my ſmall ability will ſuffer me) to endite ſome ſuch thing, that ſhall be to the inſtruction, though not of learned men, yet at the leaſt of the vulgar ſort, whoſe argument alwayes ſhall be ſuch as it ſhall delight all learned wits; though they do not learne any great thing out of it:

But to ſpeak of this preſent book of Arithmetick, I dare not, nor will not ſet it forth with any words, but remit it to the judgement of al gentle Readers, and namely, ſuch as love good learning, beſeeching them ſo to eſteeme it, as it doth ſeeme worthy. And ſo either to accept the thing for it ſelfe, either at the leaſt to allow my good endevour. But I perceive I need not uſe any perſwaſions unto them, whoſe gentle nature and favourable mind is ready to receive thankefully, and interpret to the beſt all ſuch enterpriſes attempted for ſo good an end, though the thing do not alwayes ſatisfie mens expectation. This conſidered, did bolden me to publiſh abroad this little book of the Art of numbring, which if you ſhall receive favorably, you ſhall encourage me to gratifie you hereafter with ſome greater thing.

And as I judge ſome men of ſo loving a mind to their native Country, that they would much rejoyce to ſee it proſper in good learning, and witty Arts; ſo I hope well of all the reſt of Engliſhmen, that they will not bee unmindfull of his due praiſe, by whoſe meanes they are helped and furthered in any thing. Neither ought they to eſteeme this thing

of

of so little value, as many men of little discretion oftentimes do. For who so setteth small price by the witty device and knowledge of numbring, he little considereth it to be the chief point, (in manner) whereby men differ from all brute beasts: for as in all other things (almost) beasts are partakers with us, so in numbring we differ cleane from them, and in manner peculiarly, sith that in many things they excell us again.

The Foxe in crafty wit exceedeth most men,
A Dogge in smelling hath no man his peere;
To foresight of weather if you looke then,
Many beasts excell men; this is cleere.
The wittinesse of Elephants doth letters attaine,
But what cunning doth there in the Bee remaine?
The Emmet foreseeing the hardnesse of winter,
Provideth victuals in time of summer.
The Nightingale, the Linet, the Thrush, the Larke,
In Musicall harmony passe many a Clarke.
The Hedghogge of Astronomy seemeth to know,
And stoppeth his Cave where the wind will blow.
The Spider in weaving such Art doth show,
No man can him mend, nor follow I trow.
When a house will, fall the Mice right quicke,
Flee thence before; can man do the like?

Many things else of the wittinesse of Beasts and Birds might I here say, save that another time of them I intend to write wherein they exeel in manner al mē, as it is daily seen: but in nūber was there never beast found so cūning, that could know or discerne one thing from many, as by daily experience you may well consider, when a Bitch hath many whelps, or a Hen many chikens: & likewise of other whatsoever they be, take from them all their young

saving

ſaving only one, and you ſhall perceive plainly, that they miſſe none, though they will reſiſt you in taking them away, and will ſeeke them again if they may know where they be, but elſe they will never miſſe them truly; but take away that one that is left, and then will they cry and complaine; and reſtore to them that one, then are they pleaſed again. So that of number this may I juſtly ſay, it is the only thing almoſt that ſeparateth man from beaſts. He therefore that ſhall contemne number, declareth himſelf as brutiſh as a beaſt, and unworthy to be counted in the fellowſhip of men. But I truſt there is no man ſo foule over-ſeene, though many right ſmally do it regard.

Why the Author wrot in Dialogue wiſe.

Therefore will I now ſtay to write againſt ſuch, and returne againe to this my Booke, which I have written in the form of a Dialogue, becauſe I judge that to be the eaſieſt way of inſtruction, when the Schollar may aske every doubt orderly, and the Maſter may anſwer to his queſtion plainly.

Howbeit I think not the contrary, but as it is eaſier to make another mans worke, then to make the like; ſo there will be ſome that will find fault, becauſe I write in a Dialogue: but as I conjecture thoſe ſhall be ſuch as do not, cannot, or will not perceive the reaſon of right teaching, and therefore are unmeet to be anſwered unto, for ſuch men with no reaſon will be ſatisfied.

And if any man object, that other Books have been written of Arithmetick *already ſo ſufficiently, that I needed not now to put Pen to the Booke, except I will condemne other mens writings: to them*

I an-

I answer: That as I condemne no mans diligence, so I know that no one man can satisfie every man: and therefore like as many do esteeme greatly other Books, so I doubt not but some will like this my Book above any other English Arithmetick hitherto written; and namely, such as shall lack instructers, for whose sake I have so plainly set forth the Examples, as no Booke that I have seene hath done hitherto: which thing shall be great ease to the Rude Readers.

Therefore (gentle Reader) though this Book can be but small aid to the learned sort, yet unto the simple ignorant (which needeth most help) it may be a good furtherance and meane unto knowledge.

And though unto the King his Majesty privatly I do it dedicate, yet I doubt not (such is his clemency) but that he can be content, yea, and much desirous, that all his loving Subjects shall take the use of it, and imploy the same to their most profit. Which thing if I perceive that they thankefully do, and receive with as good will as it was written, then will I shortly with no lesse kindnesse set forth such introductions into Geometry and Cosmography, as I have at times promised, and as hitherto in English hath not been interprised, wherewith I dare say all honest hearts will be pleased and all studious wits greatly delighted.

I will say no more, but let every man judge as he shall see cause. And thus for this time I will stay my Pen, committing you all to that true fountaine of perfect number, which wrought the whole world by number and measure: he is Trinity in Unity, and glory. Amen.

Here followeth a Table of the whole Contents of this Book.

The Contents of the first Dialogue.

The

The Contents of the third part.

A Collection of such Tables as are contained in this Treatise.

M W

The Contents.

Before

Before the Introduction of Arithmeticke, it were very good to haue ſome underſtanding and knowledge of theſe Figures and Notes.

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&c.	&c.	*&c.*	&c.	&c.	*&c.*

John Crowder his booke
ano dom 1645

Before the Introduction of Arithmeticke, it were very good to haue some understanding and knowledge of their Figures and Notes.

i	1	one	xx	20	twenty
ii	2	two	xl	40	fourty
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ix	9	nine	D	500	fiue hundred
x	10	ten	DC	600	sixe hundred
xi	11	eleuen	M	1000	a thousand
xii	12	twelue	MD	1500	a thousand fiue hund.
&c.	&c.	&c.	&c.	&c.	&c.

A

A Dialogue between the Master and the Schollar: teaching the *Art and use of* Arithmetick with Pen.

The Schollar speaketh.

SIR, such is your authority in mine estimation, that I am content to consent to your saying, and to receive it as truth, though I see none other reason that doth leade mee thereunto: whereas else in mine owne conceit appeareth but vaine, to bestow any time privately in learning of that thing, that every childe may, and doth learne at all times and houres, when he doth any thing himselfe alone, and much more when he talketh or reasoneth with others.

Master. Lo, this is the fashion and chance of all them that seeke to defend their blinde ignorance, that when they thinke they have made strong reason for themselves, then have they proved quite contrary. For if numbring be so common (as you grant it to bee) that no man can doe any thing alone, and much lesse talke or bargaine with other, but

hee

hee shall still have to doe with number: this proveth not number to bee contemptible and vile, but rather right excellent and of high reputation, sith it is the ground of all mens affaires, in that without it no tale can be told, no communication without it can bee continued, no bargaining without it can duely be ended, or no businesse that man hath, justly completed. These commodities, if there were none other, are sufficient to approve the worthinesse of number. But there are other innumerable, farre passing all these, which declare number to exceed all praise. Wherfore in all great workes are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to bee enhaunsed? Why are Astronomers so greatly advanced? Because that by number such things they find, which else would farre excell mans mind.

Scholar. Verily, sir, if it be so, that these men by numbring, their cunning doe attaine, at whose great works most men doe wonder, then I see well I was much deceived, and numbring is a more cunning thing than I took it to be.

Master. If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude number, and answer to this question: How many yeares old are you?

Scholar.

Scholar. Mum.

Maſter. How many dayes in a weeke? How many weekes in a yeare? What lands hath your Father? How many men doth hee keep? How long is ſince you came from him to me?

Scholar. Mum.

Maſter. So that if number want, you anſwer all by Mummes: How many miles to London?

Scholar. A poake full of plumes.

Maſter. Why, thus you may ſee, what rule number beareth, and that if number be lacking it maketh men dumbe, ſo that to moſt queſtions they muſt anſwer Mum.

Scholar. This is the cauſe, ſir, that I judged it ſo vile, becauſe it is ſo common in talking every while: For plenty is not dainty, as the common ſaying is.

Maſter. No, nor ſtore is no ſore, perceive you this? The more common that the thing is, being needfully required, the better is the thing, and the more deſired. But in numbring, as ſome of it is light and plaine, ſo the moſt part is difficult, and not eaſie to attaine. The eaſier part ſerveth all men in common, and the other requireth ſome learning. Wherefore as without numbring a man can doe almoſt nothing, ſo with the helpe of it you may attaine to all things.

Scholar. Yea, ſir, why then it were beſt to learne

learne the Art of numbring, first of all other learning, and then a man need learn no more, if all other come with it.

Master. Nay, not so: but if it be first learned, then shall a man be able (I meane) to learne, perceive, and attaine to other Sciences; which without it he could never get.

Scholar. I perceive by your former words, that Astronomy and Geometry depend much on the helpe of numbring: but that other Sciences, as Musicke, Physicke, Law, Grammer, and such like, have any helpe of Arithmeticke, I perceive not.

Master. I may perceive your great Clerklinesse by the ordering of your Sciences: but I will let that passe now, because it toucheth not the matter that I intend, and I will shew you how Arithmeticke doth profit in all these somewhat grosly, according to your small understanding, omitting other reasons more substantiall.

First (as you reckon them) Musicke hath not onely great helpe of Arithmeticke, but is made, and hath his perfectnesse of it: for all Musicke standeth by number and proportion: And in Physicke, beside the calculation of criticall dayes, with other things, which I omit, how can any man judge the pulse rightly, that is ignorant of the proportion of numbers?

Musicke. Phisick.

Law. And as for the Law, it is plaine, that the man

man that is ignorant of Arithmeticke, is neither meet to be a Iudge, neither an Advocate, nor yet a proctor. For how can he well understand another mans cause, appertaining to distribution of goods, or other debts, or of summes of money, If he be ignorant of Arithmeticke? This oftentimes causeth right to be hindered, when the Iudge either delighteth not to heare of a matter that hee perceiveth not, or cannot judge for lacke of understanding: this commeth by ignorance of Arithmeticke.

Now, as for Grammer, me thinketh you should not doubt in what it needeth number, sith you have learned that Nouns of all sorts, Pronounes, Verbes, and participles are distinct diversly by numbers: besides the variety of Nounes of Numbers, and Adverbes. And if you take away number from Grammer, then is all the quantity of Syllables lost. And many other wayes doth number helpe Grammar. Whereby were all kindes of Meters found and made? was it not by Number? Grammer.

But how needfull Arithmeticke is to all parts of phylosophy, they may soone see, that do reade either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations, depend of Arithmeticke. It is the saying of Aristotle, that hee that is ignorant of Arithmeticke, is meete for no Science. And Plato his Master Philosophy.

wrote

wrote a little sentence over his Schoolhouse doore, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing hee would have all his Scholars expert in Geometry, much rather he would the same in Arithmetick without which Geometry cannot stand.

Divinity. And how needful Arithmetick is to Divinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the commodities of Arithmeticke in civill acts, as in governance of Common-weales in time of peace, and in due provision and order of Armies, in time of war, for numbring of the host, summing of their wages, provision of victuals, viewing of Artillery, with other Armour; beside the cunningest point of all, for casting of ground, for encamping of men, with such other like: And how many wayes also Arithmeticke is conducible for all private Weales, of Lords and all possessioners, of Merchants & all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receivers, Stewards, Bailiffes, and such like, whose offices without Arithmeticke, are nothing: if I should (I say) particularly repeat all such commodities of the noble Science of Arithmeticke, it were enough to make a very great book.

Armies.

Scholar. No, no sir, you shall not need: For I doubt not, but this, that you have said, were enough

enough to perswade any man to thinke this Art to be right excellent and good, and so necessary for man, that (as I thinke now) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so farre changed since, by hearing these few commodities in generall? by likelyhood you would be far changed if you knew all the particular commodities.

Schollar. I beseech you Sir, reserue those commodities that rest yet behinde unto their place more convenient: and if ye will bee so good as to utter at this time this excellent treasure, so that I may be somewhat inriched thereby, if ever I shall be able, I will requite your paine.

Master. I am very glad of your request, and will do it speedily, sith that to learne it you be so ready.

Schollar. And I to your authority my wit doe subdue, whatsoever you say, I take it for true.

The duty of a Scholar.

Master. That is too much, and meete for no man to be beleeued in all things, without shewing of reason. Though I might of my Schollar some credence require, yet except I shew reason, I doe it not desire. But now sith you are so earnestly set this Art to attaine, best it is to omit no time, lest some other passion coole this great heate, and then

Perseverance in study.

C per[...]

you leaue off before you see the end.

Schollar. Though many there be so vnconstant of minde, that flitter and turn with euery winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indeed, and I trust you will increase rather then goe backe. For better it were neuer to assay, then to shrinke and flie in the mid-way: But I trust you will not do so, therefore tell me briefly: What call you the Science that you desire so greatly?

Schollar. Why sir, you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to haue studied a Science, and yet cannot tell how it is named.

Schollar. Some call it Arsemetricke, and some Augrime.

Master. And what doe these names betoken?

Schollar. That, if it please you, of you would I learne.

Master. Both names are corruptly written: Arsemetricke for Arithmeticke, as the Greekes call it, and Augrime for Algorisme, as the Arabians found it: which doth betoken

ken the Science of Numbring: for Arithmos in Greeke is called Number; and of it commeth Arithmeticke, the Art of Numbring. So that Arithmeticke is a Science or Art teaching the manner and use of Numbring: This Art may be wrought diuersly, with pen or with counters. But I will first shew you the working with the pen, and then the other in order.

Schollar. This I will remember. But how many things are to be learned to attaine this Art fully?

Master. There are reckoned commonly seuen parts or works of it.

Numeration, Addition, Substraction, Multiplication, Diuision, Progression, and Extraction of roots: to these some men adde Duplication, Triplation, and Mediation. But as for these three last they are contained under the other seuen. For Duplication and Triplation are contained under Multiplication, as it shall appeare in their place: And Mediation is contained under Diuision, as I will declare in his place also.

Schollar. Yet then there remaine the first seuen kinds of Numbring.

Master. So there doth: Howbeit if I shall speak exactly of the parts of Numbring, I must make but fiue of them: for Progression is a compound operation of Addition, Multiplication and Diuision. And so is the Ex-

tractions of roots. But it is no harme to name them as kindes severall, seeing they appeare to haue some severall working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Schollar. Then you will that I shall name them as seven kindes distinct. But now I desire you to instruct mee in the use of each of them.

Master. So I will, but it must bee done in order: for you may not learne the last so soone as the first, but you must learne them in that order, as I did rehearse them, if you will learn them speedily, and well.

Schollar. Even as you please. Then to begin; Numeration is the first in order: what shall I do with it?

Master. First, you must know what the thing is, and then after learne the use of the same.

Numeration.

N*Vmeration is that Arithmeticall* skill, whereby wee may duely value, expresse, and reade any *number* or *summe* propounded: or else in apt *figures* and *places* set downe any *number* knowne or named.

Schollar.

Schollar. Why? then me thinketh you put a difference betweene the value and the figures.

Master. Yea so do I. For the value is one thing, and the figures are another thing: and that commeth partly by the diversitie of figures, but chiefly in the places wherein they be set.

Schollar. Then I must know heere three things: the value, the figure, and the place.

Master. Even so: but yet adde Order to them as the fourth. And first marke, that there are but ten figures that are used in Arithmeticke; and of those ten, one doth signifie nothing, which is made like an o, and is A Cypher. privately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying figures, and bee thus Figures. figured.

1. 2. 3. 4. 5. 6. 7. 8. 9.
And this is their value:
i. ii. iii. iiii. v. vi. vii. viii. ix.

But here you must marke, that every Figure hath two values: One alwayes certaine, that it signifieth properly, which it hath of his forme; and the other uncertaine, which he taketh of his place.

A place is called the seate or roome that a Figure standeth in. And looke how many

figures are written in one summe, so many places hath that whole number. And that must be called the first place that is next to the right hand, and so reckoning by order towards the left hand, so that that place is last that is next to the left hand. As for example: If there stood before you six men in a row, side by side, and you should tell them as they stand in order, beginning with the man that were next to your right hand:, then hee that were next him should be called the second, and so forth to the furthest from your right hand, which is the first and the last.

Schollar. Sir, I perceiue you well: so might I reckon Letters or any other thing. As if I should write eight Letters after this order, a, b, c, d, e, f, g, h, then must I say, h, is the first, g, the second, f, the third, e, the fourth, d, the fifth, c, the sixth, b, the seuenth, a, the eight.

Master. That is well done. And after the same sort use hereafter, that what I declare by one example, do you expresse by another: and so shall I perceiue whether you understand it or no. And so passe ouer nothing, till you perceiue it well and be expert therein.

Schollar. Sir, I pray you how many of these places be there in all?

Master. There is no certaine number of them, but they are sometimes more and sometimes fewer, according to the summe that is expressed,

expressed. For so many as the figures are, so many are the places: and the last place is so called, not because it is last of all other, but it is the last of that present summe, and it may be the midale place in another summe.

Schollar. Me seemeth I perceive this very well, as touching the order of reckoning of the places; but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diversity of places, which you called the value vncertaine.

Value vncertaine.

Master. But first let me heare whether you know perfectly the certaine value.

Value certaine.

Schollar. Yes sir, as you wrote them, so I marked them.

Master. How write you then fiue?

Schollar. By this figure 5.

Master. And how six?

Schollar. Thus 6.

Master. Write these three numbers, each by it selfe, as I speak them, vii. iiii. iii.

Schollar. 7. 4. 3.

Master. How write you these foure other, ii. i. ix. viii.

Schollar. Thus (I trow) 2. 1. 6. 8.

Master. Nay, there you misse; look on mine example againe.

Schollar. Sir, true it is, I was too blame, I take 6 for 9, but I will beware hereafter.

Master. Now then take heed, those ce-

taine values every Figure representeth when it is alone written without other Figures joyned to him. And also when it is in the first place, though many other doe follow: as for example, This figure 9, is ix, standing now alone.

Schollar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your selfe to be alone.

Schollar. So it is. Then 9. without moe figures of Arithmeticke betokeneth ix. whatsoever other letters be about it.

Master. Even so, and so doth it, if it be in the first place joyned with other, how many soever doe follow, as in this example, 3679. You see 9. in the first place, and doth betoken nine, as it were alone.

Schollar. I perceive that, and doth not 7. that standeth in the second place, betoken vii. and 6. in the third place, betoken vi. and so 3. in the fourth place betoken three?

Master. Their figures be as you have said, but their values are not so. For as in the first place every figure betokeneth his owne value certaine onely, so in the second place every figure betokeneth his owne value certaine, tenne times: as in the example 7. in the second

place

place is seven times ten, and is lxx. And in the third place every figure betokeneth his owne value an hundred times, so the 6. in that place betokeneth vi.C. and in the fourth place every figure betokeneth his owne value a M. times, as in the aforesaid number 3, in the fourth place standeth for 3. M. and in the fifth place every figure standeth for his owne value x M times, and in the sixth place a C M times, and in the seventh place a M M times, and in the eight place, x M M, so that every place exceedeth the former ten times.

Schollar. As thus: if I make this number at all adventures, 91359684, here are eight places. In the first place is 4. and betokeneth but foure: in the second place is 8. and betokeneth ten times 8. that is 80. in the third place is 6, and betokeneth six hundred: in the fourth place 9, is nine thousand, and 5 in the fifth place is x M times 5. that is fifty M. So 3. in the sixth place is a C M times 3. that is C C C M. Then 1. in the seventh place, one M M. and 9. in the eight place, ten thousand thousand times 9 that is, xc M M. But now I cannot easily nor quickly read it in order.

A generall rule.

Master. That shall you practice by this meanes First, put a pricke over the fourth figure, and so over the seventh. And (if you have so many) over the tenth, thirteenth, sixteenth, and so forth, still leaving two figures betweene each two prickes. And those two

roomes

Ternaries. roomes betweene the prickes are called Ternaries.

Then begin at the last pricke, and see how many figures are betweene him and the end, which cannot passe three, reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands, as your numbers haue pricks.

After that, come to the next three figures and sound them as if they were apart from the rest, and adde to their value so many times thousands, as there are prickes betweene them and the first place of your whole number. And so doe by euery other three Figures following, if you haue moe. As in example, 9135968 4, this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the seuenth place, and then no more (for your places come not to ten) as thus: 9 1̇ 3 5 9̇ 6 8 4.

Now goe to the last pricke ouer 1, and take it and the figure 9 that followeth it, and value them alone.

Schollar. 91, that is xci.

Master. So it is, then adde for the number of your pricks twice M.

Schol. That is, xci thousand thousand.

Master. So it is. Then take the three other figures from one to the next pricke, and value them.

Schol.

Schollar. 359. that is, CCC.lix.

Master. Now adde for the one pricke, that is betweene them and the first place, M.

Schol. CCC.lix. thousand.

Master. Then come to the other three figures that remaine.

Schol. 684. that is, vi C lxxxiiii.

Master. Now have you valued all. And at the end of the last number you shall adde nothing, because there remaineth no pricke nor number after it; yet proue it in another number, as thus, 230864089105340.

Schol. 230864089105340, I haue pricked them as you taught me; but I am in doubt whether I haue done well or no, because of the Ciphers; for I remember you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a figure in setting of the pricks, and againe, I know not wherefore they serue.

Master. That will I tell you now. Indeed they are of no value themselues, but they serue to make vp the number of places and so make the figure following them to be in a further place, and therefore to signifie the more value. as in this example 90. the Cypher is of no value, but yet hee occupieth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is xc. so that two ciphers thrust the figure following them into the third place, and so forth.

The Cyp

Schollar.

Schollar. Then I perceiue in the example aboue, I haue pricked well enough: for though that Cypher that is pricked signifie nothing, yet must he haue the pricke, because he came in the thirteenth place. Then will I proue to number that summe. First, there is 230, M, M, M, M, and then followeth 864, M, M, M. And what shall I now do? There is a Cypher in the third place, and no figure after him, but they that I haue reckoned.

Master. Hee did serue for them that you haue already reckoned, to make them in a place further then they should be, if he were away: and therefore now ye shall let him go. And so doe alwayes when hee occupieth that place next before any pricke, which is the last of that Ternary, and a Cipher in the last place doth nothing.

Schollar. Then shall I say but 89. M. M.

Master. So, but go forth.

Schollar. 105. thousand. Now are all my prickes spent, and yet remaine 340. so that I must value them, CCC.xl. onely.

Master. Now can you reckon after this sort: and remember that euery such roome so parted, is called a Ternary or Trinity; for you haue numbred or valued the summe most truly, and by the aid of the pricks each denomination is distinct most plainly,

Schollar. What call you Denomination?

Master.

Master. It is the last value or name added to any summe. As when I say, an hundred two and twenty pounds. Pounds is the Denomination. And likewise in saying 25 men: Men is the Denomination, and so of other: But in this place (that I spake of before) the last number of every Ternary is the Denomination of it. As for the first Ternary, the Denomination is vnites, and of the second Ternary, the Denomination is thousands: And of the third Ternary, thousand thousands: or Millions: of the fourth, thousand thousand thousands, or thousand Millions: and so forth.

Schollar. And what shall I call the value of the three Figures that may be pronounced before the Denomination? as in saying: 203000000, that is, Two hundred three Millions. I perceiue by your words, that Millions is the Denomination; but what shall I call CCiii. ioyned before the Millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value, or number. Numerator. Summe value.

Schollar. Now is there any thing else to be learned in Numeration? or else haue I learned it fully?

Master. I might shew you here who were the first Inventors of this art, and the reason of all

all these things that I haue taught you, but that I will reserue till yee haue learned ouer all the practise of this Art, lest I should trouble you with ouer many things at the first.

Three kinds of numbers.

But yet this you must marke, that there are three kindes of numbers, one called Digits, another Articles, and the third Mixt numbers.

Articles.

Digits.

A Digit is any number vnder ten, as these: 1.2.3.4.5.6.7.8.9.

And 10 with all other that may be diuided into ten parts iust, and nothing remaine, are called Articles, such as are 10.20.30.40.50, &c. 100. 200, &c. 1000, &c.

Mixt.

And that number is called Mixt, that containeth Articles or at the least one Article, and a Digit, as 12. 16. 19. 21. 38. 107. 1005. and so forth: and for the more ease of vnderstanding and remembrance, marke this The Digit number is neuer written with more than one figure, but the Article and the Mixt number are euer written with more then one figure. And thus they differ that the Article hath euermore this Cipher 0 in the first place; and the Mixt number hath euer there some Digit.

Scholar. By these last words I perceiue it much better then I did before, and now (I think) I will neuer misse to know those three asunder.

Master. If you remember now all that I haue

haue said, you haue learned sufficiently this first kinde of Arithmeticke called Numeration. Howbeit I will exhort you now to remember both this that I haue said, and all that I shall say, and to exercise your selfe in the practise of it, for rules without practise, are but a light knowledge, and practise it is, that maketh men perfect and prompt in all things.

Vse maketh mastery.

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you, so must you practise to marke, note, or write downe with apt Figures and in due places, any number onely named or recited to you, or if your selfe imagined; as for a proofe. How note you or write downe this summe, fiue thousand two hundred fifty and seauen?

Schollar. This troubleth mee now, whether I should begin at the first or at the last. For reason (me thinketh) should cause me to begin at the first; and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list; but the more ease for your hand is to begin with the last, that is to say, as I did speake them, yet for the more surety, a while you may begin with the first, repeating my words backward thus: Seuen, fifty, two hundred, fiue thousand: or else sounding them all by their digit or

value

valuer, as thus: ſeven. five, two, five: for that way is eaſieſt: But then muſt you look well whether there be any cipher in your ſum, that he may be ſet in his place: As if the laſt valuer of your ſumme (as you ſpeake it) bee aboue 9, then is there a Cipher in the firſt place. And if it be an hundred, or aboue, then is there two Cyphers, one in the firſt place, and another in the ſecond, and ſo forth.

But becauſe this thing is ſuch that cannot be ſet forth without many words, I think beſt here now at the end of Numeration to adde a Table eaſie and ready for the firſt exerciſe of it.

Loe this is the Table.

The right side or hand.

The names of Digits, values certaine, or values.

The denominators of the place or value uncertaine.	Nine.	Eight.	Seven.	Six.	Five.	Foure.	Three	Two.	One.	Ciph.	The order of places.
Vnites.	9	8	7	6	5	4	3	2	1	0	First.
Tennes.	9	8	7	6	5	4	3	2	1	0	Second.
Hundreds.	9	8	7	6	5	4	3	2	1	0	Third.
Thousands	9	8	7	6	5	4	3	2	1	0	Fourth.
X. Thousands.	9	8	7	6	5	4	3	2	1	0	Fift.
C. of Thousands.	9	8	7	6	5	4	3	2	1	0	Sixt.
Millions.	9	8	7	6	5	4	3	2	1	0	Seventh.
X. of Millions.	9	8	7	6	5	4	3	2	1	0	Eight.
C. of Millions.	9	8	7	6	5	4	3	2	1	0	Ninth.
M. of Millions.	9	8	7	6	5	4	3	2	1	0	Tenth.
X. M. of Millions	9	8	7	6	5	4	3	2	1	0	Eleventh.

The left hand or side.

This Table (as ye may see) hath eleven places, and in each of them are set all the Digits, whose certaine value is written on the right hand of the Table, and the value uncertaine on the left hand, so that by this Table you may learne both how to expresse any number that you list, if that it exceed not

eleven places) that is to say, X C. thousand millions, and so may you by helpe of it, value all sums proposed under the said number.

For example : take the summe that I proposed before, which was five thousand, two hundred fifty and seven. And if you will expresse it, take the first number (as I speak it) which is five M. whose valuer or certaine value is v, and his uncertaine value or denomination is M. First, you shall seek at the right hand of the valuer 5. Then seeke along under the title of denomination toward the left hand till you finde thousands, and under it, right at the foot of the Table is the number of the place that is in the fourth, wherein you must write your digit or valuer 5.

Afterward come to the second part of the number two hundred, whose valuer is 2, and his denomination C. Seeke two at the right hand of the Table and go along under the denomination toward the left hand, till you come under C, then looke to the foot of the Table, and there you shal see the number of the place that is to say, the third, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall finde 5 in the second place for your fifty, and 7 in the first place for your seven. And thus you may doe with other numbers.

Schollar. Master, I thank you heartily. I perceive you seek to instruct me most plainly

and

and briefly, and not to hide your knowledge with subtle words, as many do. For this rule is so plaine, that I can desire it no plainer. And though it seeme somewhat long, yet I perceive it to be a sure way.

Master. So it is, and though it be long, yet it is neither too long, neither too plaine for yong learners that lacke practise: for this Table is in stead of a teacher to them that lacke one. But now I trust I have said enough of Numeration: which after you have well practised, then may you learne forth.

Scholar. Yet I pray you in one thing to tell me your judgement. Why do men reckon the order of the places backward, from the right hand to the left?

Master. In that thing all men do agree, that the Caldees, which first invented this Art, did set these figures as they set all their Letters, for they write backward, as you terme it, and so do they read. And that may appeare in all Hebrew, Chaldee and Arabicke books; for they be not only written from the right hand to the left, and so must be read, but also the right end of the book is the beginning of it, whereas the Greeks, Latins, and all Nations of Europe, doe write and reade from the left hand toward the right: and all their books begin at the left side. Why numbers are written backward.

Schollar. That reason hath satisfied me.

Master. It neither satisfieth mee, neither liketh me wel, because I see that the Chaldees

and Hebrewes do not so use their owne numbers, as at another time I will declare. But this plaine reason may best satisfie you presently, that seeing in pronouncing of numbers we keepe the order of our owne reading, from the left hand to the right: and againe, we doe ever name the greater numbers before the smaller: it was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places containing the greater numbers, to proceed toward the left hand.

Scholar. This reason is to me so plaine, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied; hoping that practice shal make me fully ready and expert in it. And in the meane season I desire to learne the other kindes of Arithmetick:

Master. That is well said: but what should you next learne? can you tell?

Scholiar. I remember you said that Addition was next.

Master. Even so, and what that is, must you first know.

Addition.

Addition.

A*Ddition is the gathering together and bringing of two numbers or more into one sum.* As if I have 160 Books in the Latine tongue, and 136 in the Greeke tongue, and would know how many they bee in all, I must write these two numbers one over another, writing the greatest number highest, so that the first figure of the one being under the first figure of the other, and the second under the second, and so forth in order.

When you have so done, draw under them a right line, then will they stand thus:

Now begin at the first places toward the right hand alwayes, and put together the two first figures of these two	160 136

numbers, and looke what commeth of them write under them, right under the line. As in saying 6 and 0 is 6, write 6 under 6, as thus:

	160 136 6

And then go to the second figures, and doe likewise: as saying 3 and 6 is 9, write 9 under 6 and 3, as here you see.

	160 136 96

And likewise do you with the figures that be in the third place, saying, 1 and 1 be 2, write 2 under them, and then will your whole summe appeare thus:

	160 136 296

So that now you see that 160, and 136, doe make in all 296.

Scholar. What? this is very easie to do, me thinketh I can do it even since.

There came through Cheapeside two droves of cattell: in the first was 848 sheepe, and in the second was 186 other beasts.

Those two summes I must write as you taught me, thus: Then if I put the two first figures together, saying, 6 & 8, they make 14. That must I write under 6 and 8, thus: 1034

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 848
 186
----
  14
```

☞ Master. Not so: and here you are twice deceived. First, in going about to adde together two summes of sundry things, which you ought not to doe, except you seeke onely the number of them, and care not for the things. For the summe that should result of that addition, should be a summe neither of sheepe, nor of other beasts, but a confused summe of both. Howbeit sometimes yee shall have summes of divers denominations to be added, of which I will tell you anon: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 and 8, under 6, 8, which is unpossible; for how can two Figures of two places be written under one Figure and one place.

Scholar. Truth it is; but yet I did so understand you.

Master. I said indeed, that you should write that under them that did result of them both together: which saying is alwayes true, if that summe dos not exceed a digit. But if it be a mixt number, then must you write the digit of it under your Figures as you have said before: and if it be an article, then write o under them, and in both sorts you shall keep the article in your minde; and therefore when you have added your second Figures, which occupy the place of tens, you shall put that one thereto, which you kept in your minde; for though it were ten indeed, yet in that place it is but as one, because that every one of that place is tenne, for that it is the place of tens. And in like manner, if you have in the second place so great a number that it amounteth above 9, then write the digit, and reserve the article in your minde, ever adding it to the next place following, and so of all other places, how many soever you have. And if you have a mixt number when you have added your last Figures, then write the digit under the last Figures, and the article in the next place beyond them: so shall your number resulting of Addition, have one place more than the numbers which you shall adde together. A place.

Scholar. Now do I perceive you, and the reason of this, is, (as I understand) because that no one place can containe above 9, which is the greatest figure that is, and then al tens or

or Articles must be put to the next place following: for every place (as I may see) exceedeth the other place next before him by 10.

Now (if it please you) I will returne to my example of [illegible]. But I remember you said I might not adde summes of sundry things together, and that I may see by reason.

Master. Truth it is, if you seeke the due summe of any thing, but if you onely seeke a bare summe, and have no respect to the thing, then were it better to name the summe onely without any thing: as in saying 848, without naming sheepe or any thing else. And likewise 186, naming nothing.

Now let me see how you can adde those two summes.

Schollar. I must first set them so that the two first figures stand one over another, and the other each one over his fellow of the same place: then shall I draw a line under them both. And so likewise of other figures, setting alwayes the greatest number highest, thus as followeth.

A place.

Then must I adde 6 to 8, which make 14, that is a mixt number, therefore must I take the Digit which is 4, and write it under 6 and 8, keeping the Article 1 in my minde, thus:

848
186
4

Next that, I doe come to the second figures, adding them together, saying 8 and 4 make 12, to the which I put the one reserved in my minde, and that maketh 13. of which

which number I write the digit 3 under 8 and 4, and keepe the Article in my minde, thus:

848
186
34

Then come I to the third figures, saying, 1 and 8 make 9, and 1 in my minde maketh 10. Sir, shall I write the Cypher under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keepe the Article in my minde.

Master. What needeth that, seeing there follow no more figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last figures, I must write the Cypher under them, and the Article in a further place after the Cypher, thus:

848
186
1034

Master. So now you see, that of 848, and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I prove by this example. There are two armies of Souldiers: in the one are 106800, and in the other 9400. How many are there in both armies say you?

Scholar. First, I set them one over another, beginning with the first numbers on the right hand, thus:

106800
9402

But the nether number will not match the over number.

Master.

Master. That forceth not.

Scholar. Then do I adde 0 to 0, and there amounteth 0, that must I write under the first place thus;

106800
9400
0

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, and there ariseth 0, which I write under the second place thus;

106800
9400
00

Then I come to the third place, saying, 4 and 8 make 12, of which I write the digit 2, and keepe the article 1 in my minde, thus;

106800
9400
200

Then I adde 9 to 6, which make 15, to that I adde the article 1 that was in my minde and it is 16, I write 6 under 6 and 9, and keepe one in minde, thus;

106800
9400
6200

Master. Why doe you not write both Figures, seeing you are come to the last couple of numbers?

Scholar. Nay, reason sheweth me, that I must adde that article that is in my minde unto the next figure of the over summe, though there be no more in the nether summe.

Master. That is well considered: then doe so.

Scholar. Then say I, 0 in the over summe and 1 in my minde maketh 1: that write I under

under 0. Then followeth there yet one more in the over summe, which hath none to be added to it, for there is none in the nether sum, nor yet in my minde: therefore I think I must write that even as it is.

Master. Yea.

Scholar. Then doth my whole summe appeare thus:

106800
9400
116200

Master. If you mark this you have learned perfectly the common Addition of all summes which are of one denomination: so that ye observe this also, that in Addition you must have two numbers at the least: or else how can you say that you do adde? And ever let the greatest number be written highest, for that is the best way, though it be not necessary.

And forget not this, that (if you have many numbers to adde together) you shall have oftentimes an article of a greater value than 10, sometimes 20, sometimes 30, sometimes more, yea (peradventure) 100. Therefore as you did with the article 10, so do with them, reserving them in your minde, and adding to the number next following so many, as their valuer or value certaine is: that is to say 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and so forth of other like. So that if the article be 100, then must you set downe the 0 and keep 10 in minde, to be carried to the next row of figures or place, if any such

happen

happen to come. For your better understanding take this example for all.

I would adde these thirteene summes into one, which I set after this manner: then doe I beginne and gather the summe of the first row of Figures, which come to 107, (for I take 9 there tenne times, and that is 90) then 9 and 8 is 17, that is in all 107, of which summe I write the 7 under the first row of Figures, and then for that 100 is ten tennes, I keepe tenne in minde, which ten I must adde unto the next row of Figures, which are in the second place:

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389

which second row of figures (when they are added together with that ten that I had in my minde) make in all 125, of which summe write the digit 5 under the second row, and then (for that 120 containeth twelve tennes) I keepe twelve in minde to be added to the third place or row of figures: which being added together, make in all 60, the cypher 0 I set downe under the row of Figures in the third place.

And the figure 6 I keepe in minde to be added to the row of figures in the fourth place, which (when they are added together) make 29. The figure or digit 9 I set downe under the fourth place. And because it is my last worke, I set downe the 2 also that I have in

my

my minde to the 9 in the fift place; so those summes doe make in all 29057.

¶ But (for your more ease in worke) when you have an addition of so many summes to be added together, you were best part that summe into two or three parts, and worke them severall, and so put their additions together, and this were the best thing you could doe when over many summes fall to be added.

4885
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389
29057

Scholar. This seemeth somewhat hard by the reason of so many numbers together.

Howbeit, I thinke (if I doe often prove, even with the same example, either by working of it alone, or else by parting it as you said even now) that I shall be able to doe so shortly with any other summe.

Master. So shall you. For it is often practice that maketh a man quick and ripe in all things; but because, as well in great sums as in small, there may chance to be some errour, I will teach you how you shall prove whether you have done well or no.

Scholar. That were a great help and ease. The proofe of Addition.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not having regard to their places, but as though they were all unites: and

and still (as your number encreaseth above 9) cast away 9. Then goe forth, ever casting away 9 as often as it amounteth thereto: and so do till you have gone over all the numbers that you intended first to adde; and whatsoever remaineth after such addition and casting away of 9, write it in some void place by the end of a line, for the better remembrance: and thus is the first part of your worke prooved. Then secondly, put together the figures that result of the addition under the line, still casting away 9 also. And then that that remaineth write at the other end of the line; and if those two figures be like, then have you well done; but if they be unlike, then have you missed. As for example, in this present summe. The first figure of the over line is 9, let him go, then 8 and 8 is 16, take away 9, there resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there resteth 2. Then come to the next row, whose first and second numbers are 9, therefore overpasse them both, and take the 5 to the 2, which did remaine in the first row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that, goe to the third line, whose two first numbers you may let passe because they are nines: then take the two figures of 2, which (with the other two that remained, in the second row) make 6. Then goe to the fourth row, whose two first numbers

numbers let go, and take the 6 to the 6 that remained, and that maketh 12: take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so goe through all the other numbers, and you shall finde that there remaineth 5, after you have cast away 9, as often as you can finde it: therefore write 5 at the end of the line in a void place, thus:

5 ————————

Then gather all the figures of the totall summe, which is under the lowest line, and cast away 9 as aften as you can finde it: as thus: 7 and 5 make 12, take away 9, there resteth 3, to that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5. which 5 you must write at the other end of the line that you made in the void place, thus:

5 ———————— 5

And then you see that these two figures be like, whereby you may know that you have done well, and so you may prove in any other.

Schollar. (If it please you) I will prove in another summe.

Master. With a good will.

Schollar. Then will I take one of your former examples, which was this.

First in the highest line 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that followeth, and that maketh 6. Then come I to the second line, where I finde first 4. which

106800
9400
116200

which with 6 maketh 10, from that I take 9, and there resteth one, the next figure is 9, and therefore I let him alone, so finde I 1 remaining, which I set at the end of a line, thus:

1 ———— ——

Then I come to the totall sum, and there I finde that all the figures put together make ten, from which I take nine, and there resteth 1 also, which I put at the other end of the line, thus: 1———— 1

And because they be like, I know that I have well added.

Master. So you know now both how to adde two summes or more together, and also how to prove whether you have done well or no; and now I will teach you how to adde summes of divers denominations together: which thing can never be but when the one Denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort (as you did them that were of one denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of divers denominations.

If you have a summe of divers denominations, then looke that you set every denomination by himselfe, with some note or figure of his denomination, as they are wont to be written. Then write your other summes so under that first, that every one be set under the other of the same denominations: As for example, if your denominations bee pounds,

pounds, shillings, and pence, write pounds under pounds, shillings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, me thinketh it needeth not to warne me of it; for it were against reason so to confound summs: but yet if you had not spoken of it, peradventure I should have beene deceived in it.

Master. If you do say it is plaine, I will speake no more of it, but with an example make the matter to appeare evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is altogether: Therefore must I first set downe my great sum, and then the other, every one under his denomination agreeing to the greatest sum, as here you see with a line under them.

li.	s	d
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwaies be set next to the right) and adde them together: and (if the sum will make 1 or 2 or 3 of the next denomination) then must I keepe it in my minde till I come to that place, and under that first place must I note the residue (if there remaine any of the same denomination:) but, if there remaine none, then need I to write under it nothing. And this is all that you must marke in this Addition: for all other things are like to the

manner of Addition before mentioned: *Therefore the chiefest point of this Addition is, to know the values of Common Coines and rated summes.* As how many ſhillings be in a pound, how many pence in a ſhilling, of which (and of other like things) I will inſtruct you hereafter in teaching of Reduction. But now I may not diſturbe your wit from the thing that we are about.

Therefore let us returne to that former example which I propoſed of the Debtors: which ſummes when I had ſet orderly, they ſtood thus with a line under them.

Then to adde them into one ſumme I muſt begin at the right hand where the ſmalleſt denomination is, and adde them together, firſt ſaying, 6 and 8 make 14. Now, ſeeing theſe 14 are pence, which containe one ſhilling and 2 pence: the 2 pence I ſet downe under the line of pence: and the one ſhilling I keepe in my minde, to carry to the next row being the place of ſhillings.

li.	ſ.	d.
22	6	8
5	16	6
4	3	0
		2

Then doe I adde the ſhillings together, ſaying, 1 in my minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the ſecond place which ſtandeth for 10, make 26, which is 1 pound 6 ſ. The 6 ſ. I ſet down under the place of ſhillings, as appeareth in the example. And the 1 pound I keepe to carry to the pounds.

li.	ſ.	d.
22	6	8
5	16	6
4	3	0
	6	2

Then

Then come I to the pounds, adding them all together, saying, 1 that I keepe and 4 make 5, and 5 make 10, and 2 make 12. The figure or digit 2 I set downe right under that place or row of pounds where I gather them, and the article 1 keep to carry to the next place, saying, 1 in minde and 2 is 3, which 3 I set down directly under the 2. And then appeareth my whole summe thus.

li.	s.	d.
22	5	8
5	16	6
4	3	0
32	8	2

And thus must you doe with any such like summes whatsoever, whether they be money, weight, or measure, which (if you practise divers summes) you shall be well acquainted with the feat of Addition.

But now, can you tell how to prove this Addition, or such other like of divers denominations, and to try whether you have well done or no?

Scholar. I would I could.

Master. That shall you doe by this means: You must make a Crosse which shall have as many lines as you have sundry Denominations in your Addition: As if you have but two Denominations then you may make it thus: that the over part and nether part may serve for one Denomination. And if you have three Denominations (as pounds, shillings, and pence) then must you make Proofe of Addition of divers denominations.

three lines, thus: The upright line may serve for pounds, and the highest thwart line for shillings & the lowest for pence, as for example the sum which we last wrought.

li.	s	d
22	6	8
5	16	6
4	3	0
22	6	2

5
6 — 6
2 — 2
5

For the proofe of the which, because it containeth three denominations, I must make a crosse of three lines as in the example before. Then I reckon first at the right hand the pence, 6 and 8 make 14, from which I take 12 for the next Denomination, that is to say, a shilling, and there resteth 2, which I must write at one end of the nether thwart line.

After that I gather the sum of the shillings 3, 16, 6, which maketh 25, to whom I put 1 that I tooke of the pence, and that maketh 26, from those I take 20, the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds, 4, 5, and 2, which make 11, to them I adde the one that came of the shillings, and they make 12, from whence I cast 9, and there resteth 3, that I joyne to the 2 in the next place, and they make 5, which 5 I set at the Crosse

also. And thus is my first part of my worke proved.

That done, I come to the totall summe under the line, and examine it, beginning at the pence, where I finde but 2, and cannot take 9 from him: therefore I set him at the other end of the nether thwart line: then I come to the shillings, where I finde onely 6 which (because it is lesse than nine) I set it at the other end of the line of shillings, that is, the overmost thwart line.

Last of all, of the 32 li, I take three times 9, which is 27, and there remaineth 5, which I write under the upright line: either else I may reckon them simply without any respect of their valuation or place: saying, 2 and 3 make 5, which (because it is lesse than nine) I set under the upright line as before. Then I consider every number, comparing it to the number that is against it: and because I find them to be every one like his match, I know that I have well done.

Scholar. This crosse I perceive doth serve for these 3 denominations, pounds, shillings, pence: but what if I had l. s. d. ob. and q.

Master. These lines (as I have said) doe serve for three denominations, such as they be, as here three doe serve for pounds, shillings, and pence: but if you have no pounds in your sum, then may they serve for shillings, pence, and halfe penies: yea, for d. ob. and q. or in weight for C, q, and l, or in measure for Elles,

 Quarters,

Quarters, and Nailes, if you haue no greater Denomination: so that you remember that the upright line serueth for the greatest denomination, and the highest thwart line for the next, and the lowest for the least.

And so if you haue foure Denominations, you must make your crosse with so many lines. And (if your summe be of more denominations) make so many lines in your crosse. And thus will I make an end of Addition, sauing that here (for the better understanding of this Rule) I haue set you downe certaine examples both of money, weight, and measure, with their workes and proofes.

Examples of Addition.

li.	s	d
23	10	4
45	6	8
37	2	9
25	13	6
131	13	3

li.	s	d
130	17	10
28	6	8
13	13	4
120	0	0
292	17	10

The proofes

	5	
4		4
3		3
	6	

	4	
8		8
1		1
	4	

C.	q.	li
34	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards.	q.		nayles.
17	3		3
35	2		1
26	1		3
54	2		0
134	1		3

3
2 — | — 2
4 — | — 4
3

8
1 — | — 1
8 — | — 3
8

Subtraction.

Scholar.

Then have I learned the two first kindes of Arithmeticke: now (as I remember) doth follow Subtraction, whose name (me thinketh) doth sound contrary to Addition.

Subtraction.

Master. So it is indeed: for, as Addition increaseth one grosse summe, by bringing many into one: so contrariwise, Subtraction diminisheth a grosse sum by withdrawing of other from it. *So that Subtraction or Rebating is nothing else but an Art to withdraw and abate one summe from another, that the remainer may appeare.*

Scholar. What do you call the remainer?

Master. That you may perceive by the name.

Scholar. So me thinketh: but yet it is good to aske the truth of all such things, lest in trusting to mine owne conjecture, I be deceived.

Master. So it is the surest way. And (as I see cause) I will still declare things unto you so plainly, that you shall not need to doubt. Howbeit, if I doe overpasse it sometimes, (as the manner of men is to forget the small knowledge of them to whom they speake) then doe you put me in remembrance your selfe,

selfe, and that way is surest.

And, as for this word that you last asked me, take you this description: The Remainer is a summe left after one Subtraction made, which declareth the excesse or difference of the two other numbers, as if I would abate or subtract 14 out of 18, there should remaine 4, which is called the Remainer, and is the difference betweene those two numbers 14 and 18. Remainer.

Scholar. I perceive then what Subtraction is: Now resteth to know the order how to worke it.

Master. That shall you doe by this meanes. First, you must consider, that if you should go about to rebate, you must have two sundry summes proposed: the first, which is your grosse sum, (or sum totall) and it must be set highest: and then the rebatement (or sum to be withdrawne) which must be set under the first, (whether it be in one parcell or in many) and that in such sort, that the first figures be one just over another, and so the second, and third, and all other following, as you did in Addition: then shall you draw under them a line, and so are your sums duely set to begin your working.

Then begin you at the right hand (as you did in Addition) and withdraw the nether number out of the higher, and if there remaine any thing, write that right under them beneath the line: and if there remaine nothing

(by

(by reason that the two figures were equall) then write under them a Cypher of nought. And so doe you with all the other Figures, evermore abating the lower out of the higher, and write under them the Remainder still, till you come to the end. And so will there appeare under the line what remaineth of your grosse summe, after you have deducted the other sum from it, as in this example.

I received of your Father 48 s. of which I have laid out for you 36 s. now would I know what doth remaine. And therefore I set my number thus in order. First, I write the greatest sum, and under him the lesser, so that the Figures at the right side be even one under another, and so the other, thus.

$$\begin{array}{r} s \\ 48 \\ \underline{36} \end{array}$$

Then doe I rebate 6 out of 8, and there resteth 2, which I write under them right beneath the line, thus.

$$\begin{array}{r} 48 \\ \underline{36} \\ 2 \end{array}$$

Then I go to the second figures, and do rebate 3 out of 4, where there remaineth 1, which I write under them right, and then the whole summe and operation appeareth thus.

$$\begin{array}{r} 48 \\ \underline{36} \\ 12 \end{array}$$

Whereby it appeareth, that if I withdraw 36 out of 48, there remaineth 12.

Scholar. Now will I prove in a greater summe, and I will subtract 2367924 out of 3468946, those sums I set in order thus:

3468946
2367924

Then doe I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write under them. Then go I to the second figures, and withdraw 2 out of 4, & there remaineth 2, which I set under them also, then I take 9 out of 9, and there resteth 0, which I write under them (for you say that if the Figures be equall, so that nothing doth remaine, I must write this Cypher 0 under them.)

Master. It was well remembred: now go forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1, which I write under them also. Then in the fifth place I take 6 out of 6, and there resteth 0, (for it I write under them the Cypher 0.) Then in the first place 3 rebated from 4, there remaineth 1, which I write under them, and likewise in the seventh and the last place 2 taken from 3, there is left 1, which I write under them: so have I done my whole working, and my summes do appeare thus. Whereby I see, that (if I doe rebate 2367924 out of 3468946) there remaineth 1101022.

3468946
2367924
1101022

Master. This is well done. And that you may be sure to perceive fully the Art of Substraction,

traction, let me see how you can subtract 52984732 out of 8250003456.

Scholar. First, I set downe the greatest sum, and after that, I write under it the lesser number, beginning at the right side, and then my figures will stand thus.

	8250003456
	52984732
	8197018724

Note. Then take I 2 from 6, and the rest is 4, which I write under them. Then do I withdraw 3 from 5, and there remaines 2, which I write under them. Then take I 7 out of 4, but that I canot, what shall I now do?

Master. Marke well what I shall tell you now, how you shall doe in this case, and in all other the like: if any figure of the nether sum bee greater than the figure of the sum that is over him (so that it cannot bee taken out of the figure over him) then must you put 10 to the over figure, and then consider how much it is, and out of that whole sum withdraw the nether figure, and write the rest under them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example where I should have taken 7 out of 4, and could not, I put 10 to that 4, which maketh 14, from it I take away 7, and there resteth 7 also, which I write under them.

Master. So have you done well: but now must you marke another thing also: that

(when-

(whensoever you doe so put ten to any figure of the over number) you must adde one still to the figure or place that followeth next in the nether line: as in this example there followeth 4, to which you must put 1, and make him 5, and then go on as I have taught you.

825000 3456
5298 4732
01873 2

Scholar. Then shall I say 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot be: therefore I must put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written under them: and because of that 10 added to the 3, I must adde one to 8 that followeth in the nether line, and that maketh 9, which I should take out of 0 and cannot: therfore I put thereto 10, & that maketh 10, from 10 I take 9, and there remaines 1, which I write under them.

Thus doe I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure above, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written under them.

Then come I to the next figure, which is 2, and to him I doe adde 1, which maketh 3: that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doe take 3, so there remaineth 7 to bee written under them: likewise doe I put 1 to 5, that

that 5, and make it 15, from which I rebate 6 and there remaineth 9, which I write under them. Now have I spent all the nether Figures, and what shall I do more?

8250003456
52984732
8197018724

Master. You should have added one to the next figure following (if there had beene any) because you added 10 to the last figure before of the over line: but being there is no figure following, you must adde that one to the place following, and then deduct that one from the number above.

Scholar. Then shall I say, because I borrowed 10 to the over 5, I must put 1 in the next place beneath, that is under 2, then must I subtract that 1 from 2, and there resteth 1 to be written under that in the ninth place. Now I have no more to subtract, for there is not any figure remaining beneath, neither yet any Vnite to be added, because I borrowed not 10 to the figure last before: and yet is there 8 remaining in the over line, which I thinke (by reason) should be set at the end of the figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed.

Scholar. But Sir, I beseech you, shall I alwayes when any number so remaineth alone, as thus 8 did, write him under the line straight

straight against his owne place?

Master. Yea, what else? Whether they be one or many: and this well remembred, you have sufficiently learned Subtraction: Howbeit, because of certaine things that might deceive you, if you did not take good heed to your working, I will propose to you another example of many numbers to be subtracted, as thus: I received of a friend of mine to keepe 2869 crownes, of which at one time I delivered him againe 500, at another time 368, at another time 440, at another time 80, and another time 64, now would I know how many doe rest behinde. Therefore first I set downe my grosse sum, and underneath it I set al the parcels thus, and under them a double line.

$$\begin{array}{l} 2869 \ \textit{Crownes received.} \\ \hline \left.\begin{array}{r} 500 \\ 368 \\ 440 \\ 80 \\ 64 \end{array}\right\} \textit{delivered.} \\ \hline\hline \end{array}$$

Then first I begin at the first place, & gather together the sum of all those lines (save the overmost) in their first figures: and so I do with all the figures of the second place, and so forth, as I did in Addition, save that I leave out the highest row of numbers (as the line warneth me) and that summe so gathered betweene the double line, is the summe delivered in all: which summe I doe afterwards subtract out of the highest row of numbers, and the remainder do I set under the nethermost line: Note.

as

as for example.

I set the summes as before: then doe I gather the first figures of all the places delivered, together: where I finde but 4 and 8, that maketh 12, (for three Cyphers increase no summe in Addition, as you learned before:) of the 12 therefore doe I write the Digit 2 betweene the double line, and keepe the Article in my minde till I come to the second place, where I finde 6, 8, 4, 6, that maketh 24, to them I put the Article in my minde, and it is 25, of which I write 5 under the second place, and keepe the Digit 2 in my minde for the third place, where I finde 4, 3, 5, that makes 12, to the which I adde the 2 in my minde, and it maketh 14, thereof I write the 4 under the third place, and because there remaines no moe figures to be added, I write the Digit in the fourth place, as you see in the example, and so it appeareth, I have delivered in all a thousand foure hundred fifty two crownes.

2869	*Crownes received.*
500	
368	
440	*Delivered.*
80	
64	
1452	*Delivered in all.*
1417	*Rest behinde.*

Then come I to the subtracting of this summe betweene the lines, for by Addition it is equall to the five parcels over it. Therefore I proceed to subtract it from the overmost summe, saying, 2 from 9 remaine 7 to be written under them beneath the lowest line.

line. Then in the second place I take 5 from 6 and there resteth 1 to be written under them. Then in the third place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2 remaineth 1. And thus I see that after those five summes are subtracted from 2869, the Remainer is 1417.

Scholar. This I perceive: but is there no shorter way and more speedy?

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest summe. But if in quantity those numbers added together, exceed the highest summe or upper number, then shall you (as before hath been taught you) imagine to borrow 10, 20, or 30 moe, as need shall require, and put them to the upper number, to helpe to further the abatement, reserving or restoring the Articles that you borrowed to the next place again: and so still go forward till you have ended your worke: as for example. In the last summe proposed. I gather first in the first place 4 and 8 that maketh 12, which 12 I should deduct or take out of 9 in the upper number above the line, but I cannot: that therefore I ad unto 9 an article of 10, & maketh the upper number 19, from whence I take 12, then there resteth 7, then for the Article 10 I adde to the next place of money delivered; saying, 1 that I bring and 6 make 7, & 8 make 15, & 4 make 19 and 6 make 25.

An abri[dge]ment of the former manner [of] Subtra[ction].

which 25 I should take out of 6 in the upper number, but I cannot. Therefore I adde 2 tens or 20 unto 6 in the upper number, and that maketh 26, then 25 out of 26, resteth 1, then the tens which I borrowed, or have in minde, I adde to the next row or sum delivered; saying, 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14, then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to bee added, the one that I borrowed, or have in minde, I rebate from 2 in the upper line, and there remaineth 1, which I set downe in the remainer line; and so my summe appeareth (as before) to be 1417 Crownes.

Loe thus have you now a shorter way.

Scholar. I like both wayes well, and I perceive both well: yet, as in the working seemeth somewhat long, so in the other it leadeth very much (me seemeth) to remembrance, and therefore may cause errour quickly, except a man have a quicke and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will give me leave) I will try what I can doe in a like summe, to worke it the shortest way: whereupon I would subtract out of 40301964, these three parcels.

Therefore

Therefore I ſet them firſt in due order; then I gather the parcels of the firſt place, which are 8. 2. 1. that is a

40301964	*Charge.*
20003428	*Diſch.*
10002432	
10101461	
43	

11, which I ſhould take or deduct out of 4, which is over him, but I cannot: therefore I adde an article, or one ten to 4, which maketh 14, then 11 out of 14, there reſteth 3 to be written under the firſt place between the two lines.

Then come I to the ſecond place, ſaying, 1 that I borrowed to have in my minde, and 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6: therefore I adde 10 to 6, which maketh 16, and then 12 from 16, reſteth 4, which I write under the ſecond place between the two lines.

Then come I to the third place, ſaying, 1 that I borrowed, or have in minde, and 4 make 5, and 4 is 9, and 4 make 13, which I ſhould take out of 9 that is over them, but I cannot: therefore I adde 10 to 9, which make 19, then 13 out of 19, reſt 6.

Then come I to the fourth place, ſaying, 1 in minde, and 1, is 2, and 2 is 4, and 3 make 7: which, becauſe it cannot be taken from 1, I take it from 11, and there reſteth 4.

After that, I come to the fifth place, where are onely three Ciphers, which make nothing,

unto which I adde 1 in minde, then should I take that (that is to say) 1 from the figure over them, which is also a Cipher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 under them. Then in the sixt place I finde but 1, and 1 in minde make 2, which I take out of 3 over him, and the remainer is 1: that must be written betweene the two lines in the sixth place. So I go to the seventh place, where I find onely Ciphers, & in the grosse summe over them a Cipher also: therefore must I write the remainer (which is nothing) with a Cypher also. Then in the eight and last place, I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is over them, there will nothing remaine. And that must be noted with a Cipher betweene the two lines (as I have often said) and so have I ended my worke, and the figures stand as followeth.

But Sir, I remember you taught mee that Ciphers should not come in the last place, for because they serve onely to increase the value of other figures which follow them and serve not those figures that go before them: and now in my Example I have set two Ciphers in the two last places.

Master. I commend you for your remembrance. And truth it is, you should not have set them here, but onely because that I would make you plainly to perceive the art of Subtraction,

traction. Therefore ſeeing that you do now perceiue it, whenſoeuer you would write downe a Cypher, looke whether any other figures be yet behinde: and if not, then let go the o alſo, for it needeth not to write him in the latter places, where no other figuer doth follow, except it be (as I did now ſuffer you) to teach the uſe of Subtraction the plainer.

Therefore your figures muſt ſtand thus when the worke is ended

40301964	*Charge.*	
20003428		
10002432	*Diſch.*	
10101461		
194643	*Reſt.*	

Scholar. Sir, I do thinke with that that you taught me before, and by theſe two ſummes that you taught me laſt alſo, that now I could ſubtract any ſumme.

Maſter. So may you, if you haue marked what I haue taught you. But, becauſe this thing (as all other) muſt be learned ſurely by often practiſe, I will propound here two examples to you: wherein if you often exerciſe your ſelfe, you ſhall be ripe and perfect to ſubtract any other ſumme lightly, for in them is contained all the obſeruances of whole numbers. And becauſe you ſhall perceiue ſomewhat both how to do it, and alſo whether it be well done when you haue proued to do it; therefore haue I written vnder them both, the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354		103145	
10249	*Paid.*	102597	*Paid.*
163		101024	
20766	*Paid in all.*	02198	*Rest.*
9840	*Rest to pay.*		

Scholar. Sir, I thank you: but I think I might the better do it, if you did shew me the working of it.

Master. Yea, but you must prove your self to do some things without my aid, or else you shall not be able to do any more than you are taught: And that were rather to learn by wrote (as they call it) than by reason. And again, there is nothing in these examples, or any other of whole numbers, but I have taught you the rules of them already.

Scholar. Then I trust, by practise, to attain the use of it. And is this all that I shall learn of Subtraction?

Master. Yea, saving that (as you have seen in Addition) there are numbers of divers Denominations, in which the working is not much unlike: yet (without some instructions be given of it) it might seem to a learner more difficult than indeed it is. Therefore I will briefly shew you the use of it onely, by an example or two.

A cer-

A certain man owed to me 14 l, 12 s, 8 d.
of which he paid me at one time 4 l, 6 s, 8 d.
at another time 3 l, *at another* 2 l, 3 s, 4 d.
and last of all 6 s, 8 d.

Now would I know what remaineth unpaid yet? therefore I set my summes thus, every one in their due place: As pounds under pounds, shillings under shillings, pence under pence

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar. Sir, I pray you why do you write 2 l', for the common speech used rather to say, 40 s.

Master. We must here use the Denomination that is greatest in any summe, so that we may not write according as we use to speak, saying, 16 d, 18 d, or likewise 7 groats, 8 groats, 24 s, 40 s, 48 s, and such other: but we must write every Denomination that is in any summe by it self.

Note how the pen differeth from the common order of counters.

Namely, shillings and pounds. So must we write for the last summes now named, 1 s, 4 d; 1 s, 6 d; 2 d, 4 d; 2 s, 8 d; 1 l', 4 s; 2 l', 8 s, and so forth of other like.

Scholar. So that we may not write in Arithmetick, pence, when the summe amounteth to shillings, nor shillings, when the summe maketh pounds. Now, (if it please you) end your example.

Master. When my summes are so set as I shewed, then (according to the rules of Addition) I gather all the particular summes which hee payd mee into one totall summe, directly to bee set under them betweene the two lines, not medling with the 14 l, 12 s, 8 d, as the line warneth mee: therefore must I beginne with the smallest Denomination, saying, 8, 4, 8, is 20, pence, which maketh one shilling and 8 pence, the 8 d I set downe under the place of pence, and the one shilling I keepe in mind to carry to the next denomination of shillings. Then come I to the shillings, and say, one that I bring or have in minde, and 6 is 7, and 3 is 10 and 6 makes 16, which, because it containeth not one pound, I set directly under the place of shillings. Then come I to the pounds, whose parcels are 2, 3, 4, that is in all 9, that 9 do I set downe directly under the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l 16 s. 8 d.

li	s	d	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	
4	16	0	Rest.

Now for the worke of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say, from the

the 14 li 12 s 8 d.

Therefore to performe the worke, I say, 8 d, out of 8 d. remaineth or resteth nothing, therefore in the place of the rest or remaine, right under the denomination, I set downe 0. Then commeth to the shillings, where I finde 16, which should be taken out of 12, but I cannot: therefore I imagine to borrow 1 of the next Denomination, that is, of the 14 li, and put that one pound so borrowed unto 12 s, that maketh 32 s.

Now 16 s out of 32 s, resteth 16 s, which 16 s I set downe directly under the place of the rest.

Lastly, comming to the pounds, saying, one pound in minde that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remaine, appeare to be 4 li 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as

they are set in order, beginning with the pence, saying, 8, 4, 8, make 20 d. which 20 d. you should take out of the 8 d. above the line, but you cannot, therefore shall you borrow 1 of the next denomination,

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

on, that is to say 1 of the shillings, and put it to the 8 d. that maketh 20 d. now 20, out of 20 d. resteth 0, which Cypher I set downe directly under them.

Then one shilling that I borrowed or had in mind, and 6 make 7, and 3 make 10, and 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I doe borrow one l. and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly, I came to the pounds, saying, 1 l. in minde, or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4,

So doth my remainer or rest appeare as before to be 4 l. 16 s. 0 d.

Scholar. Then doe I perceive very well, and if there bee no other things to be learned in Substraction, then may I come to Multiplication, for that you reckoned to be next in order.

Mast. We have done indeed with the Art of Subtraction, as touching the working.

But yet before we goe to Multiplication, I will

will instruct you how to examine your worke, whether it be well done or not. For the performance whereof, if you marke what I said in Addition, you may easily perceive what is to be done for the proofe of Subtraction, which is best made by the aid of Addition, thus.

Proofe of Subtraction.

Draw under the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did subtract before, together, and write that that amounteth under the lower line: and if the summe that commeth thereof, bee equall to the highest of the Substraction, then is the Substraction well wrought, or else not. As you may see for example in the summes set downe before, and first in summes of one Denomination, whereof one was this.

Where the number 52984732 is substracted from 825003456, and the Remainer is 81970 18724 Now to proove whether it be truely wrought or not.

	825003456
	52984732
	819701 8724

I adde the Remainer and the number substracted, together, beginning at the right hand; and first I say, 4 and 2 is 6, which is set under the line.

Example in a sum of one denomination.

The number given.	825003456
The number to subtract.	52984732
The Remainer.	8197018724
The proofe.	125000 3456

Then againe in the second place, I say, 2 and 3 is 5, which I write under, next that in the

third

third place, 7 and 7 are 14, of which I write the Digit 4, and keepe the Article 1 in my mind. Then in the fourth place 8 and 4 is 12 and 1 in my minde maketh 13, whereof I write downe the Digit 3, and keepe the Article 1 in my minde. Again in the fifth place, 1 and 8 is 9, and 1 in my minde is 10. Whereof I set downe 0 and keepe the 1 in my minde. And so going on to the rest (as it is taught in Addition) when I have made an end, I see that the lowest line of numbers and the highest be alike: wherefore I know that I have well done.

So likewise the proofe is to be made in numbers of divers Denominations; as for example in our summe of that kinde which in the first forme of working, stood thus; (all the particular numbers to be subtracted, being drawne into one.)

Example in a sum of divers denominations.

Where, in the title of pence, I find 8 and 0; the 8 I set downe directly under in that of pence.

Then in the place of shillings I find 16 and 16 which make 32 shillings, wherein is contained 1 li and 12 s. the 12 s. I set downe directly under them in the due place

	li	s	d
	14	12	8
	4	6	8
	3	0	4
	2	3	0
		6	8
Paid in all	9	16	8
Rest.	4	16	0
Proofe.	14	12	8

of

of ſhillings, and one pound I keepe.

Then comming to the pounds, I ſay 1 that I keepe, and 4 is 5, and 9 is 14, which 14 in due order I ſet downe directly under them as this figure ſheweth directly. And the whole ſumme is 14 li, 12 s. 8 d. agreeing with the upper number above. So I finde the worke is good, and the Subtraction well wrought.

The ſame thing is to be done for the latter forme of Subtraction (where the particular ſummes are not gathered together into one groſſe.) For the Remainer and all the particular ſummes ſubtracted, being added together, if the ſumme that commeth thereof bee equall to the higheſt number above, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſums which ſtood thus.

Firſt in the title of pence, I adde 8,4,8, that maketh 20 d, which containeth one ſhilling and 8 pence.

The 8 I ſet down under the loweſt line in the row or title of pence, and that one ſhilling I keepe to carry the next Denomination or place of ſhillings.

Example of a proof in the latter forme of Subtraction.

li	ſ	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

Then returning to the ſhillings, ſaying: one

one in minde, or that I keepe, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound, 12 s. the 12 s. I set downe under the title of shillings, and 1 pound I keepe or have in minde to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then do I write 14 under the pounds, and so have I ended the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I have well done.

And thus have I taught you the Art of Subtraction, and the meanes to probe whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multiplication.

MVltiplication is an operation wherby two summes produce the third: which third summe so many times shall contain the first, as there are Vnites in the second. And it serveth in stead of many Additions. As for example: When I would know how many are 30 times 48, if I should adde 48 thirty times, it would be a long work. Therefore was this work of Multiplication devised, which shall do that at once, that Addition should do at many times. Multiplication, what it is.

Scholar. I perceive the commodity of it partly, but I shall not see the full profit of it, till I know the whole use of it. Therefore Sir, I beseech you, teach me the working of it.

Master. So I judge it best, but because that great summes cannot be multiplied, but by the Multiplication of Digits, therefore I think it best to shew you the way of multiplying them. And when I say, 9 times 8, or 8 times 9, &c. And as for the small Digits, under 5 it were but folly to teach any rule, seeing they are so easie, that every childe can do it: but for the Multiplication of the greater Digits, thus shall you do. Multiplication of Digits.

First, set your Digits one right over the

other then from the uppermost downwards, and from the nethermost upwards; draw straight lines, so that they make a crosse, commonly called Saint Andrews crosse, as you see here. Then looke how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I must write those Digits thus.

The difference.

Digit difference

8

7

Then do I looke how much 8 doth differ from 10, and I finde it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference

8 2

7

After that I take a difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

Digit difference

8 2

7 3

Then doe I draw a line under them, as in Addition thus.

Last of all, I multiply the two difference, saying 2 times 3 make 6, that must I ever set under the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other digit (not from his owne) as the lines of the Crosse

warne

warne me, and that that is left, must I write under the digits. As in this Example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write under the digits, and then there appeareth the multiplication of 7 times 8 to be 56. And so likewise of any other digits, if they be above 5, for if they be under 5, then will their difference be greater than themselves, so that they cannot bee taken out of them. And againe, such little summes every childe can multiply, as to say, 2 times 3 or 4 times 5, and such like.

Digit	*difference.*
8	2
7	3
5	6

Schollar. Truth it is. And seeing me seemeth that I understand the multiplying of the greater digits, I will prove by an example how I can do it. I would know how many are *9* times *6*.

Master. It is all one in value to say *9* times *6*, or *6* times *9*: but yet the order is best to put the lesse sum first, saying, *6* times *9*, and so of all other summes.

Schollar. Then would I know how many are 6 times *9*: therefore I set the digits thus, and make the crosse, thus.

9

6

Then do I set their differences from 10 at the right side, the difference of 9, which is 1, against it, and the difference of 6 which is 4, against it also, as in this example.

9 ✕ 1
6 ✕ 4

And under them draw a line; Then do I multiply the differences together: saying, 1 time 4 maketh 4, that 4 do I write under them thus.

9 ✕ 1
6 ✕ 4
―――――
    4

Then take I one of the differences from the other digit, as, 1 from 6, or else 4 from 9, and each wayes there resteth 5, which I do write under the digits thus. And so appeareth the multiplication of 6 times 9 to be 54. Thus I see the feat of this maner of multiplication of digits.

9 ✕ 1
6 ✕ 4
―――――
5   4

Master. Now might you go straight to the multiplication of great numbers, save that both for your ease and surety in working, I will draw you here a Table, whereby shall appear the multiplication of all the Digits; and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table, when you would know the product in any multiplication of Digits, seek your first or last Digit in the greater figures, and from it go right forth towards the right hand, till you come under the number of your second Digit which is in the highest row, and then the number that is in the meeting of the rows of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then goe right forth toward the right hand, till you come under 9 of the highest row, in which place where you so come under the other digit (as here for example you come under 9) is alwayes contained the off-come or product, which you seek,

and that place we terme to be in the common angle, in respect of the two numbers so taken on the outsides; as here in that common angle, where the rows of little squares directly proceeding from 7 and 9 do meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

Schollar. This is very good and ready. And so may I find the multiplication of any digits: but now how shall I do in greater sums?

To multiply greater summes.

Master. *When you would multiply any summe by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number under it, for that is the place of the Multiplier or Multiplicator, that is to say, the number by which the Multiplication is made, and is in English alwayes put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times, is that which must be multiplied.*

Multiplier

Times.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser under it, as in Addition. And under them must I draw a line. As for example. If I would multiply 264 by 29, I must set them thus.

$$\begin{array}{r} 264 \\ \underline{29} \end{array}$$

¶ Of which numbers thus set downe to be multiplied, may be formed a question, as thus. There are 29 men, and each man hath

264 Lambes. The question is, how many Lambes they have in all.

To the performance whereof, I must multiply every figure of the higher row, by every figure of the nether row: and that that amounteth, I must set under the line, as thus:

First I do multiply 4 by 9, saying: 9 times 4 (or 4 times 9, which is all one) and that maketh 36, as the Table before of digits doth declare; of that 36 I must write the 6, that is the Digit, under the 9, and the Article 3 I keepe in minde to carry to the next place.

```
 264
  29
----
   6
```

Then come I to the second figure of the higher row, which is 6, and say: 9 times 6 make 54, and with the 3 in my minde make 57, the 7 I set downe under the 2, and 5 I keepe in minde.

```
 264
  29
----
  76
```

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in minde, maketh 23: wherefore because it is the last worke of the Multiplier, I set downe in order as you see:

```
 264
  29
----
2376
```

And so I have ended the first figure, of the Multiplier. Wherefore I give it now a fine dash with my pen.

Then begin I with the next figure, and multiply it into all the higher fi- gures, as thus:

264
28
2376

Firſt, 2 times 4 make 8, that 8 do I write under the ſecond place: for evermore the Digit, or firſt figure of the Multiplication that amounteth of the figure of the higher number, muſt be ſet under the Multiplier of it, the other in their order toward the left hand.

Scholar. I underſtand you thus, that the Digit of the ſumme amounting of the Multiplication of the firſt figure of the higher row, by the firſt figure of the lower row, or Multiplier, muſt be ſet under the firſt place: and that that amounteth of the ſame firſt figure by the ſecond Multiplier, muſt be ſet under the ſecond place, and ſo of the other, if there be more Multipliers.

Maſter. So mean I indeed; and if there amount but a Digit, then muſt it be ſet under the Multiplier.

And now to go forth: I multiply by the ſame 2, the ſecond figure of the higher row, which is 6, ſaying, two times 6, make 12, whereof I write the digit 2 under the third place, and the Article 1 I keep in minde.

264
28
2376
28

Then do I multiply the laſt figure of the higher ſumme by that ſame 2, ſaying, two times 2 is 4, and with the 1 that I have in minde maketh 5, which 5 I write under the fourth place. And ſo

ſo haue I ended the whole Multiplication: wherefore I alſo giue the 2 a daſh with my pen, thus: and ſo I do euer as ſoon as I haue diſpatched any Digit, by which I multiply: and the ſummes ſtand thus.

```
  264
   29
 ----
 2376
 528
```

Then muſt I draw a line under all thoſe ſummes that mount of the multiplication, and muſt adde all them into one ſumme, as in the example you may ſee.

```
  264
   29
 ----
 2376
 528
 ----
 7656
```

Where in the firſt place I finde but 6, and therefore write I it under the line. Then in the ſecond place, 8 and 7 make 15, whereof I write 5, and keep one in my minde, and ſo forth as you learned in Addition. And ſo appeareth the whole ſumme to be 7656, which amounteth of the Multiplication of 264, by 29, and that is the juſt number of the Lambs that 29 men had.

Scholar. If there be no more to be obſerved in it, then can I do it, I ſuppoſe, as by this example I ſhall proue.

¶ There is a piece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards ſquare there is in all this piece of ground: which numbers I ſet down with the greater aboue, and the leſſer under, as you ſee.

```
 1365
  236
 ----
```

 Then

Then doe I multiply 5 by 6, ſaying, 6 times 5 make 30, of which I write the Cipher in the firſt place and the Article 3 I doe kéepe in minde to carry to the next place.

1365
236
0

Then doe I by the ſame 6 multiply the ſecond figure of the higher ſumme, which is 6, ſaying, 6 times 6 make 36, and 3 in my minde make 39, of which I write the 9 under the ſecond place, and the Article 3 I kéepe in minde.

1365
236
90

Then do I multiply the third figure, which is 3, by the ſame 6, and that maketh 18, and 3 in my minde make 21. The 1 I ſet downe, and kéep 2 in minde.

1365
236
190

Then come I to the laſt figure of the higher ſumme, and multiply it by 6, ſaying, 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I write under the fourth place. And ſo haue I ended the firſt Multiplier, and daſh him ſlightly with my pen.

1365
236
8190

Then beginne I with the ſecond Multiplier, and ſay, firſt 3 times 5 that maketh 15, of which I ſet the 5 under the ſecond place, becauſe that the Multiplier is there, and the Article 1 I kéep in minde.

Then

Then come I to the second Figure that is 6, and mulitply it by 3, which maketh 18, and with one in minde maketh 19, the 9 I set down under the third place, and 1 I keepe in minde.

1365
236
8190
95

Then come I to the third Figure, which is 3, and multiply it by 3, saying, 3 times 3 make 9, and with one in mind make 10, the Cypher I set under the fourth place, and the Article 1 I keepe in minde.

1365
236
8190
095

And then comming to the last figure 1, I multiply it by 3: and it maketh 3, and with the one in minde, it maketh 4, which 4 I set in the fift place, and then I have ended two of the Multipliers, and the summes stand as you may see in the latter end of the page going before, and then I give 3 his dash.

1365
236
8190
4095

Then come I to the third Multiplier, and multiply it into every figure of the higher sum, and first I say, 2 times 5 make 10, of which I set the Cypher under the Multiplier in the third place, and the Article 1 I keepe in minde.

1365
236
8190
4095
0

And so multiplying the second figure 6 by

that

that same 2, there amounteth 12, and 1 in my minde maketh 13, whereof I write the Digit 3 under the fourth place, and the Article 1 I keepe in mind:

1365
236
8190
4095
30

Then do I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and the one in minde make 7, which 7 I set downe under the fifth place, as appeareth by the example.

1365
236
8190
4095
730

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the sixth place, and then doth the summe stand thus.

1365
236
8190
4095
2730

And so have I ended the whole Multiplication.

But now (as you taught me) to know what this whole summe is, I must adde all those parcels together, and then under the line will appeare, as you may see, the grosse or totall sum is, 322140. Whereby I know there is so many yards square in that peece of ground.

1365
236
8190
4095
2730
322140

Master. This is well done.

Scholar. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Master.

Master. It is to be proved by 9, as Addition was, but the surest proofe is by Division, and therefore I will reserve that proofe by Division, till you have learned the Art of Division. And anon I will shew you how it is commonly proved.]

¶ *But first, for your further instruction in this exercise of Multiplication, I will with one example more try your cunning, and so make an end: And the question is this. I would know how many dayes it is since the Nativity of our Lord and Saviour Jesus Christ, unto this yeere* 1630. *Which to performe; you must multiply this present yeare* 1 6 3 0, *by the dayes in the whole yeere, which are* 3 6 5.

Schollar. Now for that you have given me so much light into the question, you shall see I will handsomely finish the worke, for according to your former instruction, I set them downe with a line under them, thus.

$$\begin{array}{r} 1630 \\ \underline{365} \end{array}$$

Then say I, 5 times 0 is 0, which I set downe under the first place, as heere appeareth. Then say I, 5 times 3 make 15, the digit 5 I set downe in the second place under 3, and the Article 1 I keepe in minde to bee added to the next Multiplication. Then saying, five times 6 make 30, and one in minde 31, the one I set downe in the third place, and 3 I keepe in minde. Then comming to the last figure, I say once 5 is 5, and 3 in minde make 8, that 8 doe I set downe under the

the fourth place: and thus haue I ended my firſt Multiplier, and therefore I giue it a daſh with my Pen.

Then come I to the ſecond Multiplier, which is 6, and do likewiſe multiply it into the upper number, ſaying, 6 times 0 is 0, which I ſet downe in the ſecond place right under his Multiplier: then ſay I, 6 times 3 make 18, the 8 I ſet downe under the third place, and 1 I keepe in minde. Then ſay I, 6 times 6 make 36, and 1 I keepe in minde, make 37, the digit 7 I ſet downe in the fourth place, and 3 I keepe in mind: Then ſay I, 6 times 1 is 6, or once 6 is 6, and 3 in mind make 9, which I ſet down next, & ſo haue I ended two Multipliers; wherefore I daſh the 6 with my Pen.

1540
365
7950
9540

Then I begin to multiply the third Multiplier into the over number, ſaying, 3 times 0 is 0; the 0 I ſet downe in the third place right under his Multiplier. Then ſay I, 3 times 3 make 9, which I ſet down in order next: then ſay I, 2 times 6 is 18, the 8 I ſet downe, and 1 I keepe. Laſtly, I ſay, once 3 is 3, and 1 I keepe is 4, which I ſet downe orderly next: And ſo haue I ended the Multiplication, and my figures ſtand thus.

	or thus.
1630	1630
365	365
8150	815
9780	978
4890	489
594950	594950

Maſter,

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe it will be 594950; which is the grosse or totall summe of that Multiplication, and declareth the number of dayes since our Lord and Saviour his incarnation, unto the end of 1630 *yeares, besides 407 dayes, and twelve houres for leape yeers.*

Scholar. This is marvellous, me thinke, that such great matters may so easily be atchieved by this Art, which heretofore I ever thought had béene impossible, as infinite sorts of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemy than ignorance, for this is one of the least of ten thousand things that may bée done by this Art, as hereafter you shall be able to justifie.

Schollar. The manner of Multiplication I perceive, if there be no more in it.

Master. *Yes, there are other formes and helpes for ease, and shorter labour of the worke of Multiplication, but I will remit them till you have a little tasted Division, where also the like helpe into Division may be used: and so therefore under one example for both, will I shew you both ease in Multiplication, and also in Division.*

But sith the other formes and workings doe nothing differ from these works in effect, but onely in setting of the numbers, I will over-passe them till a more méete place and time.

time. And now will I instruct you in Division, so that you thinke your selfe sufficiently to perceive what I have taught you.

Schollar. Yes Sir, I thanke you, but I doe not perceive how to examine my worke, to try whether I have well done, or no: therefore as you promised me ere-while, I pray you first shew me how I shall prove it.

Master. *That is commonly used by the proofe of 9, as you learned before in Addition, saving that it differeth from that forme in divers respects: As for example.*

First, you must make a crosse after this manner. X

Proof of Multiplication.

Then must you examine your summe that should be multiplied, and look what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and looke what amounteth thereof, if it be under 9, write at the higher part of the crosse: but if it be above, 9, then take thence 9 as often as ye can, and write the rest at the head of the crosse: As for example, wee will prove the example you put forth of the piece of ground that contained 1365 yards in length, and 236 yards in breadth.

Therefore first I cast away all the nines from the summe to be multiplied, saying, 5 and

and 6 make 11, cast away 9 rest 2: then 3 and 2 make 5, and 1 is 6, that 6 I write at one side of the crosse, thus.

6

Then doe I examine the Multiplier, which is 236, wherein when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse.

2 6

Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the crosse. Then doe I examine the grosse summe, amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining; that 3 I set at the foot of the crosse, and then I see it doth agree with the other 3 at the top of the crosse, and so know I that I have done well: for if they two did differ, then were my work in vain, and the Multiplication false.

3
2 X 6
3

This is the common proofe: but the most certaine proofe is by Division, of which I will anon instruct you.

A sure Proofe of Multiplication.

Scholar. Sir, what is the chiefe use of Multiplication?

Master. The use of it is greater than you can yet understand: howbeit, these plaine commodities it hath, that if you would resolve any great and whole valus into many small and lesse portions, as if you would change pounds

pounds in shillings and pence, or any other greater or smaller parcels by Multiplication, ye shall doe it spéedily and easily. Also if you should néed to adde one summe to it selfe, or to any other often times, you shall doe it by Multiplication much more spéedily, readily, easily, and surely, than by often and sundry Additions. Take you these commodities grosly shewed for an answer at this time, and hereafter I will more abundantly make you to perceive the use of it.

Division.

Diviſion.

Schollar.

WEll Sir, then in Diviſion I pray you to inſtruct me. But me thinketh by the name of it, that it ſhould be all one with Multiplication: for I call that Diviſion, when any thing is parted into divers and many parts.

Maſter. You take it as it is taken commonly: howbeit, if you marke well, you ſhall perceive that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrary wayes, it bringeth many parcels into few, but yet ſo, that theſe few taken together, are equall in value to the other many: for by Diviſion, pence are turned into ſhillings, and ſhillings into pounds: As for example, of 120 ſhillings, it maketh 6 pounds, ſo are 120 turned into 6, which is a ſmaller number: but then if you conſider the Denominators, you ſhall ſee that they are ſuch, that one of the latter is equall to 20 of the firſt, and ſo in value the ſums are one, though in number they doe differ, and the latter ſumme is the leſſer, and ſo it is alwayes in Diviſion, howbeit, yet in the working,

the summe is parted by another, and thereof doth it take the name.

Scholar. I thinke I shall better understand the reason of the name when I know the use of the worke, therefore now would I gladly learne that.

Master. *Division is a distributing of a greater summe by the unites of a lesser: Or, Division is an Arithmeticall producing of a third number, in respect of two propounded numbers; which third number shall so often containe an unite, as the greater of the two propounded numbers can containe the lesser.* So that as Multiplication did seeme to serve instead of many Additions, so Division may seeme to be in place of many Subtractions: Because that third number briefly expresseth how many times the lesser of your two propounded numbers may bee subtracted from the greater: as in practise will more plainly appeare. Therefore (as you may perceive) unto Division are required three numbers: the first, which should be divided, and that must (generally) be the greater: and the second, by which the other must be divided, and that is (generally) the lesser, and is called, the Division: And the third, which answereth to the question (How many times?) and therefore is called the quotient.

Division what it is.

The first must be first written, and the second so set under it, that the last figure of the lower number be right under the last of the

A generall rule for placing the figure.

the higher, contrariwise to the work of other kinds of Arithmetick: for in them the two first figures were set ever meet one under the other: but in Division, the last figures must be set meet, except it chance so that the last figure of the Divisor be greater then the last of the higher number, for then you shall set the last of the Divisor under the last save one of the higher number, as for example. An exception.

If you should divide 365 (which are the summe of the daies of a yeare) by 28, which are the daies of a common moneth, then should you set them thus.

365
28

But if you should divide these 365 dayes by 52, which is the number of weekes in one yeare, then should you set them thus.

365
52

Likewise, if I would divide the same 365 by 4, which is the summe of the quarters of yeares; then must I set them thus.

365
4

Schollar. Sir, this do I understand, but now how should I do to divide the one by the other?

Master. You must begin with the last figure next the left hand, and see how many times the last figure of the Divisor may bee taken out of the last figure of the other number, and that shall you note within a crooked line toward your right hand. As for example, I would divide 365 by 28, then

28, then set I those two summes thus.

365 (
28

And I looke how many times I may finde 2 (which is the last figure of the Divisor) in 3, (which is the last of the number to be divided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

Quotient number.

1
~~3~~65 (1
~~2~~8

Then come I to the next figure of the Divisor, and take it likewise so many times out of the figures that bee over it, and looke what doth remaine, that I must write over them, and cancell them, as in this example.

Therefore now doe I take once 8, out of 16, and there remaineth 8, which I must set over the 6, and cancell or crosse out the 16, and the 8 of the Divisor: and then will the figures stand thus. And so I have once wrought.

~~1~~8
~~36~~5 (1
~~28~~

Scholar. So I perceive that you take the neather figure, not onely out of the other that is right over him, but out of that with the other, also

also that remaineth before, and are written toward the left hand.

Master. So must you doe: for you must so take the Divisor out of the over number, that there remaine not over it so great a summe as it selfe is, for then were your worke in vaine.

But yet againe here must you marke, that when you seeke how many times the last figure of the Divisor may bee found in the number over him, that you looke also whether you may as often finde all the figures following in those that are above them (considering all the remainers, if there be any) if not, take your Quotient lesse by one, and then prove againe, and so still till you finde a meete Quotient: and by that meete Quotient must you alwayes multiply your Divisor, and set the product under your Divisor, so that the first figure stand under the first figure of your Divisor, and the second under the second, and so forth: and then subtract that product from the number to bee divided that standeth directly over it, as you have seene me do.

When you have thus wrought once, then must you begin againe, and write your Divisor anew, nearer toward the right hand by one place, as in this example, you shall set 2 under 8, and 8 under 5, thus.

Then (as before) seeke how many times you may take

	18
365	(1
288	
2	

your Divisor out of the number over him now.

Scholar. That may I do here 4 times.

Master. Truth it is, that you may finde 2 foure times in 8; but then mark whether you can find the figure following so many times in the other that is over him. Can you finde 8 foure times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

ark how consi- r this nde of manner.

Master. Well then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following; make 25, in which summe I find 8 three times also: and therefore I take 3 as a true quotient, and write it within the crooked line of the quotient before the 1, thus.

```
   2
  28
 385 (13
  88
   2
```

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8, and write over it the 2 that doth remaine, thus:

Then do I take 8 as many times out of 25, saying: 3 times 8 make 24. and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and over the 5 set 1, thus.

Or, you might (after you found 3 to bee a fit quotient) straightway have multiplied the whole Divisor 28, by that at once: which giveth 84, which being set under 28, and duly subtracted from 85, of the number divided, giveth 1, the remainer of the whole Division, as before you had. Worke which way you list, here you see also the forme.

2
281
385(13
288
2

And now have I done with the dividing, for I cannot finde my Divisor 28 no more in the over summe.

Scholar. No, except you would part the 1, that remaineth into 28 parts.

Master. That is well said, and so must we do in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in Division of whole numbers, and will hereafter teach you particularly of broken numbers, called Fractions. Now if you do perceive the order of Division, then do you divide this summe 136280 by 452.

Scholar. First, I set downe the number that should be divided, then do I set the Divisor under the last figure of the over number. Then will it be thus.

136280
452

Master. Can you take the last of your Divisor (which is 4) out of one, which is the last of the over number?

Schollar. I had forgotten, because the last of the Divisor cannot bee taken out of the last of the over number, in so much as it is the greater, therefore must I set the Divisor one place more forward toward the right hand, thus.

13628o
452

And then must I looke how often I may finde the last figure of the Divisor (that is 4) in 13, which I may doe 3 times, therefore doe I say, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my summes a croked line, and write before it my Quotient 3, and I cancell 13 and 4, and over the 3 I set the 1 that remaineth, and then the figures stand thus.

1
13628o (3
452

Then I multiply the same Quotient into every figure of the Divisor, and withdraw the summe that amounteth out of the numbers over them, as first I say, 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write over the 6 that 1 that remaineth, thus.

21
13628o (3
452

Then doe I say likewise, 3 times 2 make 6, which I take out of 12, and there resteth 6, there-

therefore I cancell the 12 and the 2, over and the 2 I write the 6 that remaineth thus

xx6
x36x80 (3
45x

Then should I set forward the Divisor into the next place toward the right hand thus.

xx6
x36x80 (3
45xx
45

Master. But you may see that over the is no figure, therefore I must set the Divisor yet forwarder by another place.

And marke, whensoever it chanceth so, that you should set forward the Divisor, and that it cannot stand there, because there is no number over the last place, or if there be any, it is lesser then the last figure of the Divisor, then must you remove the Divisor, yet once againe: and because that his first place of removing served not to subtract him so much as once, therefore you shall write in the Quotient a Cipher, and if you should by chance need to doe so oft times, for every time write a Cipher in the Quotient. The reason of this will I shew you hereafter.

Schollar. Then must I set my summes thus.

And because I removed the Divisor, so that I overskipped one place, I must write a Cypher in the quotient: and then must I seeke a new quotient, as

xx6
xx6x80 (30
45xx
45

in

in this example I must say; How many times 4 is there in 6? (and with it can be but once) therefore doe I write 1 in the quotient: and then say I, 1 time 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 over them thus.

2
1163
1362180(301
43222
455
4

Then say I againe, once 5 out of 28, remaineth 23: I let the 2 stand as it did, and over that 8 I set 3, cancelling the 8 and the 5 under it thus.

2
1163
1362180(301
45222
455
4

Master. You might as well have said, once 5 out of 8, and so remaineth 3, but now go forward:

Scholar. Then once 2 out of 0 cannot be: What shall I now do?

Master. Borrow of the next number that is behinde (for there is 230) and do as you learned in Subtraction in a like case.

Scholar. Then must I borrow 1 of the 3 comming behinde next, and make that

to be 10, and then take I 2 out of 10, and there resteth 8. And because I borrowed one of the 3, I must cancell the 3, and write 2 over it: then doth the figure stand thus.

```
    22
   22628
  136280(301
   45222
    455
     4
```

Master. Now have you done, and yet remaineth 228, and your quotient sheweth you, that if you divide 136280 by 452, you shall finde your Divisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I divided 365 by 28, the quotient was 13, and 1 remained, whereby I knew that in a yeare (which containeth 365 dayes) there are 13 moneths, reckoning 28 dayes (or 4 weekes) just to a moneth, and 1 day more.

Scholar. Why then do we call a yeare but twelve moneths?

Master. Of that at a more convenient time will I fully instruct you: but now it is not convenient to intangle your minde with other things then do directly pertaine to your matter. Therefore if you remember what you have heard, you have learned a short manner of Division, which I would have you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholar. Then I pray you tell mee how

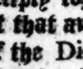
I

I will examine and try my worke, whether I have done well or no, that though no man be by me to tell me, yet I may perceive it my selfe.

Proofe of Division.

Master. Some men (yea and commonly most, doe try it by the rule of 9, as in all the other kindes, save that their order is, First, they cast away 9 as often as they can out of the Divisor, and that remaineth they set at one side of a crosse, as in our first example the Divisor was 28, from which you may take 9 three times and 1 remaineth: which they set by a Crosse, thus.

Then they likewise examine the quotient, (which in our example is 13) and from thence they cast away 9 as often as they can, and the remainder they set at the other side of the Crosse, and then they multiply together those two remainers: and to it that amounteth they adde the remainer of the Division, if there were any from that whole summe they withdraw 9 as often as they can, and the rest they set at the head of the crosse, as in our example the quotient is 13, from which take 9, and there remaineth onely 4, and therefore must you set 4 at the other side of the Crosse, thus.



Then multiply 4 by 1, and it yeeldeth but 4, thereto adde the remainer of the Division (which was 1) and it will be 5, which summe doth

doth not amount to 9, and therefore must be set wholly at the head of the crosse, as you see here.

5
4 X 1

And this number on the head of the Crosse is the first prœfe, to which if you finde another like in the number that was diuided, then you haue done well.

Therefore now shall you likewise examine the whole summe that was diuided, and take away 9 as often as you can, and that that remaineth, set at the foot of the Crosse: and if it be equall to that in the head of the Crosse, then haue you done well, else not.

As in our example the whole summe was 365, which maketh 14, from that take 9, and there resteth 5, which set at the foot of the crosse, thus.

5
4 X 1
5

And you shall see that they agree: therefore haue you well done.

Now will I likewise examine our second example, where the diuisor was 452, which maketh 11; from thence I take 9, and the 2 that remaineth I set at the right side of the crosse, thus.

X 2

Then examine I the quotient, which was 301, where I finde but onely 4: that I set at the other side of the crosse, thus:

4 X 2

Then I multiply 4 by

2, and it maketh 8: to that I adde the remainer of the Division (which was 228, and it maketh 12) and they two make 20, wherein I find twice 9, and 2 remaining: that 2 must I set at the head of the Crosse thus:

$$\begin{array}{ccc} & 2 & \\ 4 & X & 2 \end{array}$$

Then I examine the whole number to be divided, which was 136280, where I finde twice 9 and 2 remaining which I set at the foot of the Crosse thus:

$$\begin{array}{ccc} & 2 & \\ 4 & X & 2 \\ & 2 & \end{array}$$

the proofe of Division more certaine by Multiplication.

And because it doth agree with the figure at the head of the Crosse, I know that the Division was well wrought.

Master. This is the common proofe. Howbeit, the more certaine working is by the contrary kind: as, to prove Division by Multiplication thus:

Multiply the quotient by the divisor, and if the summe that amounteth, be equall to the summe that should be divided, then have you well divided: else not.

Howbeit, this must you marke, that if there remained any thing after the division, that must you adde to the summe that amounteth of the Multiplication. As in our first example our quotient was 13, and the divisor was 28, Now multiply the one by the other, and the sum will be 364: to that if you adde the 1 that remained after the division,

then

then will it be 365, which was the summe that should be divided: and therefore I know that I have well done.

Scholar. Now will I prove the same in the second example, whose divisor was 452 and the quotient 301: these do I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280, which was the whole sum to be divided: and therefore I perceive that I have well done.

Master. This is the surest way, to examine division by Multiplication: and contrariwise the surest proofe of Multiplication is by division.

And therefore (according to my promise) now will I shew you how you may prove Multiplication by division.

When you have ended Multiplication, and would know whether you have well done or not, set the grosse summe that amounteth, of the Multiplication overmost, and divide it by the Multiplier: and if the quotient be the same number that should be multiplied, then have you well wrought, else not, as in that example where we multiplied 264 by 29, the grosse sum was 7656.

Proofe of Multiplication by Division.

Now if you will know whether that Multiplication be true, you shall divide that 7656 by the Multiplier, 29, and you shall perceive that the quotient will be 264, and that is a token that you have well wrought.

Scholar.

Scholar. By your patience I will prove that, and first set downe the grosse summe and the multiplier, not after the rule of Multiplication, but after the rule of Division, for now that number is become the Divisor, that was before the Multiplier, I should set them therefore thus.

7656
29

Then shall I seeke how many times 2 in 7, that may be three times, and one remaineth: but then may not 9 be found so often in 16, therefore must I take a lesser quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then doe I cancell 7 and 2, and over 7 I write 3, and in the quotient I set 2: so the figures stand thus.

3
7656 (2
29

Then say I forth, two times 9 make 18, which I abate out of 36, and there resteth 18: then cancell I 3, and over him set 1, and likewise I cancell 6 and 9, and over them I set 8: so that thus stand the Figures.

1
38
7656 (2
29

Then I set forward the Divisor by one place, and seeke a new quotient, that is to say, how many times 2 are in 18, which I finde to bee 9 times: but then can I not finde 9 so many times in 5, therefore I take a lesser quotient, as to say 8: but yet that is too great: for if I take 8 times 2 out of 18, there remaineth

remaineth but 2, and I cannot find 8 times 9 in 25: therefore yet I take a less quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say 6, then say I, 6 times 2 make 12, that I take out of 18, & there remaineth 6, so I cancell 18, and the 2, and write 6 over 8 thus:

16
38
7656(26
299
2

Then say I forth 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus:

1
16
381
7656(26
299
2

Then must I set forth the Divisor againe & seeke a new quotient, which will be 4: for though I may find 2 in 11, 5 times, & 1 remain, yet I cannot find 9 so often in 6, therefore I set the Figures thus:

1
16
381
7656(264
2999
22

And the 4 in the quotient I multiply into the Figures of the Divisor saying, foure times 2 makes 8 which I take out of 11, and there rests 3 therefore I cansell the 11, & the 2, and set 3 over the first place of 11, thus:

1
163
381
7656(264
2999
22

And then doe I say forth 4 times 9 maketh 36, which I take from 36, and there remaineth nothing,

I is

so that the quotient of this Division also (where 7656 is divided by 29) is 264: Which doth declare, that if 264 be multiplied by 29, the summe will be 7656. And thus I perceive now how both Multiplication is proved by Division, and Division also by Multiplication.

Master. Now have I ended the five common kinds of Arithmetick. For (as touching Mediation Duplation, Triplation, and such other) they are no severall kinds of Arithmeticke, but are contained under the other. For Mediation is contained under Division, and is nothing else but dividing by 2: and so are Duplation and Tripulation contained under Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation is multiplying by 3, of which I will onely propose an example for the rules you have heard already.

An example of Mediation.

If you would mediate or divide into 2, this summe 4531010, you shall set 2 for the divisor, and worke as you learned before, as thus:

4531010(
2

Then I finde 2 in 4 two times, therefore my quotient must be 2: so I cancell 4 and 2, and remove the Divisor forward thus, as the worke requireth, and as before in division hath beene declared.

4531010(2265505
2222222

Which mediation or division by 2 being finished

finished, you shall have for your quotient 226550$\frac{5}{}$, which is the halfe of 453101, as you may trie by duplation; for double that quotient, or multiply it by 2, and the same number will amount. Duplation.

I will no longer tarry about these, seeing they are but members of the other kinds. But heere now (according to my promise) I will teach you certaine easie formes both of Multiplication and of division. And first of Multiplication.

If you would therefore multiply any summe by 10, you shall need to do no more but adde a Cypher before his first place; as for example 36 multiplied by 10, make 360. Easie formes of Multiplication.

Likewise if you would multiply any summe by 100, put two Cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This do I well perceive, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmeticke also: and as to our matter now, look, as I have told you, that you both remember it, and also often practise it.

And now you have learned how to multiply easily by 10, 100, 1000: and of like manner may you do with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 3000, and such like, where there is one Cypher in the first place,

or many orderly in the first places, you shall take away those Cyphers and multiply the summe onely by the other figure, or figures, (if they be many) and then at the beginning of the summe that amounteth, you shall set so many Cyphers as you tooke away.

Example of 2873, which I would multiply by 300. First I omit the 2 Cyphers from the Multiplier, and I multiply the summe by three onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or tooke away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beside the Cyphers, I must onely take away the Cyphers, and multiply by the other figures, as I learned before: as if I would multiply 93648 by 25000, I should take away the three Cyphers, and multiply the same by 25, and then at the beginning of that totall summe should I adde the 3 Cyphers againe.

Master. Even so: but if it chance the number that should be multiplied, or both the summes, as well the number that should be multiplied, as the Multiplier, to have Cyphers in their first places, evermore omit the Ciphers & work by the rest. But remember to restore as many Cyphers to the amounting summe as you bated before, as in this example 130200 shall be multiplied by 206, I shall onely take away two Cyphers from the greater

ter number, and then multiply 302 by 206, and afterward adde the two Cyphers againe. But if I would multiply the same 30200 by 2060, I shall not onely take away the two Cyphers from the number that should be multiplied, but also I may take away the one Cypher from the Multiplier, and then must I adde 3 Cyphers to the summe that amounteth: but take heed that you take away no Cypher that commeth after any signifying figure, as in the last example, you may not take away that in the fourth place of the higher number, neither that in the third place of the Multiplier: howbeit, yet thus you may do: If one Cypher or more come in the midst of your summes you may multiply by the other Figures, and overskip them: but so, that you give every figure his due place: as thus, I will multiply 3026 by 2004, therefore I set them thus:

3026
2004
12104
52

And thus I do multiple them. First 4 times 6 make 24, I set the 4 under the first place, and keepe the 2 still in my minde. Then say I againe, 4 times 2 maketh 8, and the 2 that is in my minde maketh 10, I set downe the Cypher 0, and keepe the article 1 in my minde: Then 4 times 0 is 0, and the 1 in my minde maketh 1, I set downe the figure 1, and say againe, 4 times 3 is 12, I set downe 2, and keeping the 1 still in my minde (having no more places of the upper number to multi-

tiply it withall) I put it downe next 2 in the fifth place.

Now when I come to the next place (being a Cypher 0) I let it go, because it multiplieth nothing and likewise the second Cipher.

But then, when I come to the 2, and multiply it into the 6 of the over number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the Multiplication be set right under the Multiplier, and the other orderly toward the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as this example sheweth.

	3026
	2004
	12104
	6052
	6064104

Which is indeed wrought so much the sooner and shorter by overskipping of the 2 Cyphers: which otherwise (if the same Example were wrought at length) it would have had 2 workings more, as by the same example here also set downe doth appeare.

	3026
	2004
	12104
	0000
	0000
	6052
	6064104

Scholar. Sir I thanke you, for I see great ease in this way of Multiplication: (and if you can shew me such like in division) you shall greatly further me.

Master.

Master. Yes, I will teach you some easie wayes in division also, and first this: If you would divide any summe by 10, you shall onely with your pen make a square line betweene the first figure of your summe and the second, and then have you done: for the whole number that followeth the line, standeth for the quotient, and the figure that is before the line, is the remainer: as for example, 3648 divided by 10 364 |8 Easie formes of Division

Where 364 is the quotient, and betokeneth that so many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be divided into 10, but by breaking it into fractions, wherewith I will not meddle yet.

And so likewise if you would divide any summe by 100 with your pen, you shall cut away the two first figures, and if you would divide by 1000, you must cut away the three first figures, and so of any other divisor, whose last figure is 1, and the other ciphers, look how many ciphers the divisor hath, and so many figures at the beginning shall you cut away with the square line, and they stand alwayes for the remainer, because they are lesse then the divisor, and cannot be divided by it, and the other Figures that are behinde the line stand for the quotient.

But now if your divisor have any other figure in his last place then 1, and in all his other places have cyphers, looke how many

Ciphers they be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that Figure that is in the last place, as if it were the whole Divisor.

Example of 64284, which I would divide by 300, here must I cut away the two first figures (for so many Cyphers my Divisor hath) and must divide the rest by 3, which is the figure in the last place of the Divisor. First therefore I part away the two first figures, and the summe standeth thus:

642|84 (2
3|00

Then do I divide 642 by 3, and the quotient is 214: for in 6 I find twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before doth make 12, wherein I finde 3 foure times: and this is a ready way to turne shillings into pounds: for sith one pound doth containe 20 shillings, I must divide the whole number of shillings by 20. Therefore easily do it, I see that my Divisor hath one Cypher, and therefore I cut away one figure from the beginning of the whole summe of shillings and then I do mediate or divide by 2 the other figures or summe that followeth.

Scholar. I will put an example.

If you would divide 64287 shillings by 20: that is to say, if I would turne so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that is 6428 by 2, so shall the quotient be 3214,

whereby

whereby I know that 6428 shillings make 3214 pounds, and 7 shillings remaining.

Master. Now prove by Multiplication whether you have well done or no.

Schollar. The quotient is 3214, which I do multiply by the Divisor 2, and it doth amount to 6428.

Master. Hereby you may perceive not onely that you have well done, but also how by division you may turne shillings easily into pounds: and contrariwise by Multiplication you may turne pounds into shillings.

But here shall you see amongst divers men divers formes of such division: but if you marke what I have told you, you shall perceive easily all the wayes. For some men do not cut away so many of the first figures of the summe that they would divide, as there are Cyphers in the first places of the divisor: but they set all their Cyphers orderly under the first places of the number that they would divide; and then with the other figure or figures (if there be many) they divide the rest of their summe. Another manner of the Abridgement.

Example. If they would divide 725931 by 3400, they do set their summes thus.

7	2	5	9	3	1
3	4			0	0

And then do they divide orderly till they come to the Cyphers: for there they stay and end their worke, as in this example.

They seeke how often 3 may be found in 7, which is two times, and 1 remaining: there-

therefore they set 2 in the quotient, and cancell 3, and 7, and over 7 they set the 1 that remaineth thus:

1
725931 (2
34 00

Then doe I goe forth saying, two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

14
725931 (2
34 00

Then remove they the divisor forward, and seeke how often 3 may bee found in 4, which is but once, and 1 remaineth, then set they 1 in the quotient, & cancell 3 and 4, and over them they set that 1, thus.

1
14
725931 (21
344 00
3

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancell the 4 and 5, and set 1 over them thus.

1
141
725931 (21
344 00
3

Then set they forth the divisor againe and seek how many times 3 are in 11, which they find three times, and 2 remaining: so they set 3 in the quotient, and cancell 11 and 3, and over them set 2 thus:

12
141
725931 (213
344400
33

Then doe they multi-

ply 4 by 3, which maketh 12, that withdraw they out of 29 and there resteth 17, of which the 7 must be set over the 9, and the 1 over the 2, thus:

```
    1
   12
  x417
  725931 (213
  343400
   33
```

And now are the two Cyphers next ensuing, so that the Divisor can no more be set forward, and therefore is the division ended, and the Remainer is 1731.

Now the quotient which is 213, doth declare, that if you divide 725931 by 3400, you shall finde it therein 213 times, and there remaineth 1731: so shall you finde it, if you worke as I taught you, by cutting away the 2 first figures because of the 2 Cyphers.

Note. But this must you marke (as you may perceive by this last example) that if there be left any other Remainer in the summe that was behinde the square line, that the Remainer, must be set to the latter end of the first Remainer, which was cut away with the square line: as if you would divide 725931 by 3400, after the forme that I taught you, then would your summes appeare thus.

```
    1
   12
  x41|7
  7259|31 (213
  3444
   33
```

So that 17 which remaineth after the line, must be set to the 31 (that was cut away

way with the line) in higher places, as you see here: where that 17 with the 31, do make 1731.

Scholar. Sir, is there no other forme of division in practise but this?

Master. Yes verily, there are other formes in practise, but because I love brevity, I will declare onely one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall loving friend, Master Henry Brigges, wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus:

Then if you please you may write the divisor in a loose paper that it may more easily without cancelling or defacing of the worke be applyed to, and removed from the dividend at pleasure; then apply your divisor 6 to 7, the first figure of the dividend, and inquire how oft it may bee had in 7, and seeing 6 is but once in 7, set 1 in the quotient line thus:

Write the Divisor in a loose paper, to remove at pleasure.

6)72.

Then multiply the divisor 6, by the quotient 1, and set the product 6 under 7 thus;

6)72(1

Then draw a line under 6, and subduct

6)72(1
6

6 out of 7, setting the remainer 1 under 6 thus:

```
6)72(1
  6
  —
  1
```

Then bring down the next figure of the dividend, and set it with the Remainer 1 under the line thus:

```
6)72(1
  6
  12
```

And bring the moveable divisor 6 under the 2, and as before enquire how oft 6 is in 12, and finding it to be twice in 12, set 2 in the quotient thus:

```
6)72(12
  6
  —
  12
  12
  —
  0
```

And multiply 6 by that new quotient 2, setting the product 12 under the other 12, and subducting it out of the upper number, there resteth nothing. And since the units of this product do stand under the units of the dividend, the division is ended; otherwise you should proceed as before, bringing down the next figure; removing the divisor, dividing, multiplying, subducting, &c.

Scholar, This is very easie; but if there be greater numbers propounded, is the operation the same?

Master. If the numbers be never so great, the work is the same without any difference, as shall appear by this example.

Divide 7890 by 33.

First set them thus, 33) 7890 (2 then bring the divisor under 78, and see how oft it is there found, which is twice, and therefore set 2 in the quotient, by which multiply the divisor 33, and set the product 66 under 78, and subduct it out of it thus.

Then bring the next figure 9 downe, and set it with the Remainer 12, it maketh 129, and removing the divisor 33 thereto, enquire how often 33 is contained in 129, and I finde it but thrice, (though at the first it	33) 7890 (239 $\frac{3}{33}$ 66 129 99 300 297 3

made a shew of more) therefore set 3 in the quotient, and multiplying 33 by 3, set the product under 129, subducting that product out of the number above, and proceed as before.

Then shall you finde the divisor 9 times in the Remainer, therefore setting 9 in the quotient, multiply, and subduct as before, and at the last you shall finde onely 3 remaining, which must be set above a line after the quotient, and the divisor under, as above appeareth.

Scholar, Is there no more difficulty in the

the whole Rule.

Master. Not any, although your number be never so great, as before I haue said.

¶ And here will I make an end of diuision, (sauing that I doe request you to exercise your selfe well herein by many summes. till you haue attained some expertnesse therein.)

For the reasons and conclusions thereof are so many, and so available for all sorts of men whatsoever; that if I should speake of the infinite uses thereof, I should rather lacke words then matter. And therefore recommending it to your judgment hereafter, upon your further trauell into the Art, I will here end this Treatise, representing unto you one example or simple question of diuision and Multiplication, in stead of many, which is this,

There are foure brasse Peeces. The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a hold, and thee is allowed by the Master Gunner 700 pounds of powder to be spent by these foure Peeces in this assault. The question is twofold. The first how many shot each Peece shall justly make about with this 700 pounds of powder? And lastly, how many pounds of powder ought justly to be allowed to each Peece for his true proportion.

A question of shooting in Ordnance.

Scholar.

Scholer. Why Sir, you make me smile, so beare me in hand, that these two demands may be simply resolved by Multiplication and division.

Master. Truly that they may, and that you may by and by worke your selfe with a little labour: first adde together their quantities of powder that is, 9 pounds, 5 pounds, 4 pounds and 2 pounds, all which make 20: Divide the 700 pounds of powder by that 20, and your quotient giveth 35, as here appeareth, which sheweth for most certainty that they shall make just 35 shootes about.

1
700
200 (35
2

Scholar. Sir, all this have I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To try the truth of the same, multiply the first peece that spends 9 pounds by 35, & you shall see his allowance, which is 315 pounds of powder. Multiply also the second peece that spends 5 pounds by 35, & you shall find 175 pounds his allowance: then 4 by 35 and you shall find 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall finde 70 pounds his allowance. All which foure particular summes you shall adde together by Addition, as here appeareth, and it maketh just 700 pounds, and so is the question truly absolved.

315
175
140
70
700

Scholar.

Schollar. Truly sir, these excellent conclusions do wonderfully more and more make me in love with the Art.

Master. It is an Art, that the further you travell, the more you thirst to go on forward. Such a Fountain, that the more you draw, the more it springs: and to speak absolutely in a word (excepting the study of Divinity, which is the salvation of our souls) there is no study in the world comparable to this, for delight in wonderfull and godly exercise: For the skill hereof is well known immediatly to have flowed from the wisdom of God, into the heart of Man, whom he hath created the chief image and instrument of his praise and glory.

Scholar. The desire of knowledge doth greatly incourage me to be studious herein: and therefore I pray you cease not to instruct me further in the use hereof.

Master. With a good will. And now therefore, for the further use of these two latter, that is, Multiplication and Division; I will briefly shew you the feat of Reduction.

Reduction.

Reduction, what it is.

REduction is, by which all sums of grosse denomination may be turned into sums of more subtile denomination. And contrariwise, all sums of subtile denomination, may be brought to sums of grosser denomination.

Grosse denominatiõ.

Scholar. What call you grosse denomination, and subtile denomination?

Master. That I call a grosse denomination, which doth contain under it many other subtiler or smaller: as a pound (in respect to shillings) is a grosse denomination: for it is greater than shillings, and containeth many of them. And shillings (in comparison to pounds) are a subtile denomination, for because they are lesser than pounds, and many of them are contained in one of the other: and so likewise of other things: whatsoever thing is compared to other, if it be greater, and containeth many of them, it is a grosse denomination: but if it be lesser (so that many of them are in the other) then are they called the subtile denominations: whereby you may perceive, that one denomination may be called a grosse denomination, and also a subtile, (that is to say, a great and a small) in divers comparisons. For shillings compared to pounds, are a subtile or small denomination: but compared to pence,

Subtile denominatiõ.

they

they are a grosse or great denomination.

Schollar. Now I understand the name, I pray you teach me the use.

Master. The use is easily learned, if you remember what you have learned before. For if you will reduce any summe of a grosse denomination into a summe of a smaller or subtiler Denomination, you must consider how many of that subtiler Denomination do make one of the grosser Denomination, and by that number or Numerator do you multiply the summe: as if you would reduce 20 pounds into shillings, you must consider that in a pound are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 pounds are contained 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will be 360, whereby you may finde that in 30 shillings are contained 360 pence. And thus may you reduce any grosse Denomination into a more subtiler, by Multiplication, if you know how many of the lesser do make the greater: of which thing I will anon give you a briefe Table for the most accustomed kindes of Money, Weights, Measures, and Time, and such like: whereby you may know how often each subtile denomination is contained

To reduce grosse denomination to subtile.

ned in the grosser, when you shall need it for the foresaid kinde of Reduction. And also the same shall serve you, if you would reduce any summe of a subtiler denomination, into a summe of a grosser denomination. For in such Reduction you must consider (as in the other form) how many of the smaller do make the greater: and by that number you must divide the other summe, and the Quotient will declare how many of the greater denomination are comprehended in that summe; as for example, If you would know how many shillings are contained in 3240 d, consider that 12 pence do make 1 s. you must divide that 3240 by 12, and your Quotient will be 270, whereby you know that so many shillings are in 3240 d. But if you would know further how many pounds are in these 270 shillings, seeing that every pound containeth 20 shillings: divide that 270 by 20, and it will be 13, and 10 remaining, whereby you may know, that in 3240 d. (or 270 shillings) are 13 pounds and 10 shillings. For evermore the remainer must be named by the name, or denomination of that summe that was divided, which in this place were shillings. And thus may you do with any other kindes of Denominations.

¶ Wherefore, to the intent you may have certain light or knowledge in most common coyns, weights, and measures (which is the chief and principallest thing in traffick to be knowen)

knowne) I have in each Reduction as they come in order, set downe certaine instructions incident thereunto. And first I have hereunto added this Table, wherein is comprehended, not onely our currant and common coynes, but also the most part of the usuall coynes of Christendome, with their just weights and value currant in this Realme of England, intending at the latter end of my Addition to this Booke, to write of the ordinary money used in divers place, and their common values currant for trafficke, with the manner of their exchanges from place to place, &c.

A Table of the names, and now valuation of the most usuall Gold-coyns throughout Christendome, with their severall weight of Pence and Grains: and what they are worth of currant English money this present yeer, 1630.

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	pence.
Great Soveraign.	10	0	33	0
Double Sover. K. H.	8	1	22	0
Double Sov. of Q. E.	7	7	22	0
Royall.	4	23	16	6
Half Royall.	2	11 d.	8	3
Old Noble.	4	6	14	8
Half Noble.	2	3	7	4
Angell.	3	8	11	0
Half Angell.	1	16	5	6
Salute.	2	5	6	11 ob.
2 *parts of Salute.*	1	11	4	7
George Noble.	3	0	9	9 ob.
Half George Noble.	1	12	4	11 q.
First Crown K. H.	2	9	6	11 ob.
Base Crown K. H.	2	0	5	6
Sover. H. K. best.	3	14	11	8 ob q.
Soveraign K. H.	4	0	11	0
Edward Sover.	3	15 d.	11	0
Elizabeth Sover.	3	15 d	11	0
Elizabeth Crown.	1	19	5	6
Half Crown.	0	19 d.	2	9
Unite.	0	12	22	0

Double

Doubl. Crown.	3	6	11	0
Britain Crown.	1	15	5	6
Thistle Crown.	1	7	4	40 bq
Half Crown.	0	19 d	2	9
Crosse Dagger.	3	6 d	11	0
Half Crosse Dagger.	1	15	5	6
Rose Royall.	0	21	33	0
Spur Royall.	4	10 d	16	0
The Angell.	2	23 d	11	0
Half Angell.	1	11 d	5	6

¶ *All the severall peeces of Gold heretofore mentioned, are set down according no their valuation by the Kings Majesties Proclamation for Gold. Dated the* 23 *of November,* 1611.

A Table of forraign Gold coyn, according to their ancient valuation and severall weight, in pence and grains.

The names and titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Vnicorn of Scot.	2	10	6	0
Scottish Crown.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crowns	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Riders.	2	2	3	6
Philips Royall.	2	10	10	0
Philips Crown.	2	5	5	0
Callen Gilden.	2	2	4	8
New and Gild.	2	2	5	0

The names & titles of the Gold	The weight in Pence.	Grains.	The value in Shil.	pence.
Flanders Noble	4	10	12	0
Half Fland. Noble.	2	6	5	0
Flan. Angell best	3	6	9	0
Flaund Royall orke	3	10	10	0
Carolus Gilden.	1	12	3	6
Flanders Royall.	2	6	5	0
Saxon Gilden	2	2	4	8
Flaunders Crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Half Phil Gilden.	1	1	2	1
Golden Lion.	2	16	7	8
3 parts of golden Lion	0	21	2	5
$\frac{1}{3}$ parts of golden Lion	1	19	4	11
Davids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old Andre Gilden.	2	3	4	10
Crusa long Crosse	2	6	6	0
Crusa short Crosse	2	6	6	2
Milreyes	4	20	1	4
Half Milreyes	2	10	6	8
Portague 1 ounce	2	16	68	0
Golden Castilo	2	23	8	10
Ducket of Aragon	2	6	6	6
Hungary Ducket	2	7	6	4
Double Pistolet	4	9	11	8
Single Pistolet	2	4 d	5	10
Ducket of Floren.	2	5	6	4
Double Ducket	4	11	13	0
Single Ducket	2	6	6	6
Double duc. of Rome	4	13	12	8

It is to be understood (gentle Reader) that whereas in these tables, the weight is called by the name of a penny, it is not ment a penny of silver money, but a penny of Gold-smiths weight, which containeth 24 Barly Corn. Concerning which, see Troy weight in folio 133.

So if a man have not the weight wherewith to weigh any peece of gold, he may do it with Barly-cornes, being dry, and as it is said, fol 133.

The prices of Gold which the bringers *in of forrain Gold shall receive at the Mint,* according to the Kiegs Majesties Proclamation, *Dated the 14 of* May, Anno. 1612.

For an ounce of *French* crowns, being 22 Karacts fine.	3 li, 6 s.
For every ounce of *Spanish* Pistolets, being 21 Karacts, 3 graines and a halfe fine.	3 li 6 s.
For Duckets of *Spaine*, being 23 Karacts, 1 graine fine at least the ounce.	3 li, 8 s, 8 d.
For Milreas Crusado long crosse. Crusado short crosse, the ounce.	3 li, 6 s, 2 d.
For *Hungary* Duckets being 23 Karacts, 1 graine five at least the ounce	3 li, 9 s. 2 d
For the Checkeen of *Venice*, being 23 Karects, 1 graine fine at least the ounce:	3 li, 10 s.

For

For *Barbary* Gold, being 23 Karects and di [?] graine fine, at the least the ounce, ———	3 li, 9 s.

¶ *And if the said Barbary Gold bee of lesse finenesse, abatement to be made according to the rate.*

For Sultaines being 23 Karects, 1 grain fine at least the ounce.	3 li. 8 s. 8 d.
For all other Gold, being 22 Karects fine the ounce- ——	3 li, 6 s.

¶ *And being finer, a greater price according to that rate, and being courser, a lesse, so that the bringer in supply the lesse fine, with the more fine, in such sort, that in the totall it makes good the same rate of 22 Karects fine.*

The price of Silver, which the bringers in of forraine Silver shall receive at the Mint, according to the *Kings Majesties aforesaid* Proclamation.

FOr the Ounce of *Spanish* silver money of *Civill.* ———	5 s.
For the Ounce of *Mexico* money. ———	4 s, 10 d,

For

For Ingots of Silver, being 11 Ounces, 2 d. weight fine, according to the Standard of *England*, the Ounce.	5 s.

¶ *And for other Silver of more finenesse, a better price according to that rate, and for courser a lesse: so that the bringer in supply the lesse fine with the more fine, in such sort, that in the totall it makes good the said rate of* 11 *Ounces,* 2 *pence weight fine, according to the Standard of England.*

Of Silver Coynes currant in this Realme.

The *Edward* Crowne of 5 s.
The *Edward* halfe Crowne of 2 s. 6 d.
The *Edward* shillings, halfe shilling, and the three pence.
Philip and *Maries* shilling, and halfe shilling.
The *Mary* Groat, and *Mary* two pence.
Queene *Elizabeth* shilling. 9 d. 6. d. 4. d. 3. d. 2 d. 1 d. three farthings, and halfe penny.

HEre would I now expresse the values of sundrie other Coynes of divers Countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the Statutes of this Realme. Another cause is, by reason they are so uncertaine, that they be never long at one rate

rate. And againe, they are so different in so many places, that it were matter enough for a great Booke to speake sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in use, and first of France.

nch
nes.

The most common money are Deniers Soulx, and Frankes: 12 Deniers make 1 shilling, 20 Soulx make 1 Franke: so that as you see these 3 kindes are like in the rate to pence, shillings and pounds with us; but that this is the difference, that their Denyer is but the ninth part of our penny, and so their Soulx (commonly called Souses) goe 9 to our shilling, and 9 of their Frankes to an English pound of money. So that three of their Frankes make a Noble. And by those three you may practice how to reduce French money into English money, according as I have set forth here following.

2160	Deniers make	240 d, or 20 s.
3240		360 d, or 30 s.
8352		928 d, or 3 li. 17 s. 4 d.

2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their Coynes I omit them till hereafter, that you have some understanding in broken numbers.

nders
ynes.

But now as for the Coynes of Flanders they be so changeable, that you must know them from time to time: else you cannot reduce them into our money certainly; but yet that

that you have an example of their money to exercise you withall, you shall take those that be most common: as Stivers both single and double, Groats Flemish, Carolus and Gyldens. A Flemish Groat is a little above 3 Farthings English. A single Stiver is 1 d. ob. q. halfe farthing. The double Stiver Carolus is 4 d. ob. halfe farthing. Then there is also the Carolus Gylden, which is worth 10 stivers. And the Flemish Noble is worth 3 Carolus Gyldens and 12 Stivers.

So that if you would convert Flemish money or any other kinde of money whatsoever it be, justly into sterling, you must reduce it first into the smallest part of English money that is in that Coyne. As for example: If I would reduce 368 double stivers into English money (considering that a double stiver containeth 3 d. farthing) you shall first looke how many farthings be in the double stiver and you shall finde them 13: therefore multiply the summe of the stivers by 13, and then have you their valve in farthings, which is 4784. Now, if you divide that by 4. then there will appeare the number of pence: but better it were to divide it by 48 (for so many farthings are in 1 shilling) and then will the quotient declare the summe of the shillings.

Likewise if you would reduce any summe of single stivers into English money, you must multiply the summe first by 13, and then have you reduced them into a certaine summe, that

is

is to wit, half farthings, which sum if you divide by 8, then will amount the sum of pence: or if you divide it by 96, the sum of shillings will appear.

But mark this in all Division: when ye do reduce to bring one Denomination into another, if there be any Remainer after the Division, that must be named by the Denomination of the grosse sum that was divided. As for example, I would bring 254 farthings into pence, therefore I divide that 254 by 4 (for so many farthings make a peny) and the Quotient is 63, which is the sum of the pence, and then remaineth yet 2, which are farthings still, as one may prove by dividing. And this must be marked in all Division, namely, when it is done for Reduction.

[...]te well.

[Da]nsk [Mo]ney.

¶ Touching Dansk Money, they have their Soulx, whereof 20 is a Liver: which is 2 shillings sterling. They have also their Grash, whereof 80 make a Gilden, which is 4 shillings sterling. They have also Dollors, and their common or old Dollor is 35 Grash. New Dollors they have, which be divers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Author, touching Dansk Money.

[S]panish [M]oney.

Concerning Spanish Money, whereof the most common are Cornadoes, Marveides, Marveide, 4 Marveides make a Ryall, and

11 Ryals

11 Ryals make one Ducket, so the Ducket containeth 374 Marveides, which is about 5 shillings, 10 pence, sterling. Therefore if you would convert 124 pounds, 5 shillings, sterling, into Duckets, consider that pence is the least value or Denomination named in this question: therefore reduce 124 pounds, 5 shillings, into pence, and it maketh 29820 pence: which if you divide by pence that a Ducket is worth, (which is 70) you shall have for your Quotient 426 Duckets, your desire.

In Venice they have Bettes, Souldyes, Lieures, 5 Bettes make an English penny, 60 Bettes a shilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice, which is a pound sterling. Venice money.

Thus much have I said of Money: Now will I shew you in like sort, the distinction of weights. Weights.

After a Statute made Anno 11. H. 7. there ought to be but one sort of weight. As 24 Barley-corns dry, and taken out of the middest of the Ear, do make a penny weight, 20 of those penny weights make an ounce; and 12 ounces a pound of Troy weight, by which is weighed Bread, Gold, Silver, Pearl, Silke, and such like. But commonly there is used another weight called Haberdupoise; in which 16 ounces make a pound. Therefore when you would reduce ounces into pounds, you must consider whether your weight be Troy weight or Haberdupoise; and if it be Troy weight, you must

Troy weight. A penny weight. An ounce. A Pound Troy.

Haberdupois weights.

divide

divide your ounces by 12, to bring them to pounds, but if it be Haberdupoise, you must divide them by 16. Now againe, there bee greater weights, which are called a hundred, halfe a hundred, and a quarterne, and also a halfe quarterne, &c.

A hundred weight.

Scholar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they have not common weights made to their rate, as the other have. And againe these that I did name, are not just in number as they seeme by their name: for an hundred is not just 100, but is a 112 pound. And so the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And these be the common weights used in most things that are sold by weight.

Wooll. Weights. Todde.

Howbeit there are in some things other names, as in Wooll, 28 pound is not called a quarterne, but a Todde: and 14 pound is not named halfe a quarterne, but a Stone, and the 7 pound halfe a stone. Other names because they differ in many places, and agree in few, I let them passe.

Stone.

Sack of Wooll.

But a Sacke of Wooll by the Statutes, is limited to be 26 stone.

Cheese weights.

¶ Now in Cheese, though it be sold by the hundred, and by the stone in some places, yet the very weights of it are Cloves, and Weyes, So that a Clove containeth 8 pound, and

a wey 32 Cloves, which is 256 pound, that is 12 score and 16 pound: and so much weigheth the wey of Suffolke Cheese, and the like is or should be the Barrel of Suffolke butter.

The Wey of Essex Cheese containeth six score and sixteene pound: and so much is also the Barrel of Essex butter.

Moreover this weight is used by the Apothecaries in their Physicall composition, and mixture of medecine wherein the least is a graine.

And	*20 Graines*	make	*A Scruple*	thus characted.	℈
	3 Scruples		*A Drachme*		ʒ
	8 Dragmes		*or Dragme*		
	16 Ounces		*An Ounce*		℥
			A Pound.		lb.

The Apothecaries weights

Now of weights are made other measures both for graine and liquor, For a pound in Troy weight, maketh a pint in measure, so that 8 pound or 8 pints do make a gallon: halfe a gallon is named a pottle, and halfe a pottle is called a quart, which containeth two pints: Now above a Gallon the next measure is a Firkin: then the Tertian a kilderkin, or halfe Barrel, and a Barrel. And by these measures are sold commonly Ale, Beere, wine, and Oyle, Butter and Soape, Salmon, Herrings, and Eles.

Measures for liquor. A pint. Gallon. Pottle. Quart. Ferkin: Tertian. Kilderkin. Barrell.

But as these be unlike things, so the measures of their vessels do differ, for the measures of them all are as followeth.

Of Ale	*the Firkin*	*containeth*	8	*Gallons.*
	the kilder.		16	
	the barrel.		32	

Ale measure.

L Of

eer mea- ures.

Of Beere	*the Firkin.*	*contai-neth*	9	*Gallons.*
	the kilderk.		18	
	the barrell.		36	

ope mea- ures.

Sope measures, both Firkin, Kilderkin, and Barrell should be equall to Ale measure.

Moreover the Statutes do limite the weight of every of those three vessels being empty.

A Barrell	*to weigh empty*	26	*pounds.*
Halfe a Barrell		13	
A Firkin		$6\frac{1}{2}$	

Herrings.

Herrings also sold by the same measures that Ale and Soape be sold by.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the last.

Salman & Eeles.

Salmon and Eeles have a greater measure.

Salmon & Eeles	*the Butte*	*hold-eth*	84	*Gal-lons.*
	the Barrell		42	
	halfe Bar.		21	
	the Firkin		$10\frac{1}{2}$	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels,

Wine measures.

Now as for wine vessels they are seldome smaller then Hogsheads, which are of 63 Gallons: Every Hogshead, is two Barrels: yet there are many other wine vessels, but of them all see this Table; and marke the measures one by another.

Of

Of wine and oyle	the Runlet	holdeth	$18\frac{1}{2}$	Gallons.
	the Barrell		$31\frac{1}{2}$	
	the Hogsh.		63	
	the Tertian		84	
	the Pipe		126	
	the Tunne		252	

But you shall marke that there be other kindes of Tertians: for there be Tertians, (that is to say) Thirds of Pipes, of Hogsheads, and of Barrels, as well of other things as of wine. Tertians.

Also Malmseyes, and Sacke, &c. the halfe Tun is not called a Pipe but rather a Butte. A Butte.

And thus much have I thought meet to tell you at this time.

Scholar. And is that alwayes true?

Master. I have told you how it should be, but how it is, I may not say: how they do differ dayly from their just measure, that Gaugiers can tell you better then I. But I will let this passe now, and speak briefly of the other measure.

And as of weights there did spring the liquid measures (whereof I spake last) so of the same springeth dry measures, as Pecks, Bushels, Quarters, and such like, whereby are measured Corne and like graines, also Salt Lime, Coals, and other like. And this is the order and quantity of them. Drie measures.

A peck is the measure of two Gallons.

Bushell. Quarter. Wey.

A Bushell		*foure Pecks.*
A Quarter	containeth.	*eight Bushels.*
A Wey		*six quarters.*

These are the common names and measures, but in divers places there be divers sorts.

Strike. Cornocke.

The Bushel in many places is two bushels, but then is that Bushel there called a Strike: and in some places halfe a quarter is called a Cornocke. But those diversities are too many to tell you briefly them all: and againe, sith they are against the law and statutes, I count them unmeet to be used.

Measure to mete length, bredth, & thicknesse.

But now remaineth yet another kinde of measures, whereby men mete length, bredth, and thicknesse, and those are, an Inch, a Foot, and such other: whose names and quantities this Table sheweth.

An Inch. Foot. Yard. Elle. Pearch.

3 *Graines of Barley in length make an inch.*

12 *Inches*	*make*	*a Foot.*
3 *Foot*		*a Yard.*

3 *Foot and* 9 *Inches*	*make*	*an Ell.*
5 *Yards and a halfe*		*a Pearch.*

1 *Pearch in breadth, and* 40 *in length, doe make a rodde of Land, which some call a rood, some a yardland, some a Farthendele.*

2 *Farthendels*	*make*	*halfe an acre of ground.*
4 *Farthendels*		*an Acre.*

Acre.

¶ *More,* 40 *rods in length do make a furlong,* 8 *furlongs make an English mile, which containeth* 320 *Perches.*

So that an English mile, grounded upon the Statute, is in length 1760 yards, 5280 foot,

foot, and 6 3 3 6 0 Inches.

Some what greater then the Italian mile of 1000 paces, and 5 foot to a pace.

Here might I tell you many things else touching measures, and also how to reduce strange measures to our measures, but because it cannot be well done without the knowledge of Fractions, which as yet you have not learned, I will let them passe till another time, that I have taught you the knowledge of, broken numbers.

Schollar. But yet sir of the parts of time I pray you tell me somewhat. The parts of time.

Master. You know that a naturall day hath 24 houres, and every houre hath 60 minutes. It needeth not to tell you, that 7 dayes make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeare, lacking one day, and certaine houres and minutes: but of that I shall instruct you hereafter. A day. An houre. Weeke, Moneth, yeare.

Here will I make an end of Reduction for this time, which though it be counted no kind severall of Arithmetick, you see it is no lesse needfull to be knowne, or easier to be done, then any of the other.

Scholar. Marry sir, it seemeth unto me much harder then any other sort, for it requireth the knowledge of so many things: but now sir when you see time, I am readie to learne forth, as much of Reduction as you have taught mee, I remember; but and if I doe at any time forget, I shall have re-

course to the Tables which you set forth for me.

Master. Do so you: for it will not be remembred without exercise. But in as much as you understand so much as we have intreated of, I will now instruct you in progression.

Progres-

Progression.

ALthough untill this day the most part of writers have defined progression as a compendious kinde of Addition, yet truly it is not so: for Progression (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, and that regularly and orderly, whose place is aptly chosen to be very neare, or rather next after the exposition of the foure principal parts of Arithmeticke, for in it after a most easie manner, are all the foure former parts exercised and practised: and not onely Addition, as customably is done, which custome hath been the cause, why it hath so specially beene named a kinde of Addition, and defined to be a quicke and briefe addition of divers summes, proceeding by some certaine and reasonable order. What progression is.

You shall also understand there are infinite kinds of progressions, but for you (as yet) two are sufficient to be exercised in, of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progression is a rehearsing or placing down of many numbers, number after number, in such sort, that betweene every two next numbers rehearsed or placed down, the difference, diversity, or excesse, be equall and alike. Arithmeticall Progression.

Scholar. Sir, I thank you for that you have both opened unto me what Progression is truly, and also why it is here placed.

But I pray you with an example make plaine your dinnition.

Master. Examples cannot want, seeing all reasonable creatures naturally use the order of one kinde of Arithmeticall Progression (which therefore is also named natural) whensoever they distinctly do count or number any multitude by one, saying, 1,2,3,4,5,6, whereby the proceeding from number to number, and every one surmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the same to be Arithmeticall Progression. And for the more plainnesse, I set it downe in this manner.

The common excesse. 1 1 1 1 1

The Progression. 1 2 3 4 5 6

Scholar. This is most evident. And I think that I am able to tel you now of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it proceedeth, if this order be kept iust.

Master. What say you of 3,6,9,12,15?

Scholar. They exceed each other by 3: And that may I set down in such evident order, as you did your example of naturall Progression, in this wise.

The

The common excesse.

The Progression.

Master. And doe you not also now perceive, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall: either if you will begin at the first number of any of them on the left hand, and so proceed right overthwart: or at any of the first numbers of the upper row, and go directly downward.

Scholar. I pray you let me consider the thing a little, and I will answer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceive it now very well, for the common excesse or difference betweene any two next, is continually as much as the first number of every row, either from the left hand overthwart taken, or from any of

of the uppermost overthwart rowes downeward.

Master. Now then, if of any such progression, you would speedily know the totall summe much quicklier then by common Rules of Addition: first tell how many numbers there are (which numbers here we call places, or parcels) and if they be odde, write their summe downe by it selfe; as in this example, 2,4,6,8,10,12,14, where the numbers are 7, as you may see: therefore set downe 7 in a place alone, then adde together the first number and the last, as in this example: Adde 2 to 14, and that maketh 16, take halfe of it, and multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together: As in this example, 8 multiplied by 7, maketh 56, and that is the summe of all those figures.

To know the totall summe of an Arithmeticall Progression.

Scholar. That will I worke by another example, I would know how much this summe is, 5,8,11,14,17,20,23,26,29. I tell the places and there are 9, that I note. Then I put the first number 5, and the last 29 together, and they make 34, I take the halfe of it, that is 17, and multiply by 9, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you finde it if you try it.

Scholar. How shall I try it?

Master. By your common Addition, for

if you adde all the parcels together, you shall see the same summe amount, if you did worke well. And that manner of Addition tryeth all kinds of summing any Progression.

Scholar. Then can I summe any Progression, if the number of the parcels be odde. But what if they be even: as in this example, 1,2,3,4,5,6,7,8?

Master. When the number of the parcels is even, then note that also as you did before, and likewise adde the first summe to the last, and by the halfe of the number of the places, do you multiply it: as in our example, the parcels are 4, that I note: then adding the first summe to the last, there amounteth 9, that do I multiply by halfe of the parcels, that is, by 4, and it maketh 36: which is the summe of the parcels.

But if you will take one Rule for these both, do thus: Multiply the halfe of the one by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels be odde, so their halfe cannot be taken: and that sometime it chanceth the Addition of the first number and the last, do bring forth an odde number, so that halfe of it cannot be taken: but they will never be both odde.

A generall rule.

Schol. Then I perceive this, if there be no mo[illegible]ging to it.

Master. As accustomably it hath beene taught, this hath beene the chiefe and onely

exercise

exercise in Progression used. But that you may perceive how divers wayes, and to how great profit so simple a thing (as this Arithmeticall Progression is) may bee considered and used, I will here propound you sixe propositions: of which foure of them were invented by a friend of mine, and never before thus published: and the two first were never to my knowledge written of but by three Men.

Scholar. This doth greatly encourage me to be attentive unto your words, seeing I shall not onely be instructed at your hands in the common knowne Rules of this excellent Art, but besides, that, so abundantly in other new rules enformed, as my very entrance shall seeme to passe a great many mens further study, and longer continuance. Therefore Sir, I beseech you, let me know your six Propositions.

Master. These they are.

To know the last number without proceeding by continuall Addition, till you come unto it, so that the common excesse, the first number, and the number of the places be knowne.

2 *The first number of the Progression and the last being knowne, with the common excesse to finde the number of the places.*

3 *The excesse being given, and the first or last, to know the quantity of any middle number, whose place is given from the first or last.*

4 *The totall summe being given, and the first and last, to finde out the number of the places.*

The

The totall summe of any Arithmeticall Progression being given, and the first and last, to finde out the common excesse. 5

The totall summe being given, and the mutuall excesse, with the number of the places, to give the first or last number of the same Progression. 6

Many moe considerations could I propound you in these *Arithmeticall Progressions*, but these are sufficient, to give you occasion to thinke, that Rules of knowledge and *Arts* are infinitely capable of enlargement.

Scholar. Happy were I, if I did but well understand that which is already invented and written: And yet in my simple fantasie, these things offer themselves (in manner) to be studied for about Progression, therefore I pray you to proceed to the Rules answering to these propositions.

Master. I will orderly for every of these six propositions give you Rules, and with every one an example, unlesse the plainnesse and easinesse need no farther exemplifying.

For the Solution of the first multiply the excesse by a number lesse by 1 *then the number of the places and the off-come adde to the first number, so you shall have the last number, which is sought for.*

As for example. If there were seven places in a Progression Arithmeticall, whose continuall encrease or mutuall excesse were 4, and the first number were 5, and I would know what the last and seventh number is: I multiply 6, which is one lesse then 7, (the

number

number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29, and that is the last number which I desire to know. And this you may straightway prove by continuall proceeding from 5, till the seventh place, encreasing every one by 4, as thus.

5 9 13 17 21 25 29.

Lo here, the last, being also the seventh, is 29.

Scholar. I perceive already one good property in this Rule, which in all works is to be desired: that is, it will ease one from great labour, if a Progression where propounded of a hundred or two hundred places, or moe: And also it is very easie to work, and most necessary for the totall summe finding, in a very long Progression,

Master. It is true, and therefore now let me see if you can answer me this question by this proposition.

A Merchant buyeth 50 pounds of Spices, and agreeth to pay for the first pound 4 pence, for the second 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The question is, how much he must pay for the last pound, and then how much the 50 pound commeth to?

Scholar. According to the proposition, I multiply 49 (which is lesse by one then the number of the places) by the excesse, which is

3,

3, to the product 147, I adde the first number which is 4 it maketh 151 pence, the price of the last pound. Now I adde 4, the price of the first pound, to 151 the price of the last pound, it maketh 155, which I multiply by halfe the number of the places, which is 25 the product 3875 pence is the totall summe or price of the 50 pounds of Spices, as appeareth.

49	Places 1 lesse.	151	last
3	excesse	4	first
147		155	
4	first	25	halfe places
151	the last	775	
		310	
		3875	totall summe which amounteth to
		li s d	
		16——2——11	

Master. It is truly wrought.

Scholar. Then I intreat you to proceed to your second proposition.

Master. *The second Rule is this. From the last subtract the first, the remainder divide by the common excesse, to the* Quotient *adde* 1, *and you have the number of the places, which you would know: As in this Progression.* 2 Proposition.

6 11 16 21 26 31

If I know onely 6 and 31, and that they encrease

encreaſe by 5, then according to the rule, from 31 I ſubtract 6, there remaineth 25: which 25 I divide by 5, (the common exceſſe) the Quotient commeth forth 5, to which I adde 1 that maketh 6: and ſo many are the places, as you ſee.

Scholar. This Rule is ſo eaſie, that I were much too blame, if I could not remember it.

3 Propoſition. Maſter. *The third Propoſition may alwayes thus be ſolued. Multiply the* exceſſe *by a number leſſe by one, then the diſtance of the place is from the* firſt, *or the* laſt number given: *the of-come adde to the* firſt, *if the* diſtance *be reckoned from the* firſt, *and the* firſt *alſo knowne, or ſubtract from the* laſt, *if the diſtance be from the* laſt counted, *and the* laſt given *alſo, and that which commeth forth, either in that* Addition to the firſt, *or* ſubtraction from the laſt *is the number* ſought. As for example, *I propound you this* Progreſsion.

8	15	22	29	36	43	50	57

And for the apt conſidering the manner of this queſtion, I will note over every place his diſtance from the firſt, and under every place his diſtance incluſively from the laſt, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the exceſſe where by this Progreſ-

ſion

ſion ſtandeth, be knowne to be 7, and the firſt numbers given, being 8, I would know what number ſtandeth under 4 that is to ſay, in the fourth place. I multiply 7 by 3 (which is leſſe by 1 then the number of the place proponed) that yeeldeth 21, to which I adde 8, the firſt number ſo commeth 29: which I ſay to belong to the fourth place, as ye ſee in the example it alſo doth: or if in the third place from the laſt, you would know what number in this example ſhould ſtand, the laſt number being knowne to be 57, and the common exceſſe 7, then by 2 which is leſſe by 1 than the place propounded, I multiply 7, that giveth 14, which appertaineth to the third place incluſively reckoned from the laſt, and ſo my example giveth you.

Scholar. I perceive right good uſe of this rule: for if I had forgotten what the firſt number were, and remember ſtill but the laſt the common exceſſe, and the number of the places, then might I come by the knowledge of my firſt number againe.

And me thinketh, that it differeth not much from the firſt propoſition, ſaving that which you make here a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was onely ſought, and no conſideration had in numbring the places from the laſt, as here I marke in your numbers noted under your progreſſion.

Maſter. And thinke you not, the middle

numbers of Progreſsion, ſtanding of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt by the common exceſſe, or abating from the laſt continually the common exceſſe as the very ſmall numbers in a ſhorter progreſsion would do?

Scholer. Yes ſir, that I think right well, and therefore I am glad of this new framed propoſition, and the manner of the working of it.

Maſter. *The rule of the fourth is this. Adde the firſt and the laſt together, and by the off-come divide the totall ſumme. Double the quotient, and that willbe the number of the places.*

4 Propoſition.

Scholar. Then if in a progreſsion, whoſe ſumme were 207, and the firſt number 12, and the laſt 57, if I adde 57 and 12 together, that maketh 69, and by it I divide 207, the quotient will be 3, which I double; and ſo I have 6, and ſo many muſt be the number of the places that this progreſsion ſtandeth on.

Maſter. Whether it be ſo or no, how will you trie?

Scholar. Halfe 6, which is 3, being multiplied by 69, muſt make 207, the totall ſumme; if 6 be the number of the places. For ſo the whole worke of your rule in ſumming any Arithmeticall progreſsion did enforme me. I will then multiply 69 by 3, thus.

$$\begin{array}{r} 69 \\ 3 \\ \hline 207 \end{array}$$

It commeth forth juſtly.

Maſter.

Master. I must much herein commend your promptnesse both in memory and in well applying your rule: although in manifest words it did containe no such matter.

Scholar. Sir I pray you heare me frame one example more.

Master. I am well pleased, so that ye bee short, for you make mee more longer here then willingly I would have been: but I cannot perceive how I could have omitted any thing as yet, without your great lack thereof.

Scholar, If I had received 85 pounds of certaine men, but of how many I have forgotten, yet I remember that the first gave me 7 pound, and the last 27 pound, and every payment after other did rise by a like summe. And the man for whom I received this mony, conditioned with me, that of every payment I should have twelve pence for my labour: now unlesse I can by Art finde the truth of this case, I am like to lose the most part of my reward. A question of money.

Master. I perceive you can handsomely frame an example, which should concern your owne gaine: I pray you let me see how you would do justice in this point.

Scholar. I adde the first and the last together, that maketh 34: by which I divide 85, thus:

7	
27	17
85	(2 ――
34	34

Why how now? Sir, here is a remnant of 17, in which 34 cannot bee

 had:

had: so that now I am in the dyers for doubling of my Quotient, and farewell then both my Iustice, and a good lumpe of my gaines.

Master. Ye are never the farther from the matter, though it fall into a Fraction. For you shall understand, that the Fraction which of any such work preceedeth, is ever halfe of one such, as the unites of the Quotient before are. ☞ And that you may trie, if you double that which so remaineth, for then it will be equall to your Divisor, as if ye double 17 (the remnant) it maketh 34, & your divisor also was 34 this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard example. For with it I have a generall rule for the Fraction that may hap in this worke. So that the Quotient being two and a halfe, I double that, it maketh 5, therefore should my gaine be 5 shillings. And to be sure (by your leave) I will trie it, for I will multiply halfe of 34: (which is the first and last number joyned together) by 5. thus.

$$\begin{array}{r} 17 \\ 5 \\ \hline 85 \end{array}$$

It is most true (I see) that I should leese nothing by the former working.

5 Proposition.

Master. *The fifth proposition hath this rule appertaining unto it: By the fourth rule finde the number of the places, that being done, from the last subtract the first, & the residue divide by a number lesse by 1, then the number of the places, and the quotient will shew the excesse which is sought for.*

An

An example hereof shall be this: If ye had disbursed 685 pounds to a certaine number of men, you neither can tell how many they were, or how much the ones mony exceeded his next before, but you are sure that the excesse was equall between every two next: and also you remember that the first had 19, and the last 180 pounds, how would you finde the numb:r of the name and the excesse, continually observed in the succession of their payments. Example.

Scholar. Your rule doth plainly bid, first to finde the number of the places, which I will do according to the fourth rule: I adde 19, and 118 together thus.

$$\begin{array}{r} 118 \\ 19 \\ \hline 137 \end{array}$$

By this 137, I divide 685 thus.

Seeing there is no Fraction but a whole number, being 5. I double that, and then must the number of the places be 10. Now from the last I subtract the first, as 19 from 118, thus: And so remaineth 99.

$$\begin{array}{r} \cancel{1}\cancel{3} \\ \cancel{6}\cancel{8}\cancel{5}(5 \\ \cancel{1}\cancel{3}\cancel{7} \end{array}$$

$$\begin{array}{r} 118 \\ 19 \\ \hline 99 \end{array}$$

This 99 I divide by a number lesse by one then the number of the places, and seeing the places were 10, I divide 99 by 9, thus:

$$\begin{array}{r} \cancel{9}\cancel{9} \\ \cancel{9}\cancel{9}(11 \end{array}$$

The Quotient is 11, and so was the excesse, if I have followed

your rule right.

Master. You have wrought every part of this question both well in order, and truly in the practise of your rules.

Scholar. I will then set it downe also formably, so that the number of the places, the excesse and the totall summe may straight appeare, as your first example stood.

The common excesse.
The progression.

11	11	11	11	11	11	11	11	11	11
19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from the first to the last, the common excesse is 11, I perceive most evidently: but whether the totall summe be 685. I have not yet proved, which I will now doe: I adde 19 and 118 together, that maketh 137: I multiply that by halfe the number of the places, thus.

137
5
685

All things agree most exactly, so that I am perfect enough in these rules, if I forget them not againe.

Master. Use maketh all things perfect.

6 Proposition.

Your sixth rule is this. By the number of the places divide the totall summe, double the quotient, and that will be the first and last joyned in one summe. Then by a number lesse by 1, then the number of the places, multiply the excesse, that off-come subtract from the first doubled,

doubled quotient, and the halfe of the reſidue is the firſt number. The laſt number you may diverſly finde out, as by the firſt of our ſix rules, or by ſubtracting this firſt number from the ſumme which here contained both the firſt and laſt joyntly, (or thirdly) by continuall adding the exceſſe.

Scholar. I pray you make this ſomewhat more plaine with an example.

Maſter. If every moneth in the yeere (counting them now as 13) you gained clearly 40 ſhillings more then you did the moneth next going before, and at the yeeres end you finde the whole gaine 5720 ſhillings, but ye remember not how much either the gaine of the firſt moneth or the laſt was, by this rule it may be tried out. Example of gaine.

Scholar. So that here ye ſeeme to apply the 13 moneths to thirteene places, the 40 ſhillings every one more then the other next before it, to be the common exceſſe, and 5720 ſhillings to the totall ſumme.

Maſter. It is true: by 13 then I divide 5720 in this manner.

2
25
5720 (440
1333
11

I double this quotient, ſo have I 880 for the firſt and the laſt ſumme joyned together; by 12 which is leſſe by one then the num-

ber of the places; I multiply 40 (the common excesse) so commeth 480

	40
	12
	80
	4
	480

This 480 I subtract from 880, so remaineth 400: halfe whereof is the first number which we desired to know: that is 200

And as for the last number, I can give you it three wayes. As by the first of my six rules I multiply, the excesse by a number lesse by 1 then the number of the places, as 40 by 12 that giveth 480, which I adde to the first, being 200, so shall the last be 680.

The same summe commeth forth, if ye subtract 200 from 880.

And thirdly, If I begin at 200, and so proceed, encreasing by 40, I shall at the thirteenth place have 680, as thus:

200	240	280	320	360	400	440
480	520	560	600	640	680	

Scholar. I thanke you most heartily for these sixe rules. Now if it be your pleasure I would heare and learn somewhat of Progression Geometricall.

Master. There are yet very many rules and propositions, which fall into this Arithmeticall Progression.

And for the use and practise of them, I will propone unto you certaine pleasant and necessary questions of Arithmeticall Progression, and to the performance of their workings,

such

such necessary rules and documents, as are requisite for the better understanding of them, or any such like.

A certaine Mercer sold 20 yards of Velvet to be paid in 12 weekes, by Arithmeticall proportion: that is to wit, to receive the first weeke 6 shillings, the second weeke 12 shillings, the third weeke 18 and so forth, encreasing the number of weekes by 6 shillings, till the twelfth and last weeke were expired. The question is how many pounds hee had for 20 yards of velvet.

A question of Velvet.

red. The question is how many 6
pounds hee had for 20 yards of vel- 12
vet. 18
To the performance of this que- 24
stion, and such other the like, I set 30
forth the 12 payments in such sort, 36
as for example, heere appeareth. 42
Then touching the adding toge- 48
ther of these summes, without the 54
aid of Addition, according to the 60
rules I taught you in Progressi- 66
on Arithmeticall, I note the num- 72
ber of the places, which are 12, then adding the last number of the Progression, which is 72, and the first number together, make 78; and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 shillings, and in pounds maketh 13 pounds 8 shillings. And so much hath the Mercer for his 20 yards of Velvet, which is nigh about 23 shillings, 5 pence, a yard.

Scholar. I understand this worke very well but is there any proofe for the justifying hereof,

hereof, as you haue of other workes.

Maſter. The worke of it ſelfe (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantity, is all that is demanded for iuſtifying of the ſame: yet notwithſtanding, becauſe your requeſt is reaſonable, I will propone an example for the proofe hereof.

The proof of the laſt queſtion.

A certaine man is bound to pay for 20 *yards of velvet, the ſumme of* 23 *pound* 8 *ſhillings, and it is to be paid weekely, in* 12 *weekes or termes by Arithmeticall Progreſſion.* The queſtion is therefore to know with what number the ſame Progreſsion is to be begun and continued in ſuch equall proportion Arithmeticall, that in 12 weekes the ſame may iuſtly be accompliſhed.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings, all into ſhillings, which maketh 468 ſhillings.

A generall rule.

Then adde 1 unto 12, the number of the termes, it maketh 13, which 13 you ſhall multiply by halfe the number of the termes, which is 6, it maketh 78; then diuide 468 by 78, and you ſhall finde 6 in the quotient, which is the true number that ſhall begin and continue the ſaid Progreſsion. That is to ſay, the firſt weeke 6 ſhillings, the ſecond 12 ſhillings, and the third weeke 6 ſhillings more, which is 18 ſhillings, and ſo euery weeke as they riſe, 6

ſhillings

ſhillings more then the weeke before, as is manifeſt in the queſtion aforeſaid.

A Farme is to be ſold to be payed by the weekes in a yeare, the firſt weeke to pay 4 ſhillngs, the ſecond weeke 8 ſhillings, the third weeke 12 ſhillings and ſo forth, increaſing each number by 4, till the number of 52 (which are the number of weeks in a yeare be expired.) The queſtion is, what the price of the Farme commeth to?

A queſtion of a Farme.

Scholar. I doubt not, but by that you have already taught me, to end this queſtion very well; wherefore I ſet forth the Progreſsion with his exceſſe 52 times.

Maſter. Nay ſtay a while: And here for your further eaſe (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may do in any other the like) if a queſtion were proponed of 100 or 200 places or moe, and that this queſtion, nor any other the like can be ended, unleſſe you know abſolutely what the laſt number of the Progreſsion at the 52 place is, or ought to bee) I will give you a generall rule how to know the laſt number of any Progreſsion Arithmeticall, as well as if you had ordinarily proceeded by continuall Addition, till you had come to the laſt worke, which is this.

Multiply the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſsion, and you ſhall have your deſire.

A generall rule.

Scholar.

Scholar. This rule is well worth the noting: for if I underſtand you aright, I conſider that my exceſſe is 4, which I multiply by 51, which is one leſſe then the number of the places, and it maketh 204, whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208, which you ſay is, or ſhould be the laſt number of the Progreſſion.

Maſter. This is a moſt approved truth, if there were never ſo many places.

Scholar. This rule is ſo eaſie, that I were much too blame, if I do not remember it. For by the benefit hereof, I have ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeems to paſſe a great many mens further ſtudy, and longer continuance.

Maſter. Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to give you occaſion to thinke that *Rules* of knowledge and *Arts*, are infinite capable of enlargement.

Scholar. Happy were I, if I did but well underſtand that which is already invented and written. But theſe things, in my ſimple fantaſie, offer themſelves to be greatly beneficiall unto the aide of Progreſsion. Therefore now I will go forward with your queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt

number

number of the Progression, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the totall summe or addition of this Progression: which maketh 275 pounds, 12 shillings, as appeareth here by my Tables.

Master. I like well your labour, and commend you for your diligence; I will here propone one example more, and therewithall for this time will end Progression Arithmeticall.

A certaine man bought 20 *Ells of Holland, to be paid in* 17 *weekes or termes by Progression Arithmeticall. And the first weeke to pay* 1 *shilling* 8 *pense, the second weeke* 3 *shillings,* 4 *pence, the third weeke* 5 *shillings, the fourth weeke* 6 *shillings* 8 *pence, and so forth, each weeke succeeding* 20 *pence more then the weeke before.* The question is, what the summe of his 20 Ells commeth to? A question of Holland

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, then in the other before: therefore I pray you work it your selfe, and I will diligently mark your labour.

Master. There is no more to be done in this, then in the other before; but because your request is so reasonable, be attentive unto me.

First by the generall Rules I seeke to finde out the last number of the 17 place, what this

this progression ought to bee. Therefore here in my Tables multiplying the excesse 20 by 16, which is one lesse then the number of the termes for places, and it commeth to be 320; and thereunto adding the first number of the progression, which is 20 pence, all is 340 pence, or 28 shillings 4 pence: for so much ought the last number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the first number of the progression and the last together, which make 360. Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360, which is 180, and multiplying 180 by 17, commeth to 3060 pence, which maketh as you see by Division 12 pound, 15 shillings. And so much is the buyer to pay for his 20 Elles of Holland. Which 3060 pence if you divide by 20, the number of Elles that was bought, you shall finde 12 shillings 9 pence, and so much payed he for an Elle one with another.

The proofe.

A question of debt.

A certaine man doth owe 12 pound, 15 shil. to bee payed in 27 weekes or termes Arithmeticall Progression. The question is, to know with what number he shall begin and continue the Progression in such equall proportion, as the same may be truly paid and satisfied in 17 weeks.

The

The answer.

First I reduce 12 pounds 15 shillings, all into pence, which as you see here in many Tables, make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiply by halfe the number of the weeke or termes, which is $8\frac{1}{2}$ which $8\frac{1}{2}$ multiplied by 18 cannot well be done, unlesse you were acquainted with Fractions or broken numbers, therefore you shall let that passe and multiply 17 by the halfe of 18: which is 9, (for that is all one with the Multiplication of $8\frac{1}{2}$: and the Multiplication of 9 into 17 maketh as you see 153, with which number you shall divide the 3060 pence beforesaid, and the quotient bringeth forth 20 pence, which is the first number or payment to begin the progression withall: and so each week succeeding to rise 20 pence more then the weeke before, and thereby in 17 weekes shall 12 pound 15 shillings be paid: as before was sufficiently declared. Thus much for progression Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightfull, and pleasant, that I count my self happy to be in your company.

Master.

Master. I am glad you delight so well herein, which is an Art of wonderfull dexterity to all sorts of men of what degree or profession soever they be. And now will I proceed to Progression Geometricall wherein I will be more briefe, both because I have beene so long in this part of Arithmeticall Progression, and also for that it would require the knowledge of Roots and surd numbers, (whereof ye have learned nothing) if I should frame the like propositions in them as I have done in these. Therefore I will onely teach you two practises about it, and so end the considerations and works of these Progressions.

Progression Geometricall.

Progression Geometricall is when the numbers increase by a like proportion, that is, if the second number containe the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the fift the fourth, wherefore I set these 3 examples

3	6	12	24	48
1	3	9	27	81
2	10		50	250

Here in the first example you see, that every number containeth the other (that goeth next before him) two times: and in the second example three times, and in the third example five times. Now if you will know how to finde easily the summe of any such number, do this: Consider by what numbers, they be multiplied,

multiplied, whether by 2,3,4,5, or any other, and by the same number multiply the last summe in the Progression. To finde the totall summe in any Geometricall Progression.

Scholar. I pray you worke it by this example, 2,8,32,128,512,2048, which I have framed by proceeding from 2, and continuall multiply by 4.

Master. Then must I multiply the last summe (which is 2048) by 4 also, and it will be 8192. Now must I abate from this sum the first number of the Progression, which here is 2, then resteth 8190: which summe I must divide by 1 lesse then was the number that I multiplyed by. Seeing then I multiplyed by 4, I must divide by 3, so dividing 8190 by 3, the Quotient will be 2730, which is the summe of all the Progression. And now to prove whether you can do the same, I give you these numbers to adde by this rule 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progression doth increase.

Master. In any such doubt do thus: Divide the second number by the first, and the quotient will shew you the number that engendreth the Progression.

Scholar. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Now worke as I taught.

Scholar. The last number is 46875, which I multiply by 5, and it yeeldeth 323475, from

which I abate the last number of the progression, that is 3, and there resteth 234372, which I divide by 4, for that is one lesse then 5, and the Quotient is 58593, which is the whole summe of the progression.

Master. If you remember well this, you have learned the Art of progression both Arithmeticall, and also Geometricall, which you may prove either by subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding together of all the parcels, for so will the same summe amount.

A question of Satten.

A Mercer hath 12 yards of Satten, which he valueth at 16 shillings the yard, and selleth the same 12 yards to another man to be paid as followeth: That is to wit, for the first yard to have one shilling, for the second yard two shillings, for the third yard foure shillings, for the fourth yard 8 shillings, &c. doubling each number following, till the twelfth and last yard. The question is, who hath made the better bargaine of the buyer or the seller.

First you may set downe 12, the number of the yards as you see here in this Example. And against each number the number of shillings due to be paid as the order of Duplation or Multiplication by two teacheth.

Then resorting to the adding up or summing of this progression, where I consider that the increase of this sum proceeded by the Multiplication of 2, & therefore after I have drawn

a

S	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4095	

a line under the 12, I worke and multiply the last summe by 2 also, and it yeeldeth 4096: from whence I abate the first number of the progression, which is 1, and then resteth 4095: which I should divide by one lesse then I did multiply by, but seeing it is 1, I need not to divide it: for 1 (as I have said before) doth neither multiply nor divide, therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction amounteth to 204 pounds 15 shillings, and so much hath the Mercer for his twelve yards of Satten: which is 17 pound, 1 shilling, 3 pence a yard. But I thinke you will buy none so deare.

Scholar. No Sir, by the grace of God this yeare.

A question of an horse

Master. *Then what say you to this question? If I sold unto you an horse having 4 shoes, and in every shoe 6 nayles, with this condition, that you shall pay for the first nayle one* ob: *for the second nayle Two* ob: *for the third nayle foure* ob: *and so forth, doubling untill the end of all the nayles.* Now I aske you how much would the price of the horse come unto?

Scholar. First to know the number of the nayles, I must multiply 6 by 4, and it maketh 24. Then will I do thus: I will write the number of the nayles every one in order from 1 to 24, and against each number of the nayles the summe of halfe pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next figure following appeareth.

Then do I resort to the Rule of summing up the progression, where I consider that the increase of this summe proceedeth by the Multiplication of 2, as the last example did. And therefore multiplying the last summe by 2 also, and it yeeldeth 16777216, from which I abate the first number which is 1, and then resteth 16777215, which I should divide by one lesse then I

1	1	did multiply : but seeing
2	2	that it is 1, I need not to
4	3	divide it, for 1 (as you
8	4	have before said) doth nei-
16	5	ther multiply nor divide,
32	6	therefore I doe take the
64	7	number 16777215 for
128	8	the whole summe of the
256	9	half pence, which by Redu-
512	10	ction I find to be 699050
1024	11	shillings, and 7 pence, halfe
2048	12	penie : that is 34952
4096	13	pounds, 10 shillings, 7
8192	14	pence, ob.
16384	15	Master. That is well done,
32768	16	but I think you will buy no
65536	17	horse of the price.
131072	18	Schol. No sir, if I be wise.
262144	19	Master. Well then an-
524288	20	swer mee to this questi-
1048576	21	on.
2097152	22	*A Lord delivered to a*
4194304	23	*bricklayer a certaine number*
8388608	24	*of loads of Bricke, whereof he*
16777216		*willed him to make twelve*

A question of Bricks.

walles, of such sort, that the first wall should receive two thirdels of the whole number, and the second two thirdels of that which was left; and so every other, two thirdels of that that remained: and so did tde Bricklayer: and when the 12 walls were made, there remaineth one load of Bricke.

Now I aske you, how many load went to each wall, and how many load was in the whole?

Scholar. Why Sir, it is impossible for mee to tell.

Master. Nay, it is very easie, if you marke it well. Marke well that I said, that every wall should receive two thirdels of the summe that was left. Now take away two thirdels from any summe, and you must needs grant that that which remaineth, is one thirdell of the summe last before, Example of 9, from which if you take two thirdels, there will remaine three, which is one thirdle of 9. Likewise from three bate two thirdels, and there will remaine 1.

Scholar. This is true, and now I perceive the least wall had but two load of bricke.

Master. And by the same reason may you know how many load every wall had, according as this figure following doth shew, and likewise what the whole summe of brickes was for if you make 12 summes, multiplying by 3, still from the last remainer, as you may see here on the left side of the Table, there will appeare all the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that be the summe of the loads which was delivered to the Bricklaer.

The

The remainer after every wall.		Loads due to each wall.
1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294

Summe of the loads 531440 *delivered.*

Againe, if you double every Remainer, as you may ſee at the right ſide of this Table, thoſe numbers will ſhew the ſumme of loads that went to each wall, wherby you may perceive that each wall was three times ſo great as the next leſſer.

Scholar. Lo now it appeareth eaſie enough. Now ſurely I ſee that Arithmeticke is a right excellent Art.

Maſter. You will ſay ſo when you know more of the uſe of it: For this is nothing in compariſon to other points that may bee wrought by it.

Scholar. Then I beſeech you ceaſe not to inſtruct mee further in this wonderfull cunning.

M 4 The

The Golden Rule, or Rule of Proportion direct, called the Rule of Three.

Master.

The Rule of proportion.

BY order of the Science (as Men have taught it) there should follow next the extraction of Roots of number, which because it is somewhat hard for you yet, I will let it passe for a while, and will teach you the feate of the Rule of Proportion, which for his excellency is called the Golden Rule.

The Golden Rule.

Whose use is, by three numbers knowne to finde out any other unknowne, which you desire to know, as thus.

Question of boardings.

If you pay for your board for three moneths sixteene shillings, how much shall you pay for eight moneths?

To know this and all such like questions, you shall consider which two of your numbers bee of one denomination, and set those two the one over the other, so that the undermost be it that the question is of: as in my question 3 and 8, be both of one denomination, for they both be moneths; and because 8 is the number that the question is asked of, I set the one over the other, and 8 undermost thus, with such a crooked draught of lines, Then doe I set

3
8

the

the other number which is 16, against 3 at the right side of the line, thus.

$$\begin{array}{l} 3 \diagup 16 \\ 8 \end{array}$$

And now to know my question, this must I do: I must multiply the lowermost on the left side, by that on the right side, & the summe that amounteth, I must divide by the highest on the left side: or in plainer words, thus, I shall multiply the number of which the question is asked (which is called the third number) by the number of another denomination (which is called the second) and the summe that amounteth, must I divide by the summe of like denomination (which is called the first) Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, which I divide by 3, & it yeeldeth 42 shillings, and 2 shillings remaineth, which I turne into pence, and they be 24 pence, of which third part is 8 pence, so the third part of 128 shillings, is 42 shillings, 8 pence, which sum I write at the right hand of the figure against 8 thus.

Note.

The third number. The second number. The first number.

$$\begin{array}{l} 3 \diagup 16 \text{ shillings,} \\ 8 \diagup 42 \text{ shil. } 8 \text{ pence.} \end{array}$$

Hereby I know that if three months boarding, do come to 16 shillings, that 8 moneths boarding will come to 42 shillings, 8 pence, and likewise of any other like question.

But here must you marke, that the first number and the third be of one denomination and also the second and the fourth, for which

you

you seeke : or else be of such demonination^{ns}, that you in working may bring them into on^{e}; As if a man should aske me this question.

Question of expence.

Twelve weekes iourneying cast me 14 French crowns at 6 shillings the peece, how many pounds is that in one yeare? Here you see no two numbers of one denomination, but yet in working you may turne them into like denomination: as thus, turne the one yeare into 52 weekes, and the fourth summe will be French Crowns, by the order of the working. Then to know this question, multiply the third summe 52, by the second 14 and the summe will be 728: that divide by your first number, 12, and the Quotient will bee 60. Crownes, and 8 Crownes remaining: which if you turne into shillings, they will be 48 shillings, which if you divide by your first number 12, the quotient will be 4, which signifieth 4 shillings: put those 60 French Crownes, which make 18 pounds with the 4 shillings, for the summe that answereth to the question, and it is the just expences of a yeare: And the worke will be thus.

12 Z 14

52   60 4 s.

A generall rule.

And take this evermore for a generall rule touching this whole Art, that the doubtfull or unknowne number that you would be resolved of, shall alwayes be set in the third place, Note also the first number & the third, must ever be of one nature and denomination, or else must in working be brought

to

to like denomination, and then of necessity must the other number be in the second place.

Remember also that the place of the first number is highest on the left side, and the place of the second, right against it on the right side; the place of the third number is under the first, as by those examples you have seene.

Scholar. This I trust I can do.

Master. But and if the question be asked thus: *In* 8 *weeks I spend* 40 *shillings, how long will* 105 *shillings serve me?* Here you see that 8 weeks answers himself, and saith 40 shillings. But how long time 105 shillings will serve you know not. Therefore you shall set 105 in third third place, according as I told you even now. And the first place must alwayes be of the same nature or Denomination that the third is of, which here is 40. Then must 8 needs be that other: Now multiply 105 by 8, and it will be 840, which if you divide by 40, it will yeeld 21, which is the fourth number, and sheweth how many weekes 105 shillings will serve, if you spend 40 shillings in 8 weekes.

The figure of this question is this: as if you should say: if 40 shillings serve for 8 weekes: 105 will serve for 21 weekes

Shillings.	*Weekes.*
40	8
105	21

Other diversities there be of working by this rule, but I had rather that you would learne this one well, then at the beginning

to

to trouble your minde with many formes of working, sith this way can do as much as all the other, and hereafter you shall learne the other, more conveniently.

ote. ¶ And for your further aid and instruction, to make you better acquainted with this Golden Rule, I have here proponed sixe questions, and their answers, which I thinke most convenient and meet to preferre the desirous to perfect understanding. The first foure are all branches of one question sprung out of the best tree (for a young learner to taste of) that groweth in this *Ground* of *Arts*: for that no manner of question in the Rule of Three whatsoever it be, can be proponed, but it must be comprehended under the reason or stile of one of these foure.

The Questions.

If 15 *Elles of Cloth cost* 7 pounds 10 shillings, *what comes* 27 *Elles to at that rate?* Answer, 13 pounds 10 shillings.

If 27 *Elles cost* 13 pound 10 shillings, *what are* 15 *Ells worth?* Answer 7 pound 10 shillings.

If 27 *Ells cost* 13 pound 10 shillings: *how many Ells shall I have for* 7 pounds 10 shillings? Answer; 15 *Ells*.

If I sell 15 *Ells for* 7 pound 10 shillings: *how many Ells are to be delivered for* 13 pound 10 shillings? Answer, 27 *Ells*.

If 8 *pound of any thing cost* 15 shillings 6 pence,

pence: *what money is to be received for* 49 pound? Answer: 5 pound, 1 shilling 9 d $\frac{3}{4}$.

If 4 pound *of any thing cost* 17 pence: *what money will* 8 7 6 5 *pound of that commodity cost*; Answer: 155 pound, 4 shillings, 3 pence, q.

Of all which questions, I omit the worke of purpose, that you shall whet your wit thereby at convenient leisure, to clime each branch, and gather the fruit of them, and do minde now, before we make an end of this Rule, to give you some instructions of the backer rule of three, whose order is quite contrary to this that you have learned.

Scholar. I thank you heartily for the six Questions, which I will God-willing practise at convenient times: I pray you proceed therefore to the Backer or Reverse Rule.

The

The Golden Rule, or Rule of Proportion Backward, or reverse.

Master.

IN the former Rule evermore look how much the third number is greater then the first, so much the fourth number is greater then the second. And contrariwise: looke how much the first summe is greater then the third (if it do chance so) so much is the second summe greater then the fourth.

Note this well.

The backer or reverse rule of three.

But in this rule, there is a contrary order, as this: That the greater the third summe is above the first, the lesser the fourth summe is beneath the second: and this rule therefore you may call the Backer or Reverse Rule, as in example.

Question of cloth.

If I have bought 30 *yards of cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards should I need?*

Scholar. Why, there is none so broad.

Master. I do not care for that, I do put this example onely for your easie understanding: for if I should put the example in other measures, it would be harder to understand. But now to the matter: If you would know this question, set your numbers as you did

did before: but you shall multiply now the first number by the second, and that ariseth thereof, you shall divide by the third: which thing if you do here, I meane if you multiply 30 by 2, it will be 60: which summe if you divide by 3, there will appeare 20: whereby I know, that if 30 yards of cloth of two yards broad, should be lined with Canvas of three yards broad, 20 yards of Canvas would suffice, as this figure sheweth. Note.

Breadth. Length.

2 30

3 20

And now because ye found fault with my Example, how say you, perceive you this?

Scholar. Yes Sir. I suppose.

Master. Then answer me to this question: how many Elles of Canvas of Elle breadth, will serve to line 20 *yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the summes to like Denominations.

Master. Then will I tell you, sith there is mention here of quarters, and again every one of the measures both Elles and yards may bee parted into quarters, part them so both in the breadth, and length, and then put forth the question by quarters.

Scholar. Then I shall say thus. How many quarters of Canvas of 5 quarters broad will line 80 quarters of 3 quarters broad?

Master.

Maſter. Now anſwer to the queſtion.

Scholar. Firſt I will ſet them downe in their forme thus: for 5 is ioyned with the queſtion, & is therefore the third number: then is 3 the number of the ſame denomination, I meane becauſe they be both referred to breadth. Now I multiply 80 by 3, and it is 240, which I divide by 5, and it yeeldeth 48. Then ſay I that 48 quarters of 5 quarters broad, will ſuffice to line 80 quarters of three quarters broad.

Breadth. Length.

3 — 80

5

Maſter. Turne the quarters againe into Ells and yards.

Scholar. Then I ſay that 9 Ells and three quarters of a yard of ell-broad, will ſerve to line 20 yards of three quarters broad, as this figure ſheweth.

Breadth. Length.

3 — 80

5 — 48

¶ Maſter. *Now what ſay you to this queſtion: I lent my friend* 400 pound *for* 7 *moneths, how much money ought he to lend mee againe for* 12 *moneths to recompence my curteſie ſhewed him? can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, for I will ſet downe my numbers thus: where I multiply 7 into 400, and it maketh 2800, which I divide by 12, and it yeeldeth

Moneths. Pounds.

7 — 400

12

yeeldeth 233 pound, and there is 4 pound remaining of my Division, what shal I do therewith?

Master. Turne the same 4 pound into shillings, & then divide it by 12 as you did before.

Scholar. Well Sir, it shall be done: so have I 6 shillings for my Quotient, and yet remaineth 8 shillings upon my Division.

Master. You must also reduce that 8 shillings into pence, which maketh 96, and divide that also by your Divisor.

Scholar. So have I done, and I finde 8 pence for my quotient, and nothing is left.

Master. This must you alwayes doe when any thing remaineth upon your Division, whether it be money, weight, measure, or any kind of thing whatsoever. This Rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bound highly to esteeme Arithmeticke.

By this rule may a Captaine in war, worke many things, as Master Digges in his Stratiocos doth declare: Only now in this my simple addition, for a taste & incouragement, I will inlarge the Author with a question or two more wishing you and every my Countreymen or Gentlemen whatsoever, that by nature be any thing given to Military affaires, to be familiar and acquainted with this Excellent Art, the which he shall finde not onely at the Sea, but also in the Campe and Field-service, aboundantly to aid him, either in fortification, pay-

ing of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Instruments whatsoever, as for example.

Question of an Armie.

If it should chance a Captaine which hath 40000 *Souldiers to be inclosed with his Enemy, that he could have no fresh purveyance of victuals, and that the victuals which he had would serve that Armie but onely three moneths, how many men should he dismisse to make the victuall to suffice the residue eight moneths?*

Schollar. As you taught me, I set the numbers thus, saying; If three moneths suffice 40000, to how many will eight suffice?

Moneths	*Men.*
3	40000
8	

To know this; I multiply the first number 3 into the second 40000, and it yeeldeth 120000, which summe I divide by 8, and there will be in the quotient 15000, which if I do subtract from 40000, the remainder will declare that he must dismisse 25000 as this figure sheweth.

Moneths	*Men.*
3	40000
8	15000

A question of a Fort

¶ Master. *Now answer mee to this question: If* 136 *Masons in a moneth bee able to build a Fort to preserve the Souldiers from the Enemy, and such expedition requireth that I would have the same finished in eight dayes: how*

how many workemen say you is there to be appointed.

Scholar. As you taught mee; I set the numbers thus, saying: If 28 dayes require 1 3 6 Masons, what number of mem by the like proportion will 8 dayes require?

28 — 136
8

To know this, I multiply the first number 28 into 136, and it yeeldeth me 3808: which I divide by 8, and my Quotient is 476: which is the just number of Masons that shall supply this worke. And now me thinke these questions are very easie.

Master. Truly if you take delectation herein, you shall finde this Art not onely easie but wonderfull pleasant and profitable. Now therefore one question more I will propose, and so leave off this Rule in whole numbers, untill we come to the use of it in broken numbers: for had you the understanding of broken numbers perfectly, not onely in this Rule, but in all other, the question that in the sight or appearance seemeth to bee 100 times more harder to absolve, may thereby be wrought as soone, or sooner than this.

Scholar. Your words do greatly incourage me to be studious to attaine whole numbers: but might I one attaine to be a practitioner in broken numbers, I should thinke my selfe happy.

Master. *What say you then to this question? If 48 Ioyners in two dayes make 200 light horsemen staves (esteeming they worke but 12 houres a day) and such need requireth that 384 Ioyners are set to the finishing of those 200 staves; in what time say you, will they make them up?*

Scholar. I see here that I must turn my 2 dayes into houres. And so doing, I set my numbers thus:

48 Z 24
384

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded upon an old Proverbe, *Many hands make quicke speed.*

I multiply 48 into 24, and it amounteth to 1152, which I divide by 384, and my quotient is three houres, which is my desire.

Note. *I take this for a note worthy the marking, either in the Rule of Three, forward or backward, when the two numbers are multiplied together, the Product is of the same nature and denomination that the second number is of.*

The

48 Z 24
384 1638
1152 6 fe

The double Rule of Proportion direct.

Master.

WEll, sith you perceive now the use of this Rule, I will shew other which insue of the same. & first the *Double Rule* which is so called, because there is in it double working, by which thing onely it differeth from this. The double Rule.

Scholar. Then by an example I shall understand it well enough.

Master. *So shall you, and let this be the example: If the carriage of* 100 *weight (that is* 112 *pound)* 30 *miles do cost* 12 *pence, how much will the carriage of* 500 *weight cost, being carried* 100 *miles?* Question of cariage.

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus: If C weight cost 12 peace, how much will five hundred weight cost? Set your figures thus:

CWeight.	Pence
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you divide by one, the quotient will be still 60, that is the price of 500 weight for 30 miles.

Then begin the second worke, saying: If

30 miles cost 60 pence, how much will 100 miles cost? Set your figure thus.

Miles.	*Pence.*
30	60
100	

Then multiply 100 by 60, whereof amounteth 6000, which being divided by 30, will yeeld 200 pence. Then you may say, that so many pence shall cost the carriage of 5000 pound weight 100 miles, after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceive it also.

¶ Master. These and such other like questions of the double Rule of Three, are to bee answered much sooner, at one onely working by the Rule of proportion composed of five numbers, which anon I will shew you, and then when you have the use thereof, you may use it which way you think good.

Scholar. Sir, I thanke you much for your courtesie, And I long now till this Rule be ended, that I may see how I may behave my self with that new Rule of five numbers: for that I have ever since you taught me hitherto in the Golden Rule both forward and backeward, wrought but with three numbers only.

Master. But yet a while we will go on forward with this Rule of Three, therefore answer to this question.

Question of sowing. Thirty bushels of wheat sowed, yeelded in one yeere 360, how many will 80 bushels yeeld in 7 yeares? I meane sowing every yeare

yeare of thoſe ſeven, ſtill 80 buſhels?

Scholar. Firſt I ſay, that if 30 buſhels will yeeld 360 in one yeere, then 80 buſhels will yeeld 960 in one yeere. Then for the ſecond worke I ſay. If one yeare yeeld 960, then 7 yeeres will yeeld 6720; as theſe two figures do ſhew.

Seed.	*Encreaſe*	*Yeere.*	*Encreaſe.*
30	360	1	960
80	960	7	6720

But now Sir, if I ſet forth 30 buſhels of Corne to another man for 7 yeares, agreeing ſo that hee ſhall ſow every yeare the whole increaſe of the Corne, and I at the end of theſe 7 yeeres to have the halfe of the whole increaſe: I would know how many buſhels will there amount to my part, ſuppoſing the increaſe to be after the rate of the laſt queſtion, for 30 buſhels in one yeare to yeeld 360? Queſtion of Corne.

Maſter. In ſuch a queſtion you muſt have ſo many ſeverall workings as there bee yeares, as for example: in the firſt yeare 30 buſhels yeeds 360: then to know the yeelding of the ſecond yeare, I muſt ſay, If 30 yeeld 360, how many yeeldeth 360? Worke by your Rule, and you ſhall finde 4320. Then ſay for the third yeere. If 30 yeeld 360, how many will 4320 yeeld? you ſhall have 51840, and ſo every yeare multiplying the whole increaſe by 360, and dividing

it by 30, the increase of the next yeere will amount, as these 7 figures following do orderly declare: where I have set 7 letters for the 7 yeeres, of which the first is set without Art, because that is the increase which you do presuppose: and the last number of each other doth shew the increase of that yeare that it standeth for, which the letters do declare, so that the increase of the seventh yeere, is 1074954240 bushels: how many quarters that is, and also how many waies you may by Reduction soone finde.

a

30 — 360

b

30 Z 360
360 Z 4320

c

30 Z 360
4320 Z 51840

d

30 Z 360
51840 Z 622080

e

30 Z 360
622080 Z 7464960

f

30 Z 360
7464960 Z 89579520

g

30 Z 360
89576520 Z 1074954240

Question of mowing. Now with one question more I will prove you. *If six Mowers do mow 45 acres in five dayes, how many mowers will mow 300 Acres in six dayes?*

Scholer.

Scholar. If 45 acres require 6 Mowers, then 300. Acres require 40. Now againe, if five daies require 40 Mowers, then 6 dayes need but 33 mowers.

Master. Why do you not make mention of the 2 that remaineth in the last Division? for the last part of the question is wrought by the Backer Rule, where the first number 5 is multiplied into the second, that is 40, whereof amounteth 200, which if you divide by the third number 6, the Quotient will be 33, as you said: but then will there remaine 2 which cannot well be divided into 6 parts: howbeit you may understand by the 6 part of 2, the third part of one mans worke, which you must put to the 33; or else you must say that 33 Workemen will end all the 300 Acres in 6 dayes save 2 mens worke for one day or 2 dayes worke for one man. But such broken numbers called Fractions, you shall hereafter more better perceive, when I shall wholly instruct you of them.

Master. *Yet one question more of field matters I will propone, and so I will make an end of this double Rule of Three.*

Scholar. With all my heart. Sir I thanke you, and I will dispatch it as soone as I can, because I would faine see the order of the next Rule of 5 numbers.

Master. *If a Captaine over a band of men did set 300 pioners a worke, which in eight houres did cast a trench of 200 Rods: I demand how many* Question of entrenchings.

labourers

labourers will be able with a like trench in three houres, to intrench a camp of 3400 Rods.

Scholar. I thinke I am now in the Backe-houſe ditch: for I know not well which way to go about it. And beſides that, truly I think I ſhall never come to preferment that way, my growth is ſo ſmall.

Maſter. You know not how God may raiſe you hereafter by knowledge and ſervice into the favour of your Prince, for the availe of your Countrey.

Example for navigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the talleſt man, and yet hath made as great an adventure for the honour of his Prince and Countrey, as ever Engliſhman did.

Scholar. Sir, I thanke you for your good incouragement. My minde, though I be little, is as deſirous of knowledge, as any other: I have pondred now a little of it, and thus I ſet forth the worke.

Rod	*Men*
200	300
3400	

Saying, if 200 Rod require 300 men, what ſhall 3400 rods require? I multiply 3400 by 300, and it yeeldeth 1020000, which I divide by 200, and my quotient is 5100 men.

Then muſt I ſay for the ſecond worke, if in 8 houres 5100 men, be able to diſcharge it, how many ſhall performe the ſame in theree houres; Now if I would worke by the Golden

Golden Rule of Proportion forward, I should finde a lesse number of men : because three houres is lesse than 8 houres : but because reason teacheth me, that the lesser the time is, wherein the trench must be made, the more Labourers I ought to have, thereupon I use now the Backer Rule, as in example. And I have in my Quotient 13600. So many Pioners must I have to intrench the Campe in three houres.

Master. You have answered the question very artificially : And truly I commend you for your diligence and apt understanding : and now according to my promise, I will (in whole numbers) give you a little taste of the Rule of Proportion, compounded of five numbers.

The

The Rule of Proportion, composed of 5 Numbers.

The first part of the rule of proportion compound, direct.

THe Rule of Proportion composed, is distinct for most needfull questions, into severall parts or workings: And there belongeth unto it alwaies five numbers, whereof in this rule being the first part, the second number and the fifth, are alwaies of one nature and like denomination, which Rule is to be wrought thus: you must multiply the first number by the second, and that shall be your Divisor: Then again, multiply the other 3 numbers, the one by the other, and their product shall be your dividend.

And now according to my promise, we will first work the question of weight and carriage, which I delivered you in the double rule of three, to be absolved by this rule, which was this.

If the carriage of 1 *C weight,* 30 *miles cost* 12 *pence, what will the carriage of* 5 *C weight stand me in, being carried* 100 *miles?*

C. weight.	Miles.	Pence.	C. weight.	Miles.
1	30	12	5	100

Now mark well how these five numbers stand: Then multiply the first number by the second, as 30 by 1, which maketh but

30,

30. that number keepe for your Divisor. Then multiply the other three numbers, the one into the other: that is to wit, 12 by 5, which maketh 60: Lastly 60 by 100, which as you see here in our Tables, ariseth to 6000, which 6000 you shall divide by the product of the two first numbers, which here is 30. And you see there is found 200 pence, which is the duty that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the conclusion of the double Rule of three.

Scholar. Sir I thanke you, it is even so.

Master. *Yet note this in a generality in this rule, looke what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes your quotient.* Note this.

Scholar. Well now and if it please you, by your patience, I will see how I can end the question, next following of 30 Bushels of wheat sowed, which in one yeare yeeldeth 360 how many then will 80 Bushels yeeld in 7 yeare following every yeare of those 7 still 80 bushels, and according to your reasons I set my numbers thus.

Bush. Yeare. Bush. Bush. Yare.

30 —— 1 —— 360 -- 80 —— 7

360
80
28800
7
201600

When

Where I multiply 30 by 1, and it maketh 30 my Divisor: then multiplying the other three numbers the one into the other, as here appeareth in my tables, they make 201600, which I divide by 30: & my quotient is 6720 bushels, my desire, for so much also it came to at two workings by the Rule of three.

Master. Yet one question more I will propound unto you, and so leave this Rule, till it please God hereafter, that I may make you worke it in broken numbers.

Qrestion of Interest. *What comes the interest of* 258 pound, *for five moneths too, after the rate of* 8 pound *taken in the* 100 pound *for* 12 *moneths?*

Scholar. Sir, this is yet within the compasse of some reasonable usance. Therefore to minister equity in this case, I will see how I can worke the same, which I set downe thus, praying you if I have not done wel, to shew you mine errour.

li *moneths.* li li *mon.*	
100--12--8-258--5.	

Master. Proceed, you have done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeeldeth 1200, and the three other numbers multiplied together produce 10320, which I divide by 1200: and my quotient is 8 pounds. Then acording as you have taught me heretofore, I turne the 720 pound that I left: into shillings: and dividing it by the first number, my Quotient is 12 shillings,

lings. So I answer, that the loane of 258 pounds for 5 moneths, after the rate of 8 pound in the 100 pound for a yeare, comes to 8 pound 12 shillings.

Master. You say true, I commend your diligence: now behold the manner of the second part of this rule.

The

The backer Rule, or the second part of the Rule of Proportion compound.

Master.

IN the second part of this Rule of Proportion composed, the third number is like unto the first. And the Rule is to be wrought thus: you shall now, contrary to the last Rule, multiply the third number and the fourth together, and that Product shall be your Divisor. Then multiply the fifth by the second, and the Product thereof by the first: and that is the number that shall be divided. For example, I propound this question, for a proofe of my last question of interest.

The proofe of the last question.

A Merchant hath received 8 pound 12 shillings, *for interest of certaine money for* 5 *moneths tearme, which he received after the rate of* 8 pound *in the* 100, *for a yeare. The question is now, how much money was delivered to raise this interest.*

Behold therefore the manner, how the question is set forth.

li *moneths.* li *moneths.* li. s.
100——12——8——5——8–12

Scholar. Sir I perceive it very well: and according

according to the doctrine which you prescribed for the working thereof: if please you now it is set downe, I thinke I can follow the worke.

Master. Nay stay a while, and before you worke, marke well how I deliver a reason for the perfect understanding of this Rule, which is thus: *If* 8 pound *in* 12 *moneths do yeeld me* 100 pound, *to take* 8 pound 12 shillings *for* 5 *moneths, must needs yeeld a great deale more.* Note.

So upon the knowledge that I have in this Art, the first part of this rule is answerable to the rule of three forward: and this latter part accordeth to the rule of three backward.

Scholar. Sir, I yeeld you most hearty thanks for these your last instructions, they have given mee great light into these two rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Master. You say well, go to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings for your more ease in working. Note.

Scholar. If it please you to behold me a litle, I will quickly end it: for I have but my first, my second, and my last number to be multiplied together for my dividend: And my third

into my fourth for my Divisor.

li Moneths. Moneths. li s.

100—12———8.———5—8—12

20 20 20

2000 160 172

12 5

4000 800

2000

24000

172

48000 4128000(

168000 8 00

24000

———4128000

Which 4128000 I divide by 800, and my quotient is 5160 shillings, which in pounds yeeldeth 258, my desire.

Master. I will here for this time in whole numbers end this Rule, and I will instruct you in the Rules of Fellowship. You may at your convenient leasure for your exercise worke the same by the Rule of Three at twice. And for your aid and incouragement therein, I set downe heere a proffer how to apply it.

A

A			B	
Moneths.	li.	s	*Pound.*	li.
5	8—12		8	100
12	412 $\frac{4}{5}$ s		412 $\frac{4}{5}$	258 li

The Rule of Fellowship.

The Rule of Fellowship without time.

BUt now will I shew you of the Rule of Fellowship or Company, which hath sundry operations, according to the divers number of the Company. This Rule is sometime without difference of time, and sometimes there is in it difference of time. First I will speake of that without difference of time, of which let this be an example.

A question of company

Foure Merchants of one Company made a bank of money diversly: for the first layed in 30 pound, *the second* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which stocke they occupy so long, till it was increased to* 3000 pound. *Now I demand of you what should each receive at the parting of this money.*

Scholar. I perceive that this Rule is like the other, but yet there is a difference which I perceive not.

Master. Then will I shew it to you; First by Addition, you shall bring all the particular summes of the Merchants into one summe, which shall be the first summe in your working by the Golden Rule, and the whole summe of the gaines by that stocke shall be the second sum. Now for the third summe you shall

shall set the portion of each man one after another, and then worke by the Golden Rule, and the fourth summe will shew you each mans gaines: as in example.

30
50
60
100
———
240

The parcels of these foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the third place, thus:

240 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appeare 375 pounds thus:

And that is the gaines for the first man.

240 3000
30 375

Now for the second man, set the 50 pound that he brought, in the third place, and worke as before: and his part will be 625 pound: as this figure sheweth.

Likewise for the third man, set his money which was 60 pounds and his part of gaines will bee 750 pounds, as here appeareth.

240 3000
50 625

And so for the fourth man: if you set his sum which is 100 pound his gains will bee 1250 pound, as the work will declare.

240	Z	3000
60		750

Scholar. This I perceive: but is there any way to examine whether I have well done or no?

240	Z	3000
100		1250

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the worke well done.

Note this common proofe.

Scholar. That will I try by and by, the foure parcels are these which added together make 3000, which is the just summe of money that they gained, whereby I know the worke is well done.

375
625
750
1250
3000

Master. Well, now another example will I put to you, not of gaines, but of losse: for one reason serveth for both.

A question of losse.

If three Merchants in one ship, and of one fellowship, had bought Merchandise, so that the first had laid out 200 pound, the second 300 pound, the third 500 pound, and it chanced by tempest that they did cast over board into the Sea Merchandise of the value of 100 pound, how much should each man beare in this losse?

Scholar. If I shall do in this, as you did in the

the other question, then must I ioyne their three portions together 200, 300, 500, which maketh 1000. Then say I, if 1000 lose 100 then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by the three figures it doth appeare plaine.

1000 100 1000 100
200 20 300 30

1000 100
500 50

Master. Well sith now you have done these, I will propound a question of more importance, which shall make you not only the abler to understand this Rule, but also it will greatly aid you in the next Rule of Fellowship with time, if such need be that your money be of divers denominations.

For this may not be forgotten in all such questions: if the number be of divers kinds, you must by reduction bring it into one kind, that is to say, to the least value that is named in the question. And likewise shall you do, if the time be of divers kinds, as some yeares, some moneths, weekes, and dayes, you shall make all moneths, weekes, or dayes, according as the least name of time in the question is, as for example.

First in diversity of money. Three companions bought 2000 *sheepe and paid for them* 241 pound 13—4 *pence, of which summe one paide* 101 A question of sheepe.

pound, 10 shillings. *The second* 82 pound, 17 shillings, 10 pence. *And the third payed* 57 pound 5 shillings 6 pence, *How many sheepe must each of them have?* Answer: *The first shall have* 840. *The second* 686. *And the third* 474. *And that must you worke thus.*

Solution. First, considering that your money is of divers denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence, and so will the totall summe be 58000 pence.

Now if you turne each mans money into pence also, the first mans summe will bee 24360 pence: The second mans money will be 13746 pence.

Now to know how many sheepe every man shall have, let the whole summe of money, that is, 58000 pence be set in the first place, and in the second place set the number of sheepe, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and dividing that that amounteth by the first, there will appeare the number of sheepe that each man ought to have: as these three figures do shew.

a

58000 2000
24360 840

b

58000 2000
19894 686

c

58000 2000
13746 474

Scholar. Why do you set the money in the first place, seeing in the question you say 2000 sheepe cost 58000 pence, and not thus: 58000 cost 2000 sheepe.

Master. You remember I taught you at the beginning of the Golden Rule, that the first and third numbers must be of one name, and of like things: and evermore the number that the question is asked of must be set in the third place.

Now is the question plainly this: If foure men bought 2000 sheepe for 58000 pence, how many sheepe shall each man have?

But seeing in this question, there ought more respect to be had to the summe of money then to the summe of the persons (for in the summe of money is their proportion toward the sheep, and not in the number of persons)

If 580000 pence bought 2000 sheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceive it reasonbly, and so

shall

shall I do in all questions.

Master. Even so. But for easinesse of the worke, marke this: Whensoever the first and second numbers have Ciphers in the first places, you may both in the Multiplication and in the Division leave out those Cyphers, so that you leave out like many out of both summes, as in this question, the first number 58000 hath three Cyphers, and so hath the second, that is 2000: therefore cast away their Cyphers, and so will the first number be 58, and the second 2: set them in their places, and work according to the Rule, and you shall perceive that will be all one, saving that this is the shorter and easier way, as these three figures do shew.

Note.

a

58 2
24360 840

b

58 2
19894 686

c

58 2
13746 474

And this you see is both easier, and also the more certaine way to know the answer to this question.

Scholar. Truth it is as you say: but Sir me seemeth I might aske a further question here, not onely how many sheepe each man should have, but also what every sheepe cost.

Master. That question doth not onely be-
long

long to this Rule, but may also be discussed by Division, especially if the questions number be one onely, as thus: Divide the totall summe 58000 pence by 2000 (or 58 by 2, omitting the Cyphers, and the quotient will be 29 pence that is 2 shillings 5 pence. Howbeit, by this Rule you may do it, and best when the number of the question doth exceed 1; as if I should aske this question,

2000 sheepe cost 58000 pence, how much do 20 cost? Then shall I set my figure as before.

2000 — 58000
20

And doing after the Rule, there will amount 580 pence, that is, 2 pound 8 shilling 4 pence the price of one score: but if you will use that easie way that I did teach you now, you may change the first and second number thus.

2 — 58
20

Thus do you perceive the use of the Rule without Time.

Scholar. All this I understand very well: I pray you now instruct me in the Rule of Fellowship with Time.

The

The Rule of Fellowship with time.

Master.

The Rule of Fellowship with time.

TO the intent you may as well perceive the same Rule with diversity of time, I propose this example.

Foure Merchants made a common stocke, which at the yeares end was increased to 35145 pound. *Now to know what shall be each mans portion of gaine, you must know each mans stocke, and time of continuance.*

Question of a bank

The first man of these foure layd in 669, which he did take from the stocke againe at the end of 10 moneths. The second man laid in 810 pound, for eight moneths. The third layd in 900 pound, for seven moneths. And the fourth layd in 1040 pound, for 12 moneths.

Note. A generall Rule.

This question shall you examine as you did the other before, saving that whereas in the third place of the figure you did set each mans summe alone, heere you shall set the same being multiplied by the number of their time: and likewise in the first place of the figure you shall set the number which amounteth of their whole summes so multiplied by their time, and added into one whole sum, as thus.

The

The first mans summe is 6 6 9 pounds, which I multiply by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 pound, multiplyed by 8 (which was his time) maketh 6480. The third mans sum 900 pound, multiplyed by 7 (for that was his time) yeeldeth 6300. The fourth mans summe was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The foure summes thus multiplied by their time, must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all foure, which is 31950; and the gaine must be in the second place, which is 35145. Now to end the question, I say first, If 31950 did get 35145, what did 6690 get? Answer 7359 pounds as by this figure appeareth.

a

31950 — 35145

6690 — 7359

Likewise, the second man had to his part 7128 pound, the third must have 6930 pounds, and the fourth man shall have for his part 13728 pound as these figures do partly declare.

3.5145
3419
31726

669
810
900
1040
3419

b | c

$$31950 \quad 35145 \qquad 31950 \quad 35145$$
$$6480 \quad 7128 \qquad 6300 \quad 6930$$

d

$$31950 \quad 35145$$
$$12480 \quad 13728$$

Scholar. This I like very well: but what proofe is there of this worke?

Another proofe.

Master. The same that I taught you for the other: howbeit, there is used both for this work and the other also, this manner of proof, to adde all the portions together, and it may agree to the whole summe, then seemeth your worke well done: but this is no sure proofe.

Scholar. Yet will I prove in this example: The foure parcels are these, which if I adde together, there will amount 35145, and that was the whole summe, whereby I perceive the worke is well done.

7359
7128
6930
13728
35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then do I understand this work also very well: but what have I now to learn?

Master. There are many other excellent parts

parts behinde, of which I will not as now make mention, because that without the knowledge of Fractions they cannot bee duely taught, and much lesse understood. Therefore will I propose to you two or three questions more (that thereby you may better perceive the use of this Rule and all other the like) and so make an end for this time.

¶ *Three partners, by some ill adventure sustained the losse of* 160 pound, *whereof the first laid into the common stocke* 200 pound, *for tenne moneths. The second laid in* 350 pounds, *and the third* 100 pound, *but for how long the two latter, is unknowne. But breaking off their partnership, the first found himselfe a loser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is for how long time was the money of the two latter in company.* A question of losse.

For the solution hereof, and of such other like, you must also multiply the first mans 200 pound, that he put into the stocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme, if his mony that lost 80 pound, multiplyed by his time make 2000: what shall his mony make that lost 56 pound, and his that lost 24 pound, which two numbers I commit to the triall of the rule of Three, at two workings thus:

If 80 give 2000, what giveth 56? And againe, if 80 give 2000, what giveth 24?

80 2000 80 2000
56 1400 24 600

To conclude, if you now divide 1400, the second mans portion by 350, which was his stocke that he laid into company, you shall finde in your quotient 4 moneths, and for so long time did the second man put his money into the common stocke.

Lastly, if you divide the third mans new laying in, which was 600 by 100, which was his stocke that hee put into Company: the Quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolved.

Scholer. Sir, I have attentively beheld you working, and the more we travell herein, the more me thinke I am in love with this excellent Art.

Master. Then what say you to this Question?

A question of Canon.

There is in a Cathedrall Church 20 Canons, and 30 Vicars, those may spend by yeare 2600 pound but every Canon must have to his part five times so much as every Vicar hath: how much is every mans portion, say you?

Scholar. I pray you make the answer your selfe also, so shall I perceive best the meanes to answer to such other like.

Master. In this question, you must doe as in those beforesaid, that have diversity of

time,

ne, for here is diversity of portions. Therefore shall you multiply the number of the persons by their difference of portions: (as you did in the other by time:) Then must you multiply the 20, (which is the number of Canons) by 5, (for that is the number of their portion) so will it be 100. Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put these two summes together, and they make 130. Then say thus; If 130 spend 2600 pounds, what may 100 spend? The Rule sheweth 1000 pounds.

Againe for Vicars: If 130 spend 2600 pound; what may 30 spend? Answer, 600 pound, as these figures shew.

130	2600		130	3600
100	2000		30	600

But if every Canon should have so often times 4 pound, as the Vicar should have 3 pound, then should I multiply 20 by 4, (that were 80) and 30 by 3, (that were 90) and then both were 170. Then should the figures be set as followeth.

	li s d		li s d
170	2600	270	26000
80	1223-10,7	90	1376-9,5

But this sort is too hard for you, by reason

son of the fractions, therefore I will let it rest that place.

And by this rule you see what the 20 Canons may spend, which summe if you divide by 20, you shall see each Canons portion: and so of the Vicars, if you divide their summe by 30, the quotient will declare every Vicars portion.

The

The ſecond Dialogue.

The accounting by Counters.

Maſter.

NOW that you have learned the common kinde of Arithmeticke with the pen, you ſhall ſee the ſame Art in Counters: which feat doth not onely ſerve for them that cannot write and reade, but alſo for them that can doe both, but have not at ſometime thier pen or tables ready with them.

This ſort is in two formes commonly; The one by lines, and the other without lines. In that that hath lines, the lines do ſtand for the order of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many Counters as ſhall need for each line one; and they ſhall ſupply the ſtead of lines,

Scholar. By examples I ſhould better perceive your meaning.

Maſter. For example of the lines, loe here you ſee ſix lines, which ſtand for ſix places, ſo that the

——100000——
——10000——
*——1000——
——100——
——10——
——1——

umera-
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ounters.

nethermost standeth for the first place, and the next above it for the second, and so upward till you come to the highest, which is the sixth line, and standeth for the sixth place.

Now what is the value of every place or line you may perceive by the figure which I have set on them, which is according as you learned before in Numeration of figures by the pen, for the first place is the place of unites or ones, and every counter set in that line, betokeneth but one: and the second line is the place of 10, for every counter there standeth for 10: the third line the place of hundreds, the fourth of thousands, and so fourth.

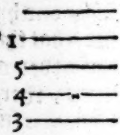
Scholar. Sir, I do perceive that the same order is here of lines, as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

Th
of
gu

Master. If you do understand it, then how will you set 1543?

Scholar. Thus as I suppose.

Master. You have set the places truly, but your figures be not meet for this use: for the meetest Figures in this behalfe, is the Figure of a counter round, as you see here, where I have expressed that same summe.



Scholar. So that you have not one Figure

for

for 2, nor 3, nor 4, and so forth, but as many Digits as you have, so many counters you set in the lowest line, and for every 10 you set one in the second line, and so of other. But I know not by what reason you set that one Counter for 500 betweene two lines.

Master. You shall remember this, that whensoever you need to set downe 5, 50, or 500, or 5000, or set forth any number whose Numerator is 5, you shall set one counter for it in the next place above the line that it hath his denomioation of: as in this example of that 500, because the numerator is 5, it must be set in a voyd space, and because the Denomination is a hundred, I know that the place is the void place next above hundreds, that is to say, above the third line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe betweene 4 and 10, for the first part of that number you shall set downe 5, and then so many counters more, as there rest numbers above, 5. And this is true both of Digits and Articles. And for example, I will set downe this summe 297965, which summe if you marke well, you need none other examples for to learne the numeration of this forme.

But this shall you marke, that as you did in other kindes of Arithmeticke, set a pricke in the places of thousands, in this worke you shall set a Starre, as you see before.

Schollar. Then I perceive Numeration: But I pray you how shall I do in this Art to adde two summes or more together?

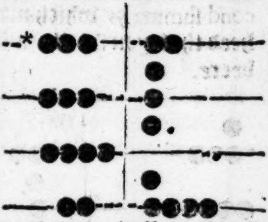
Addition.

Addition.

Master.

THe easiest way in this is to adde but two summes at once together: Howbeit you may adde more, as I will tell you anon.

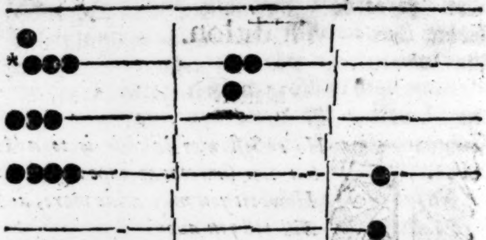
Therefore when you will adde two summes, you shall first set downe one of them, it forceth not which, and then by it draw a line crosse the other lines. And afterward set downe the other summe, so that the line may be betweene them: as if you would adde 2659 to 8342, you set your summes as you see here.



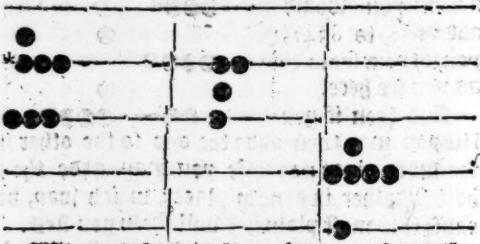
Addition of two summes

And then if you list you may then adde the one to the other in the same place: or else you may adde them both together in a new place: which way, because it is most plaine, I will shew you first.

Therefore will I beginne at the Vnites, which in the first summe is but 2, and in the second summe 9, that maketh 11. Those doe I take up, and for them I set 11 in the new roome thus.

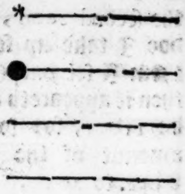


Then do I take up all the Articles under a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90: or you may say better, that in the first summe, there are foure Articles of 10, and the second summe 5, which maketh 9, but then take heed that you set them in their right lines, see heere.



Where I have taken away 40 from the first summe, and 50 from the second, and in their stead I have set 90 in the third roome, which I have set plainly, that you might well

well perceive: howbeit seeing that 90 with the 10 that was in the third room, already, doth make 100, I might better for those 6 Counters set 1 in the third line thus:



For it is all in one sum, as you may see, but it is best never to set five Counters in any line, for that may be done with one counter in a higher place.

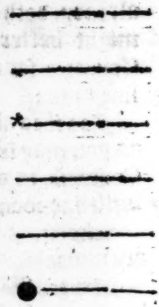
Scholar, I judge that good reason, for many are undeedfull where one will serve.

Master. Well, then will I adde forth of hundreds: I finde 3 in the first summe, and 6 in the second, which maketh 6000, them do I take up, and set in the third roome, where is 100 already, to which I put 900, and it will be 1000: therefore I set one counter in the fourth line for them all, as you see here.

Then adde I the thousands together, which in the first summe are 8000, and in the

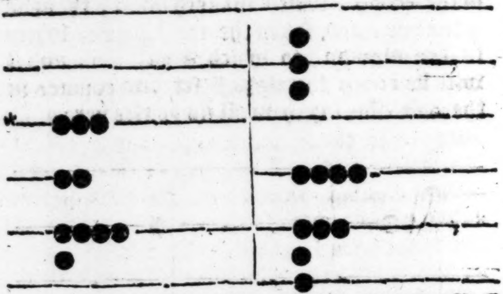
the ſecond 2000, that maketh 10000, them doe I take up for thoſe two places, and for them I ſet one Counter in the fifth line, and then it appeareth as you ſee to be 11001, for ſo many doth amount of the Addition of 8342 to 2659.

he adde ummes gether.



Scholar. Sir, this I doe perceive: but how ſhall I ſet one ſumme to another, not changing them to a third place?

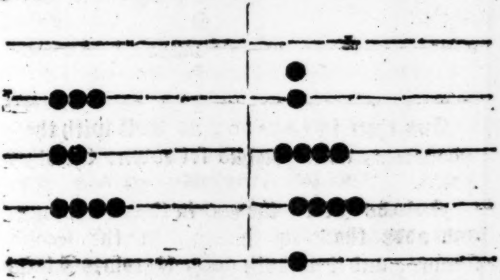
Master. Marke well how I doe it. I will adde together 65436 and 3245, which firſt I ſet downe thus:



Then doe I beginne with the ſmalleſt Denomination, which is 1 in the ſecond ſumme, and ſet it in his place: then doe I finde 5 in the firſt ſumme, and 5 in the ſecond, which put together, ſaving the two Counters, cannot

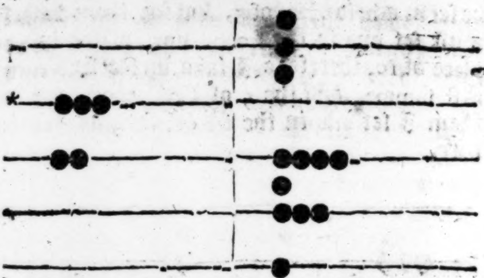
be

be set in a void place of 5, but for them both I must set one in the second line, which is the place of 10, therefore I take up the five of the first summe, and the 5 of the second, and for them I set one in the second line, as you see here.



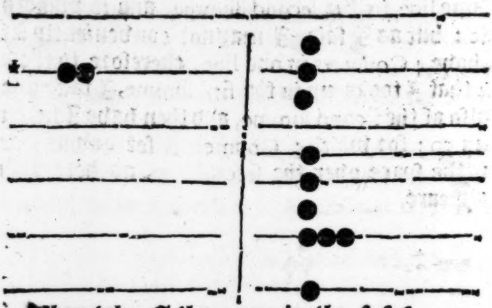
Then doe I likewise take the 4 Counters of the first summe and second line, (which maketh 40) and adde them to the 4 Counters of the same line in the second summe, and it maketh 80: but as I said, I may not conveniently set above 4 Counters in one line, therefore to those 4 that I tooke up in the first summe, I take one also of the second summe, and then have I taken up 50: for which 5 Counters I set downe one in the space over the second line, as here doth appeare.

And



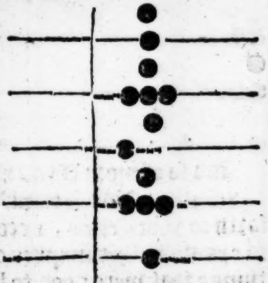
And then is there 80, as well with thoſe 4 counters, as if you had ſet downe the other 4 alſo.

Now do I take the 200 in the firſt ſumme, and adde them to the 400 in the ſecond ſumme, and it maketh 600, therefore I take up the two counters in the firſt ſumme and three of them in the ſecond ſumme and for them 5, I ſet 1 in the ſpace above, thus:



Then take I the 3000 in the firſt ſum me, unto

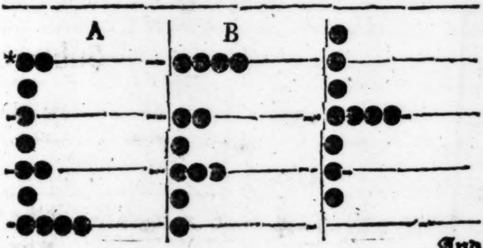
unto which there are none in the second summe agreeing, therefore I do onely remove those three counters from the first summe into the second, as here doth appeare.



And you see the whole summe that amounteth of that Addition of 65436 with 3245, to bee 68681.

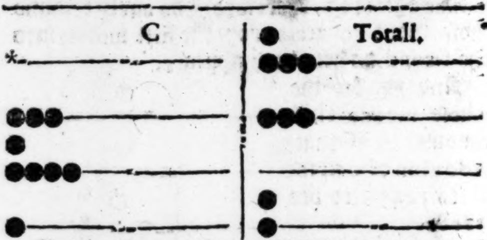
And if you have marked those two examples well, you need no further instruction in Addition of 2 onely summes: but if you have more then two summes to adde, you may adde them thus:

First adde two of them, and then adde the third and fourth, or more, if there bee so many: as if I would adde 26795, with 4286, and 1391. First I adde the two first summes thus:



And

And then I adde the third thereto thus.



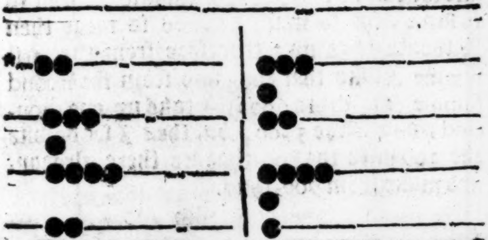
And so of more if you have them.

Schollar. Now I think it best that you passe forth to Subtraction, except there be any way to examine this manner of Addition, then I thinke that were good to be knowne next.

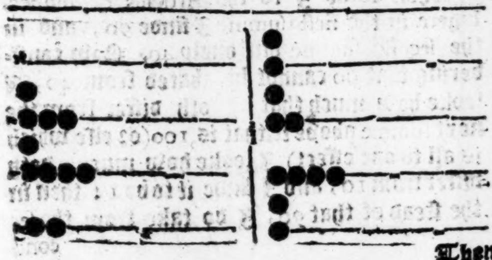
Master. There is the same proofe here that is in the other Addition by the pen, I meane Subtraction; for that onely is a sure way, but considering that Subtraction must be first knowne, I will first teach you the Art of Subtraction, and that by this example.

Subtraction.

I *Would subtract* 2892 *out of* 8746. *These summes must I set downe as I did in Addition: but here it is best to set the lesser number first, thus:*



Then shall I begin to subtract the greatest numbers first (contrary to the use of the pen) that is the thousand in this example: therefore I find amongst the thousands 2, for which I withdraw so many from the second summe (where are 8) and so remaineth there 6, as this example sheweth.

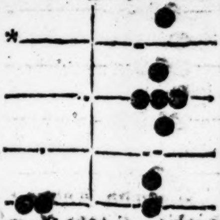


Then do I likewise with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I cannot take 2, therefore this must I do: I must looke how much my summe differeth from 10, which I find here to be 2, then must I abate for my summe of 800, one thousand and set downe the excesse of hundreds, that is to say 2, for so much as 1000 is more then I should take up: therefore from the first summe I take that 800, and from the second summe (which are 6000) I take up one thousand, and leave 5000, but then I set downe the 200 unto the 700 that are there already, and make them 900, thus.

Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot be abated from 40, I looke how much that 90 doth differ from the next summe above it, that is, 100 (or else which is all to one effect) I looke how much 9 doth differ from 10, and I finde it to be 1: then in the stead of that 90, I do take from the se-

cond

cond summe 100: but considering that is 10 too much, I set downe 1 in the next line beneath for it, as you see here.



Having that here I have set 1 Counter in the space in stead of 5 in the next line.

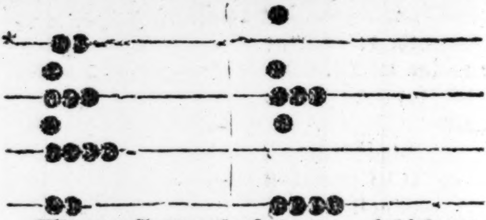
And thus have I subtracted all save 2, which I must abate from 6 in the second summe, and there will remain 4, thus:

So that if I subtract 2892 from 8746 the remainer will bee 5854.

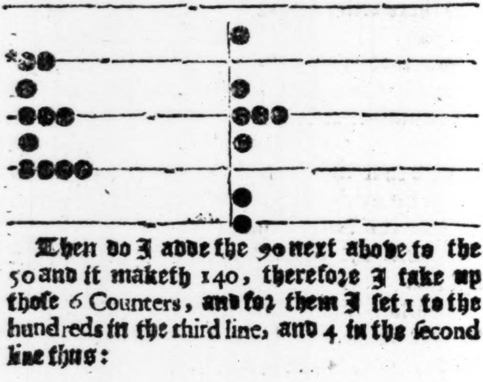
And that this is truly wrought, you may prove by Addition: for if you adde to this remainer the same summe that you did subtract, then will the former summe 8746 amount againe.

Scholar. That will I prove, and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus:

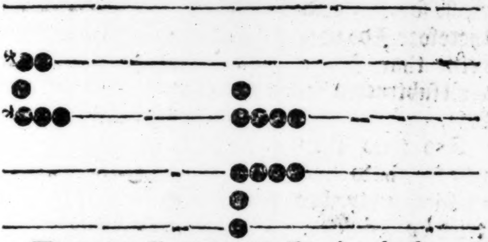
A proofe of subtraction.



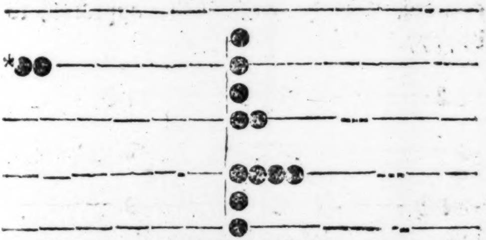
Then do I adde the first 2 to 4, which maketh 6: so take I up 5 of those Counters, and in their stead I set 1 in the space: and one in the lowest line, as here appeareth.



Then do I adde the 90 next above to the 50 and it maketh 140, therefore I take up those 6 Counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus:

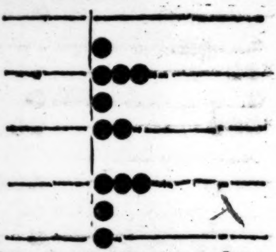


Then do I come to the hundreds, of which I finde 8 in the firſt ſumme, and 8 in the ſecond, that maketh 1600, therefore I take up thoſe 8 Counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and in the third line, as you may ſee here.



Then is there left in the firſt ſumme but onely 2000, and in the ſecond 5000, which is 7000, which I ſhall take up from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already: and there will the whole ſumme appeare as you may well ſee, to bee 8746, which was the firſt

groſſe ſumme, and therefore I do perceiue that I had well ſubtracted before.



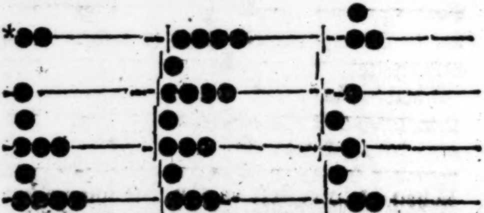
And thus may you ſee, how Subtraction may bee tried by Addition.

Scholar. I perceiue the ſame order here with Counters, that I learned before in figures.

Maſter. Then let me ſee how you can try Addition by Subtraction.

Schollar. Firſt I will ſet forth this example of Addition, where I haue added 2189 to 4988. And the whole ſumme appeareth to be 7177.

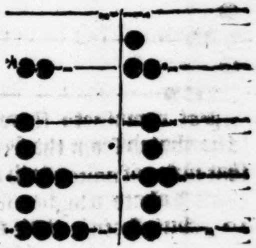
Proofe of Addition by ſubtraction.



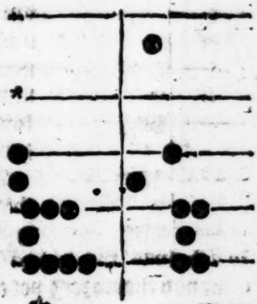
Now to try whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I haue well done, the remainer will be like that other ſumme: as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in order.

Then

Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, and then remaineth there 5000, thus.



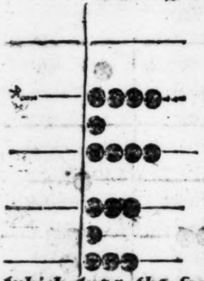
Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line, reſteth nothing, as you may ſee in this example following.



Then in the ſecond line with his ſpace over him I finde 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70, I muſt take it out of ſome higher ſumme, which is here onely 5000:

therefore I take up 5000: and ſeeing that is too much by 4920, I ſet downe ſo many in the ſecond roome; which with the 70 being there already, do make 4990 and then the ſummes do ſtand thus.

Yet remaineth therein the firſt ſumme 9, to be abated from the ſecond ſumme, wherein that place of unites doth appeare onely 7: then muſt I abate a higher ſumme, that is to ſay 10, but ſeeing that 10 is more then 9,

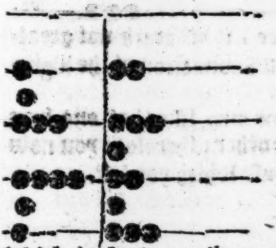


(which I ſhould abate) by 1, therefore ſhall I take up one Counter from the ſecond, and ſet downe the ſame in the firſt line, or lowermoſt line, as you ſee here.

And ſo have I ended this work, and the ſumme appeareth to be the ſame which was the ſecond ſumme of mine Addition, and therfore I perceive I have wel done.

Maſter,

Master. To stand longer about this, it is but folly: except that this you may also understand, that many do begin to subtract with Counters, not at the highest summe, as I have taught you, but at the nethermost, as they do use to adde; and when the summe to be abated in any line appeareth greater then the other, then do they borrow out of the next higher roome, as for example. Another way of Addition.



I should abate 1846 from 2378 they set the same thus:

First they take 6 which is the lower line, and his space from 8 in the same roomes in the second summe, and yet there remaineth two Counters in the lowest line. Then in the second line must 4 be subtracted from 7, and so remaineth there 3. Then 800 in the third line, and his space, from 300 of the second summe cannot be, therefore do they abate it from a higher roome, that is, from 1000, & because 1000 is too much by 200 teerefore must I set downe 200 in the third line, after I have taken up 1000 from the forth line. Then is there yet 1000 in the fourth line of the first summe which if I withdraw from the second sum, then do all Figures stand in order, thus: 532.

So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may use which you list.

Multi-

Multiplication.

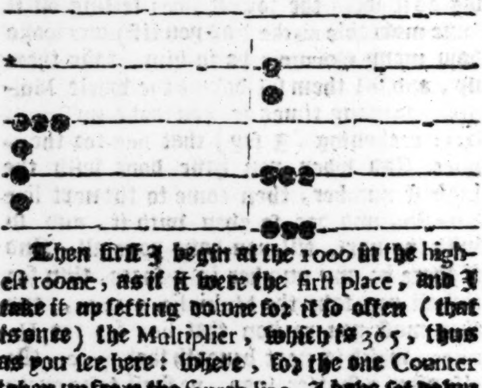
But now touching Multiplication: you shall set your numbers into two roomes (as you did in those other kinds) but so that the multiplier be set in the first roome: then shall you begin with the higvest numbers of the second roome, and multiply them first after this sort.

Take the overmost line in your first working as it were the lowest line; setting on it some moveable marke (as you list) and looke how many Counters be in him, take them up, and for them set downe the whole Multiplier so many times as you tooke up Counters: reckoning (I say) that line for the unites. And when you have done with the highest number, then come to the next line beneath, and doe so even with it, and so with the next, till you have done all. And if there be any number in a space, then for it shall you take the Multiplier 5 times, and then must you reckon that line for the Unites, which is next beneath that space. Or else after a shorter way, ye shall take onely halfe the Multiplier, but then shall you take the line next above the space for the line of unites. But in each working, if by chance your Multiplier be an odde number, so that you

you cannot take the halfe of it justly, then must you take the greater halfe, and set downe that, as if that it were the just halfe: and further, you shall set one Counter in the space betweene that line, which you reckon for the line of unites, or else onely remove forward the same that is to be multiplied.

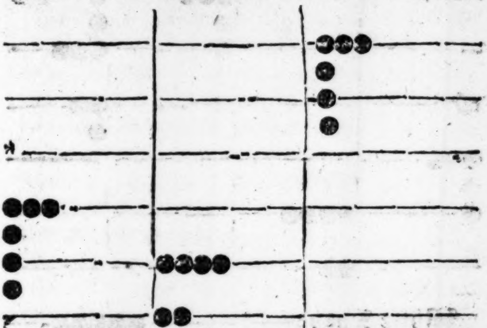
Scholar. If you set forth an example hereof, I thinke I shall perceive you.

Master. Take this example: I would multiply 1542 by 365, therefore I set my numbers thus.



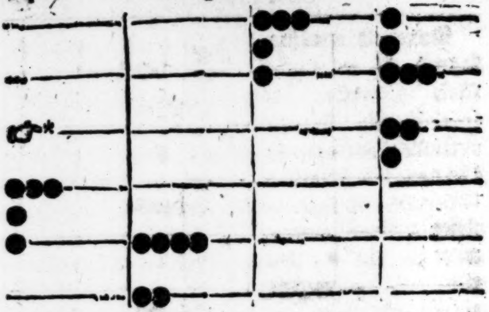
Then first I begin at the 1000 in the highest roome, as if it were the first place, and I take it up setting downe for it so often (that is once) the Multiplier, which is 365, thus as you see here: where, for the one Counter taken up from the fourth line, I have set downe other five which make the summe of the multiplier, reckoning the fourth line, as if it were the first, which thing I have marked by the starre set at the beginning.

Scholar.

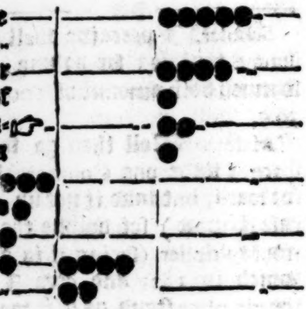


Scholar. I perceive well, for indeed this ſumme that you ſet downe, is 365000: for ſo much doth amount of 1000, multiplyed by 365.

Maſter. Well then go forth, in the next ſpace I finde one Counter, which I remove forward, but take it not up, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my Multiplier (ſeeing it is an odde number) which is 182, and here I do ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſee.

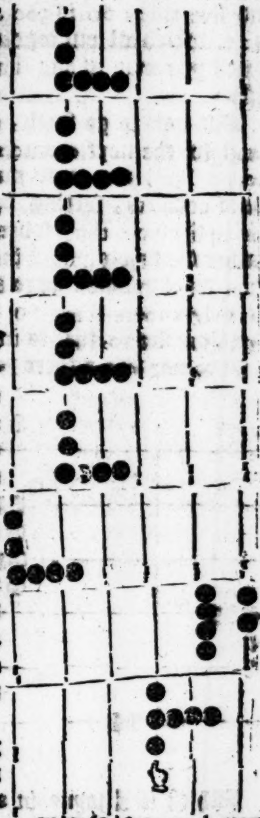


Where I haue set the Multiplication with other but for the ease of your understanding, I haue set a litle line betweene them. Now should they both in one summe stand thus.



Another forme of Multiplication.

Howbeit another forme to multiply such Counters in space, is this: first to remove the finger to the next line beneath the space, and then to take up the Counter, and to set downe the Multiplier five times, as here you see.

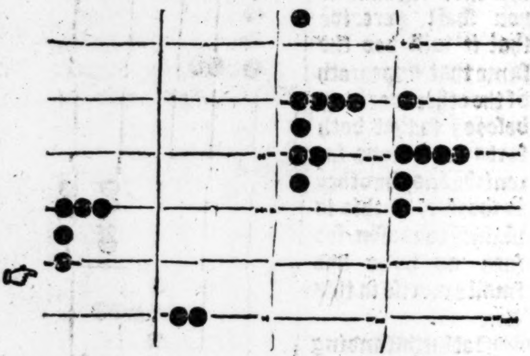


Which summes if you doe adde together into one summe you shall perceive that it will bee the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter, so this is plainer to reason, for such as have had small exercise in this Art.

Notwithstanding you may adde them in your minde before you set them downe: as in this example you might

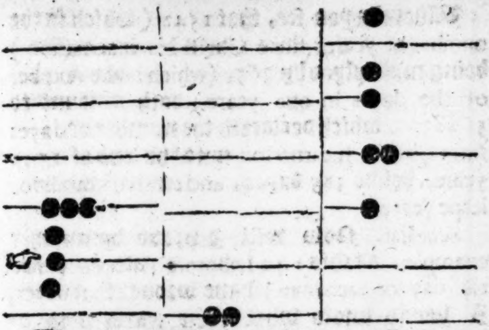
might haue said, five times 300 is 1500, and five times 60 is 300, also five times five is 25. which all put together, do make 1825, which you may at one time set downe if you list.

But now to go forth, I must remoue the hand to the next counters, which are in the second line, and there must I take up those foure counters, setting downe for them my multiplier foure times seuerally, or else I may gather the whole summe in my mind first; and then set it downe: as to say, foure times 300 is 1200: foure times 60 are 240: and foure times 5 make 20, that is in all 1460: that shall I set downe also, as here you see.

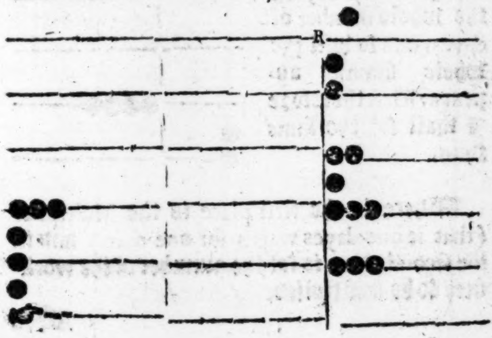


Which if I ioyne in one summe with the former numbers, it will appeare thus.

Then



Then to end this Multiplication, I remove the finger to the lowest line where are onely 2, then do I take up, and in their stead do I set downe twice 365, that is 730, for which I set one in the space above the third line for 500, and 2 more in the third line with that one that is there already, and the rest in their order, and so have I well ended the whole summe thus:



Whereby you see, that 1542 (which is the number of yeares since Christ his incarnation) being multiplyed by 365, (which is the number of the dayes in one yeare) doth amount to 562830, which declareth the number of dayes since Christs Incarnation unto the end of 1542 yeares, beside 385 dayes, and twelve houres for leape yeare.

Example of wages.

Schollar. Now will I prove by another example, as this: 40 Laborers (after 6 pence the day for each man) have wrought 28 dayes, I would know what their wages doth amount unto.

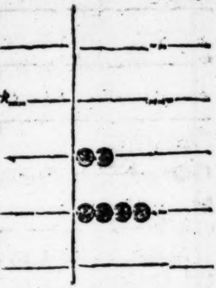
In this case must I worke doubly: first I must multiply the number of the Laborers, by the wages of a man for one day, so will the charge of every day amount.

Then secondly shall I multiply the charge of one day by the whole number of dayes, and so will the whole summe appeare: First therefore I shall set the sums thus.

Where in the first place is the Multiplier (that is one dayes wages for one man) and in the second place is set the number of the workmen to be multiplied,

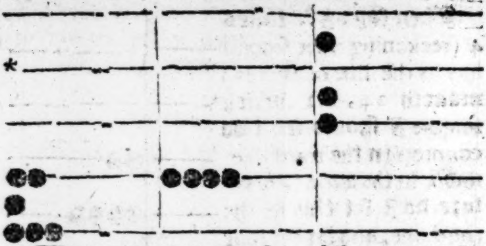
Then

Then ſay: If 6 times 4 (reckoning that ſecond line as the line of unites) maketh 24, for which* ſumme I ſhould ſet two counters in the third line, and 4 in the ſecond, therefore do I ſet two in the third line, and let the four ſtand ſtill in the ſecond line thus.

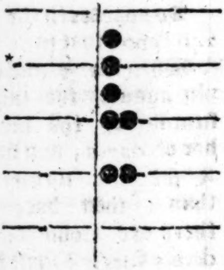


So appeareth the whole dayes wages to be 240 pence, that is 20 ſhillings.

Then do I multiply againe the ſame ſumme by the number of dayes, and firſt* I ſet the numbers thus: then becauſe there are Counters in divers lines, I ſhall begin with the higheſt, and take them up, ſetting for them the Multiplier ſo many times as I tooke up counters, that is twice, then will the ſumme ſtand thus.



Then come I to the second line, and take up those 4 Counters setting for them the Multiplier 4 times, so will the whole summe appeare thus:



So is the whole wages of 40 workemen for 28 dayes after 6 pence each day for a man, 6720 pence, that is, 560 shillings or 28 pound:

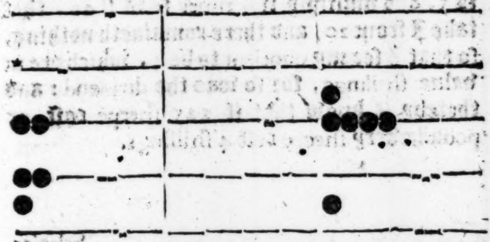
Master. Now if you would prove Multiplication: the surest way is by Division: therefore will I overpasse it till I have taught you the Art of Division, which you shall worke thus.

Division

Division.

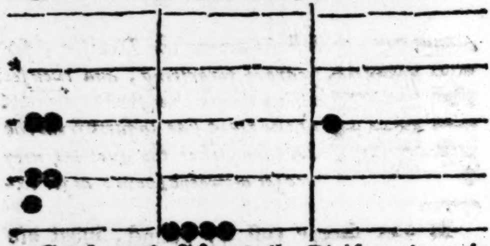
FIrst set downe the Divisor, for feare of forgetting, and then set that number that shall be divided at the right side so farre from the Divisor, that the quotient may bee set betweene them: as for example.

If 225 sheepe cost 45 pound, what did every sheepe cost? To know this, I would divide the whole summe, that is 45 pound, by 225, but that cannot be: therefore must I first reduce that 45 pound, into a lesser denomination, as into shillings, then I multiply 45 by 20, and it is 900; that summe shall I divide by the number of sheepe, which is 225, these two numbers therefore I set thus: An example of sheepe.



Then begin I at the highest line of the dividend, and seeke how oft I may have the

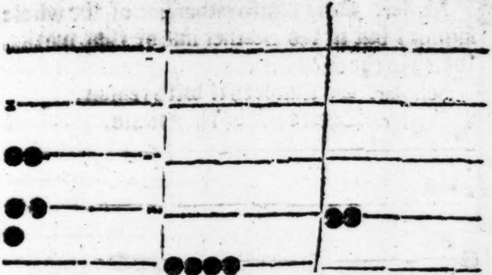
Diviſion therein, and that I may do foure times: then ſay I, foure times 2 are 8, which if I take from 9, there reſteth but 1, thus:



And becauſe I found the Diviſor 4 times in the dividend, I have ſet, as you ſee, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of the Diviſor as often out of the remainder: therefore come I to the ſecond line of the diviſor, ſaying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

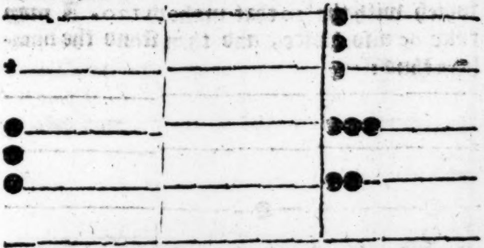
Then come I to the loweſt number, which is 5, and multiply it 4 times ſo is it 20, that take I from 20, and there remaineth nothing, ſo that I ſee my quotient to be 4, which are in value ſhillings, for ſo was the dividend: and thereby I know that if 225 ſheepe coſt 45 pound, every ſheepe coſt 4 ſhillings.

Scholar.



Scholar. This can I do as you shall perceive by the example. If 160 Souldiers do spend every moneth 68 pound, what spendeth each man? Examples of soldiers wages.

First, because I cannot divide the 68 by 160, therefore I will turne the pounds into pence by Multiplication, so shall there be 16320 pence: now must I divide the summe by the number of Souldiers, therefore I set them in order thus:



Then begin I at the highest place of the dividend, seeking my Division there, which I finde once, therefore I set 1 in the nether line.

Master. Not in the neather line of the whole summe, but in the neather line of that worke, which is the third line.

Scholar. So standeth it with reason.

Master. Then thus do they stand.

Then seeke I againe the rest, how often I may finde my divisor: and I see that in 300 I might finde 100 three times: but then the 60 will not be so often found in 20, therefore I take 2 for my quotient, then take I 100 twice from 200, and there resteth 100, out of which with the 20 that maketh 120, I may take 60 also twice, and then stand the numbers thus:

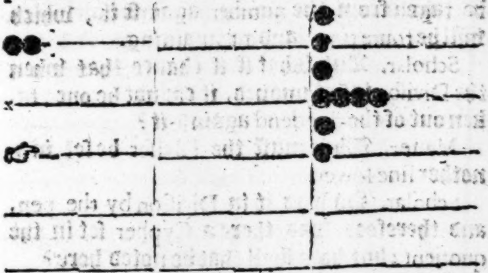
Where I have set the quotient 2 in the lowest line: so is every Souldiers portion 102 pence, that is 8 shillings 6 pence.

Master. But yet because you may justly perceive the reason of Division, it shall be good that you set your Divisor still against those numbers from which you do take it, as by this example I will declare.

If the purchase of 200 acres of ground did cost 260 pound, what did one acre cost?

First will I turne the pounds into pence, so will there be 69600 pence. Then in setting down these numbers, I shall do this: An example of purchase.

First set the dividend on the right hand as it ought, and then the divisor on the left hand against those numbers from which I intend to take him first, as here you see where I have set the divisor two lines higher then his owne place.

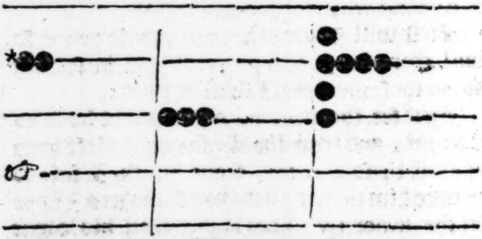


Scholar. This is like the order of Division by the pen.

Master. Truth you say, and now must I set

ſet the quotient of this worke in the third line, for that is the line of unites in reſpect of the Diviſor in this worke.

Then I ſee how often the Diviſor may bee found in the dividend, and that I find 3 times, then ſet I 3 in the third line for the quotient: and take away that 60000 from the dividend and further I ſet the Diviſor one line lower: as you ſee here.



And then ſecke I how often the Diviſor will be taken from the number againſt it, which will be foure times and 1 remaining.

Scholar. But what if it chance that when the Diviſor is ſo removed, it cannot be once taken out of the dividend againſt it?

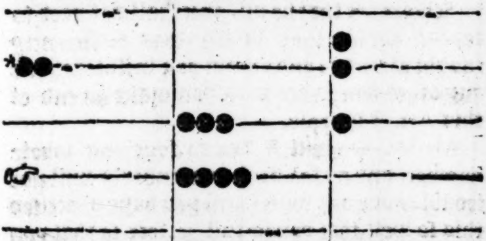
Maſter. Then muſt the Diviſor be ſet in another line lower.

Scholar. So was it in Diviſion by the pen, and therefore was there a Cypher ſet in the quotient; but how ſhall that be noted here?

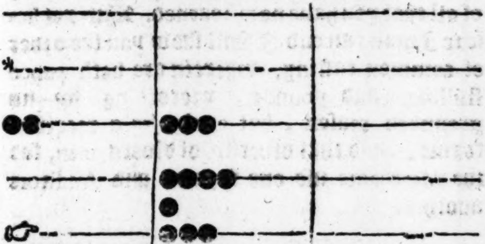
Maſter. There needeth no token, for the lines do repreſent the places, onely looke that you ſet your Quotient in that place which ſtan-

deth

deth for unites in respect of the Divisor. But now to returne to the example: I finde the Divisor foure times in the dividend, and 1 remaining: for 4 times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the Quotient, I set 4 in the second line, which is in this worke the place of unites.



Then remove I the Divisor to the next lower line, and seeke how often I may have it in the Dividend, which I may do here 8 times just, and nothing remaine, as in this forme.



Where you may see that the whole quotient is 348 pence, that is 29 shillings, whereby I

know

know that so much cost the purchase of one Acre.

Scholar. Now resteth the proofs of Multiplication, and also Division.

Master. Their best proofs are each one by the other, for Multiplication is proved by Division, and Division by Multiplication, as in the work by the pen you learned.

Scholar. If that be all, you shall not need to repeat againe that which was sufficiently taught already; and except you will teach me any other feat, here may you make an end of this Art, I suppose.

The reason of all the former rules.

Master. So will I doe as touching whole number, and as for broken number I will not trouble your wit with it, till you have practised this so well, that you be full perfect, so that you need not to doubt in any point that I have taught you, and then may I boldly instruct you in the Art of Fractions or broken numbers: wherein I will also shew you the reasons of all that you have now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence shillings and pounds, proceeding by no grounded reason, but onely by a received forme, and that diversly, of divers men, for the Merchants use one forme, and Auditors another.

Merchants

Merchants use.

BVt first for Merchants forme, marke this example here, in which I have expressed this summe 198 pounds, 19 shillings, 11 pence. So that you may see that the lowest line serveth for pence, the next above for shillings, the third for pounds, and the fourth for scores of pounds.

Merchants Accompt.

And further you may see that the space betweene pence and shillings may receive but one Counter (as all other spaces likewise do) and that one standeth in that place for 6 pence.

Likewise betweene the shillings and the pounds one counter stands for 10 shillings.

And betweene the pounds and 20 pounds, one counter standeth for 10 pounds.

But beside those, you many see at the left side of shillings, that one number standeth alone, and betokeneth 5 shillings.

So against the pounds, that one Counter standeth for 5 pound. And against the 20 pounds, the one Counter standeth for five score pounds, that is, 100 pounds: so that every side Counter is five times so much as one of them against which he standeth.

Auditors

Auditors Accompt.

Auditors Accompt.

NOw for the Accompt of Auditors, take this example.

Where I have expressed the same summe 198 pound——19 shillings——11 pence.

But here you see the pence stand towards the right hand, and the other increasing orderly towards the left hand.

Againe you may see, that Auditors will make two lines (yea and more) for pence, shillings, and all other values, if their summes extend thereto. Also you see that they set one Counter at the right end of each row, which so set there standeth for five of that roome, and on the left corner of the row it standeth for 10 of the same row.

But now if you would adde or subtract after any of both those sorts, if you marke the order of the other feate which I taught you, you may easily do the same here without much teaching: for in Addition, you must first set downe one summe, and to the same set the other orderly, and in like manner, if you have many but in Subtraction, you must set downe first the greatest summe, and from it must you

abate

abate the other, every Denomination from his due place.

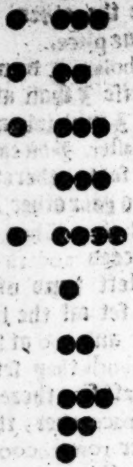
Scholar. I do not doubt but with a little practise I shall attaine these both: but how shall I multiply and divide after these formes?

Master. You cannot duely do any of both by these sorts, therefore in such case you must resort to your other Arts.

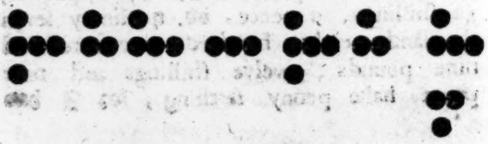
Scholar. They that use such accounts that it exceed 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand thereof, and the thousands they set in a farther row yet, and at the left side thereof they set the 5000, and in the space over, they set the 10000, and in a higher row 20000, which all I have expressed in this example, which is 97869 pounds, 12 shillings, 9 pence, ob. q Ninety seven thousand, eight hundred threescore and nine pounds, twelve shillings and nine pence halfe penny farthing, for I had

not

not told you before, where neither how you should set downe farthings, which (as you see here) must be set in a voide place sideling beneath the pence, for a farthing, one counter, ob. 2 counters, for ob, farthing 3 counters, and more there cannot be: for 4 farthings make 1 d, which must be set in his due place,



And if you desire the same summe after Auditors manner, loe here it is.



But in this thing you shall take this for sufficient, and the rest you shall observe as you may see by the working of each sort, for the divers wits of men have invented divers and sundrie wayes, almost innumerable.

THE

THE SECOND PART OF ARITHMETICKE, touching Fractions, briefely set forth.

Scholer.

ALbeit I perceive your manifold businesses doth so occupie or rather oppresse you, that you cannot as yet compleatly end the Treatise of Fractions Arithmeticall, which you have prepared, wherein not onely sundrie works of Geometry, Musicke, and Astronomy bee largely set forth, but also divers conclusions and naturall works, touching mixtures Arithmeticall fractions.

mixtures of Metals and compositions of medicides, with other strange examples. Yet in the meane season, I cannot stay my most earnest desire, but importunely crave of you some briefe preparation toward the use of Fractions, whereby at the least I may be able perfectly to understand the common works of them, and the vulgar use of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will is good, you should not need to use any importunate craving, for the attaining of that thing, whereby I may bee perswaded that I shall any wayes profit the Commonwealth, or helpe the honest studies of any good members in the same: wherefore while mine attendance will permit me to walk and talk, I am well willing to helpe you as I may.

Therefore first to begin with the explication of this name Fraction, what take you it to be?

What a Fraction is

Scholar. Marry sir, I think a Fraction (as I have heard it often named) to be a broken number, that is to say, to be no whole number but part of a number.

Master. A Fraction indeed is a broken number, and so consequently the part of another number: but that must be understood of such another number as cannot bee divided into any other parts then Fractions: for although I may take the third part of 60, or the

the fourth part of it, and so of other parts diversly, yet those parts be not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers, for the third part of it is 20, the fourth part is 15, the twelfth part is 5, and so forth of other parts, all which be whole numbers.

Wherefore properly a Fraction expresseth the parts or part onely of a unite, that is to say, that the number which is the whole or entire summe of any Fraction, may not bee greater then one: and therefore it followeth, that no one Fraction alone can bee so great, that it shall make 1, as by example I will declare, as soone as I have taught you to know the forme how a Fraction is expressed or represented in writing. What a fraction is properly.

T Nume-

Numeration.

The expressing of fractions.

BUt first to begin with expressing of a Fraction, which is the numeration of it: you must understand that a Fraction is represented by two numbers set one over the other, and a line drawne betweene them as thus, $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{10}{17}$, which foure fractions you must pronounce thus; $\frac{1}{3}$ one third part, $\frac{3}{4}$ three quarters, $\frac{2}{5}$ two fift parts, $\frac{10}{17}$ ten seventeene parts.

Scholar. I understand this forme of their expression and pronounciation: but their meaning or valuation seemeth more obscure. Yet I thinke that by the two first Fractions I understand the valuation of the two latter fractions, and consequently of other.

Master. Value them then, that I may perceive your taking of them.

Scholar. $\frac{2}{5}$ betokeneth two fift parts, that is to say, if one be divided into 5 parts, that fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing bee divided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare that one being divided into foure quarters, I must take (for this Fraction) three of

of those quarters.

If there be no more difficulty in their Numeration, then I pray you go forward to their Addition and Subtraction, and so to the other kinds of works. For I understand that the same kinds of works be in fractions, that be in whole numbers.

Master. There are the same kinds of works in both, albeit the order of them is divers, as I will anone declare; but yet more in Numeration before we leave it. You must understand that those two numbers which expresse a Fraction, have severall names, the overmost, which is above the line, is called the Numerator, and the other beneath the line, is called the Denominator. Numerator. Denominator.

Scholar. And what is the reason of their divers names? For (in mine opinion) both be Numerators, seeing both they do expresse the numeration of the Fraction.

Master. You are deceived: for one onely (which is the overmost) doth expresse the Numeration, and the Denominator doth declare the number of parts, into which the unite is divided, as in this example, when I say: divide a pound weight of Gold between foure men, so that the first man shall have $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now do you perceive that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should be divided into so many parts, I meane 15,

and by the foure ſeverall Numerators is limi- ted the divers portion that each man ſhould have, that is, that when the whole is parted into 15, the firſt man ſhall have two of thoſe 15 parts: the ſecond man three of them: the third man foure: and the fourth man ſix. And ſo may you ſee the ſeverall offices (as it were) of thoſe two numbers, I meane of the Nume- rator and the Denominator.

And hereby you perceive that a man can have no more parts of any thing then it was divided into, neither yet aptly ſo many: ſo that it were unaptly ſaid: You ſhall have $\frac{15}{15}$ that is 15, fifteene parts of any thing, ſeeing it were better ſaid, you ſhall have the whole thing.

Scholar. So doth it appeare reaſonably, for the labour is vaine to divide any thing, and then to apply the Diviſion to no uſe. And much leſſe reaſonable were it to ſay $\frac{16}{15}$: for if the whole be divided into 15 parts onely, it is not poſſible to take 16 of them, that is to ſay, more then all together.

Maſter. This is true touching the proper and apt uſe of the name of a Fraction: yet improperly (and after a vulgar acceptation for eaſineſſe in worke) both thoſe formes bee called Fractions, becauſe they be written like fractions, although they be none indeed: for $\frac{15}{15}$, and generally in ſuch other, where the Numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts: And ſo $\frac{17}{15}$ is not to be called a Fracti-

on,

on, but a mixt number, of a whole number and a Fraction, for it is as $1\frac{2}{12}$ that is one whole and $\frac{2}{12}$ parts, as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions, where the Numerator is either equall, or greater then the Denominator.

An improper fraction of a mixt number.

Scholer. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like fractions: but they needed not to call them fractions, but (as they bee) whole numbers or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions, or as improper fractions.

Now must you understand, that as no fraction properly can be greater then one, so in smalnesse under one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increase above one infinitely, so that not onely one may be divided into infinite fractions or parts, but also every fraction may be divided into infinite Fractions or parts, which commonly be called Fractions of Fractions, and they be expressed diversly: as for example, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is three quarters, of two third parts, of one halfe part. Whereby is signified, that if one be divided into two halfes, and the one halfe into three parts, and two of those

Fractions of Fractions.

 three

three parts be divided joyntly into foure quarters, this fraction of fractions doth repreſent three of thoſe quarters.

Scholar. I pray you let me prove by an example in common money, whether I doe rightly underſtand you or no. One Crowne which I take for an unite, doth containe 60 pence; therefore the halfe of it is 30 pence, $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fifteen pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crowne: and so is 3 pence $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a ſhilling.

Maſter. You perceive this well enough: yet this note I give you by the way, that the forme of expreſſing theſe fractions is voluntary, and hath no other reaſon than the will of the Diviſor, which forme many follow: for ſome expreſſe them thus $\frac{3}{4}\frac{2}{3}\frac{1}{2}$ without any figure of diſtinction between them, which forme alſo many follow. Some other do make lines betweene every fraction, and adde words of diſtinction, after this ſort, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, which forme is beſt.

Some other expreſſe them thus in ſlope forme, to diſtinct them from fractions of one whole numbers, for if they

$$\frac{3}{4}$$
$$\frac{2}{3}$$
$$\frac{1}{2}$$

were ſet in one right line thus, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, then ought it to be pronounced, three quarters and two third parts, and a halfe, which maketh almoſt two whole unites, lacking but one twelfth part. And ſo is it nothing agreeable with

with the other fraction of fractions: wherefore it is a great oversight in certaine learned men, which do expresse them so confusedly with such severall fractions, that a man cannot know the one from the other.

Therefore some men (as Stifelius*) do expresse without a line, numbers of proportion, being applied to* Addition *or* Subtraction, *because they must be taken as two, where the line in* fractions *maketh them to be taken for one: For of the* Numerator *and* Denominator *is made one number.*

Scholar. Then I perceive there be three severall varieties in fractions: first, when one onely fraction is set for one number, as $\frac{4}{5}$, that is, fourth fift parts. The second is, when there be set two or more severall fractions of one number, as $\frac{4}{9}$, $\frac{2}{5}$, that is, foure ninth parts, and two fifth parts. The third sort is fractions, of fractions, as $\frac{4}{9}$, of $\frac{2}{5}$, that is, foure ninth parts of two fifth parts. Three severall varieties.

Master. You have said well, if you understand well your owne words.

Scholar. If it shall please you I will by an example in the parts of an old English Angel, expresse my meaning.

Master. Let me heare you.

Scholar. The old English Angel did containe 7 shillings 6 pence, that is, 90 pence; Now $\frac{4}{5}$ of it is 72 pence. And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is, foure nine parts, and two fift parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that

is, foure nine parts of two fifth parts, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken foure times maketh 16.

Master. This is plainly expressed, and truly, and hereby (I doubt not) but you do perceive, that as great a difference, as is betweene 16 and 76, so much difference is betweene those two Fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you understand these varieties, I will proceed to the rest of the works; first, admonishing you, that there is another order to be followed in Fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Division, and Reduction: but in Fractions (to follow the same aptnesse in proceeding from the easiest works to the harder) we must use this order of works, Numeration, Reduction, Addition, Subtraction, Multiplication and Division.

The order of works in fraction.

Scholar. That Addition and Subtraction should go together, and Division to follow Multiplication, naturall order doth perswade; but why Reduction should be first in order here next to Numeration, and Addition, and Subtraction in the middle, I desire to understand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so go forward by degrees to the hardest:

hardest: even reason teacheth in Fraction the like order. And consider that Addition or Subtraction of Fractions can very seldome bee wrought without Reduction: and contrariwise, Reduction may be wrought without this form of Addition or Subtraction, therefore was it orderly required, that Reduction should goe before Addition and Subtraction, and this reason serveth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the easiest, I pray you declare the forme of it first by rule, and then by example.

Master. Your request is good.

Redu-

Reduction of Fractions.

Of Reduction of Fractions, there are five varieties.

THerefore will I now declare the diversities of Reduction of fractions, which commonly hath five varieties, or formes.

1 First, when there be sundrie Fractions of one intire unite; they must be reduced to one denomination, and also into one Fraction.

2 Secondly, when there be proponed fractions of fractions, they must be reduced likewise into one fraction: for otherwise they cannot be brought into one denomination.

3 Thirdly, when an improper fraction is proponed, that is to say, a fraction in form, which indeed, is greater then an unite: it must bee reduced into apt forme, expressing the unite or unites of it, and the proper fraction distinctly. And sometimes also it shall be needfull to convert such a mixt number of unites with fractions, into the forme of a Fraction, that is, into an improper fraction: which two forms I esteeme but as one, because they worke one kinde of number.

4 Fourthly, there happeneth sometimes fractions to be written in great numbers, which might bee written in lesser numbers: therefore is there a mean to reduce such great numbers into their smallest termes.

5 Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine

certaine parts, but none of like denomination with that Fraction, then may you reduce the said fractions into another, whose denomination shall expresse the common parts of that whole thing.

Scholar. This distinction in Doctrine delighteth me much, but more with hope then present fruit: for as yet I do not understand scarcely the varieties, and much lesse the practise and use of their works.

Master. Reduction is an orderly alteration of numbers out of one forme into another, which is never done orderly but for some needfull use, as in every of the said five severall formes, I will distinctly declare.

First therefore, when two, or more severall fractions of any unite be proponed; as for example $\frac{3}{16}$ and $\frac{4}{6}$, because it is hard to tell what proportion of the intire number those two fractions do expresse, therefore was Reduction divised, to be a meane whereby these severall fractions might be brought into one denomination and fraction. The first forme of Reduction.

And in these fractions, this is the Art for bringing them to one denomination.

Multiply first the denominators together, and the totall thereof you shall set twice down under two severall lines for two new denominators, or rather for one common Denominator. Then multiply the Numerator of the first fraction, by the denominator of the second, and set the totall thereof for the Numerator over the first line. Likewise multiply the Numerator of the second fraction by the denomina- How to reduce fractions of divers denominations into one denomination.

tor

ter of the first, and set that total over the second line for the Numerator of that fraction: and so are these two first fractions of severall denominations, brought to one denomination.

Scholar. If I understand you, as I thinke I do, my example shall declare the same. The fractions which you proponed were these, $\frac{3}{16}$, and $\frac{4}{6}$, whose Denominators (being 16 and 6) I multiply together, and there amounteth 96, which I set under two lines thus: $\frac{}{96}\ \frac{}{96}$.

Then I multiply the Numerator of the first fraction by the Denominator of the second saying, 3 into 6 maketh 18, that I set over the first line for a new Numerator, and it will be thus $\frac{18}{96}$.

Likewise I multiply the Numerator of the second fraction by the Denominator of the first saying 4 times 16 maketh 64, that I set for the second Numerator, and the fraction will appeare thus, $\frac{64}{96}$.

So that both Fractions brought to one denomination, must stand thus $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar. I beseech you let me examine it after my accustomed forme, by common parts of coyne or other measure.

Master. Do so.

Scholar. I have a peece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the sixteenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that

is

is $\frac{48}{768}$. Againe $\frac{1}{?}$ of the same peece of Gold is 16 pence, so that $\frac{4}{?}$ parts maketh 64 pence, that is $\frac{64}{96}$. And so I finde the summes to agree with the other before.

Master. *So have you now the Art to bring two such fractions into one denomination, And if there bee more then two, then must you multiply all the Denominators together, and set the totall thereof so many times downe as there bee fractions; and then to get for each one a new Numerator, multiply the Numerator of the* first, *by the Denominator of the* second, *and the totall thereof multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the third into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth if there were moe. So these three Fractions* $\frac{2}{5}\ \frac{3}{4}\ \frac{2}{3}$ *do make by Reduction these other three fractions of Denomination* $\frac{24}{60}$, $\frac{45}{60}$, $\frac{40}{60}$. *All which you may bring into one Fraction by adding the Numerators together, and putting the totall for the totall Numerator, reserving still that same common Denominator. And those three Fractions make one improper Fraction, thus:* $\frac{109}{60}$.

Note the Reduction of three Fractions or more, into one.

Scholar. All this I perceive, and also that this last Fraction is more then an unite, and therefore you did call it an improper Fraction.

Master.

Master. There be certaine other formes of working in this Reduction, which I will briefly touch also, to give you an occasion to excercise your wit therein.

The first variety of Reduction.

The first variety is this: When you have made and written down you common Denominator (as I have taught before) then to get a Numerator for the first, do thus: Divide the common denominator by the denominator of the first fraction, and the quotient multiplied by the Numerator of the same, yeeldeth a new Numerator for the first new fraction. So likewise doe with the second and the third, and with all the residue, if there be more.

Scholar. That will I prove in your last example of these three fractions $\frac{2}{5}\ \frac{3}{4}\ \frac{2}{3}$. When the Denominators be multiplied, they make 60: for 5 into 4 maketh 20, and 20 by three yeeldeth 60, that I set down three times thus: $\frac{}{60}, \frac{}{60}, \frac{}{60}$, then to have a Numerator, for the first, I must divide 60 by 5 (the Denominator of the first) and the quotient is 12, which I must multiply by 2 (the Numerator first) and that maketh 24 (and so have I for the first fraction $\frac{24}{60}$.

Likewise for the second fraction: I divide 60 by 4, and there commeth 15, which I multiply by 3, and so have I 45, for the second fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{40}{60}$.

The second variety.

Master. *Another way is this: If it happen so, that the lesser denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator over that lesser denominator.*

tor, & for the lesser denominator put the greater, as thus in these two Fractions $\frac{3}{12}$ *and* $\frac{2}{3}$, *three being the lesser denominator multiplied by* 4, *will make* 12, *which is the greater denominator : therefore by the same* 4 *I do multiply* 2 *which is the Numerator over* 3, *and that maketh* 8 : *under which I do put* 12, *being the greater denominator, which is also made by multiplication of* 4 *into* 3, *and so have I these two fractions* $\frac{3}{12}$, $\frac{8}{12}$, *thus shortly reduced, without altering the one fraction.*

Scholar. This I understand.

Master. *Then marke this third way : If the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be parts of any other one number, as in* $\frac{5}{12}$, *and* $\frac{7}{18}$, *although the lesser taken but twice, be too much to make* 18, *yet they both may be parts unto* 36, *therefore looke how many times twelve is in* 36, *and that quotient being multiplied by the Numerator over* 12, *the totall shall be put in stead of the Numerator over* 12, *and for* 5 *put* 15, *thus* $\frac{15}{36}$. *So likewise looke how often is* 18 *in* 36, *because it is twice, therefore by* 2 *multiply* 7, *which is over* 18, *and it will be* 14 : *set that for the Numerator, and in stead of* 18 *put* 36 ; *and then your Fractions reduced stand thus,* $\frac{15}{36}$, $\frac{14}{36}$, *in stead of* $\frac{5}{12}$ *and* $\frac{7}{18}$. The third variety.

And if you will prove whether you have wrought well or no, that may be proved by Reduction of them againe to their former denominations, which Art, shall be taught in the fourth kinde of Reduction, where greater termes of Fractions be reduced into smaller in number Proofe.

number, but no smaller in proportion. And if in such Reduction the same termes or numbers come againe that were before, then is the worke good, else not.

Scholar. Sir, I heare your words, but I do not understand many of them: which if it please you, declare.

Master. With a good will, when convenient place serveth, but that must be in the said fourth kinde of Reduction, which teacheth how to reduce Fractions of Fractions into one Fraction, and so to one Denomination.

The second forme of Reduction of Fractions of Fractions into one Fraction and Denomination.

When Fractions of Fractions be proponed, you shall multiply the Numerators of each into other, and set the totall for the new Numerator, and then multiply all the Denominators likewise, and take their totall for the new Denominator, and so are they speedily reduced.

Scholar. If that be all, then I understand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{6}{7}$ of $\frac{7}{9}$, which I would reduce to one Denomination, and proper simple fraction.

Therefore begin I with the Numerators, and multiply them together, saying, 3 into 2, maketh 6: and 6 by 6, maketh 36, which multiplied by 7, yeeldeth 252: that I set over a line for the Numerator, thus.

$\frac{252}{}$

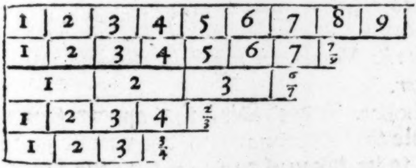
Then I multiply the Denominator, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplyed by 9, yeeldeth 756, the new

new Denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to understand yet. $\frac{252}{756}$

Master. You thinke so, and no marvell, but anon you shall learn to judge it easily, for this Fraction is no more indeed than $\frac{1}{3}$, although it be in greater termes, and therefore more stranger, and more obscure.

And this sufficeth for this Reduction, save that I will shew you by a figure of measure the just rate and reason of this kinde of fractions, and also the due understanding of their Reduction.

The entire measure parted into 9.



Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 divisions, whereof 7 are severed by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being divided into foure parts, the lowest

measure doth containe $\frac{3}{4}$, so that the same $\frac{3}{4}$ must bee named, not $\frac{3}{4}$ of the whole measure, but indeed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Schollar. This Example is so sensible, that I cannot chuse but see it. And furthermore see also, that the same fraction is equall to $\frac{3}{9}$ of the entire measure, as the lines which run up and downe doe expressly set forth. Also I see here that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{9}$. And further yet, that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Master. I am glad that you see it so well not douting but you will gather greater light of knowledge hereby.

[Margin: The third forme of Reduction of improper fractions.]

But now it is time that wee come to the third forme of Reduction, which teacheth of improper Fractions, that is to say, mixt numbers of unites and Fractions, although they appeare like Fractions, as this $\frac{26}{5}$, which doth conclude 25 unites wholly, and $\frac{1}{5}$ over. Wherefore first you shall know them, by that the Numerator is greater then the Denominator.

Schollar. Indeed Sir, that appeareth reasonable, that if the numerator doe expresse more parts to be taken of any unite, then the Denominator doth signifie that unite to bee divided into, it must needs follow that such a fraction importeth more then the whole, that is to say, the whole with certaine parts over: but what Reduction is there in it?

[Margin: Two severall wayes in this Reduction.]

Master. There bee two severall kindes of Reduction, concerning such Fractions. Sometimes

times it shall bee needfull to convert these Fractions into vnites, and the proper Fractions that will remaine. And sometimes contrariwise, it shall be meet to reduce mixt numbers, that is, vnites written with Fractions, into the forme of one simple Fraction, and so be there two wayes.

Schol. What is the mean of the first way to turne improper fractions into vnites with their proper fractions?

Master. *That is thus. Your Numerator being greater then the Denominator, must be divided by the same Denominator, and the quotient thereof expresseth the vnites: the remainer shall be put for the Numerator of the fraction that resteth, and the Denominator must be the same that was before.*

The fifth way. Reduction of improper fractions into vnites, with their proper fractions.

Schollar. For example, I take $\frac{17}{5}$, And dividing 17 by 5, the quotient will be 3, and there will remaine 2.

Master. That you must write thus, $3\frac{2}{5}$ where (you see) I have written 3 without any line, as entire numbers ought to bee written, and the 2 that remained I have set over the former Denominator with a line as a proper fraction. And this number doth signifie now three vnites, and $\frac{2}{5}$ of one.

Schollar. Then if I would by vnites here understand Crownes, so it were 3 Crownes, and $\frac{2}{5}$ that is 2 s.

Master. Even so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes

that when the reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple intire number, represented like a fraction.

The second way.

Scholar. As $\frac{15}{5}$ will make 3 just, and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the second forme of Reduction that you speak of for these sorts of Fractions?

Reduction of whole numbers either alone, or ioyned with fractions into improper fractions.

Master. *Whensoever you have any of these two sorts of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turne them into the forme of a fraction, you must multiply the whole number by that denominator which you will have to remaine still, and to the totall thereof adde the Numerator, which you have already, and all that shall you set for the new Numerator, keeping still the former Denominator: as if you have 6 $\frac{3}{4}$ which you would convert into an improper fraction, you must multiply 6 by 4, whereof commeth 24, and thereto adde the numerator, which is 3, and so have you 27 for the numerator, and 4 still for the denominator.*

Scholar. Then is $\frac{27}{4}$ equall to 6 $\frac{3}{4}$.

Note

Master. Even just, and so backward (as appeareth by the former Reduction) 6 $\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of their Reductions may be the proofe of the other worke.

Scholar. This I perceive: But now if you would turne whole numbers without fractions into any fractions, I see not how that may be done, because there is no Denominator to make the multiplication by.

Master.

Master. That is well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proofe whereof I set downe 7, which is a whole number. And if you will have this number converted into any certaine fraction, will me to doe it.

Scholar. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some Fraction.

Scholar. No, I passe not for the sort of the Fraction.

Master. Then how can you thinke that you require me to doe any thing certaine, when you leave me to doe as I list? And seeing you stand at that stay, whether thinke you that I must first intend in minde what fraction I will make of it before I can do it indeed?

Scholar. Else you should do ignorantly.

Master. Then will I limit my selfe (seeing you will not) to turne it into quarters: And therefore I multiply 7 by 4 (which is the Denomination of quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the Denominator, and the Fraction will be thus $\frac{28}{4}$

Scholar. Indeed I perceive this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then

If I would turne 8 into 5 parts, it will make $\frac{40}{5}$ which is all one with 8: for 8 Crownes turned into 5 parts, (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a Crowne.

The fourth forme of Reduction.

Master. *Seeing you understand now these three kinds of Reduction, I will declare unto you the fourth kinde, that is, when fractions be written in greater termes then they need, how they may be brought to lesser termes.*

Schol. To write any thing in greater terms then needeth, seemeth to be a fault, and so this Rule seemeth to amend that fault.

Master. It were a fault to doe any thing without nead, which after must be redressed: but in this case it is not so, neither did I say absolutely (as you doe) that it needeth not to expresse those fractions in so great termes, but that the fractions doe not need, I meane for their value to be understood: but yet it may be needfull for the case of these workes whereto they be applyed, as for example: In the first kinde of Reduction this was your owne example: $\frac{3}{12}$ and $\frac{4}{8}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus: $\frac{18}{96}$ and $\frac{64}{96}$, where the fractions (for their owne understanding) needed not to be turned out of smaller terms into greater, but yet the easinesse of working needed it.

Schollar. Sir, I understand now, not onely the difference of this need (for the fractions might better bee understood as fractions

severall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you mean by greater terms, and lesser terms where- of before I was in doubt: for I see you call the Numerator and Denominator, the termes of the Fraction.

Termes of fractions.

Master. *I am glad you understand it so well: Now when then you would value any fractions (because they may best be done when the terms are smallest) you shall reduce them to the smallest that you can, which thing you may do thus: Divide the greatest of any such two terms by the lesser, and if any thing remain by that remainer, divide the last divisor: and if any thing remaine now, by that divide the first divisor (which was before the remainer of the last division) and so continue still, till nothing do remain in the division: and then marke your last divisor, for it is the number that will easily reduce your fractions, if you divide both the Numerator and the denominator by the same Number, and put for the Numerator the quotient of his division, and for the Denominator also his quotient, that riseth by his division.*

Schol. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I divide it by 18, and the quotient is 5, and there resteth 6, what shall I do with this quotient?

Master. Nothing in this worke, but now seeing there remaineth somewhat, by that remainer must you divide the last Divisor.

Scholar. If I shall divide 18, (which was

the last Divisor) by 6, that was the remainer, so is the quotient 3, and nothing resteth.

Master. As for the Quotient, I omit him yet: but because there doth remaine nothing therefore is 6 (which was your last Divisor) that number by which you may reduce the Fraction proponed.

Schollar. Then as you taught me, I must divide the Numerator 18 by 6, and the quotient is 3, which I must put for the Numerator over a line, thus: $\frac{3}{16}$ And then by the said 6 must I divide also the Denominator 96 and the Quotient will be 16, which I must take for the Denominator, and so is the Fraction $\frac{3}{16}$. And so me thinketh this rule doth prove the work of the first Reduction,

Mast. That is true, if the first Reduction were made of fractions into their least terms and else not, without some help, as the second number in that place will declare.

Schollar. The second number was $\frac{2}{3}$ which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it againe into the least terms I must divide 96 by 64, and there remaineth 32, wherefore I must take that 32 for the Divisor, to reduce the said fractions. Then do you divide 64 by 32, and the quotient is 2, which I set for my Numerator. Again, I divide 96 by 32, and the quotient will be 3, and so I have but $\frac{2}{3}$.

Master. Muse not at the matter, for you have

have done well enough: but you thinke you have not the fraction that you looked for, that is, $\frac{4}{6}$, yet have you one equall to it, as by the parts of a shilling you may prove.

Schollar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, bee all three equall. And now I perceive that because $\frac{4}{6}$ was not written in the least termes, that it might bee therefore this Reduction brought forth not it, but that other which is written in the least terms. Now understand I this rule well. But is there any other way to work this Reduction?

Master. Yes, but first note this, that if you finde no such Divisor, to reduce the fraction till you come to 1, because one doth make no Division, therefore that fraction is already in his least termes, as by $\frac{71}{100}$ you may prove, and so $\frac{85}{98}$. and many other like.

Another way to Worke this Reduction.

But now for your better aide to finde the due proportion in least terms, with more ease for a yong learner, you shall mediate or take the halfe of the Numerator, and also of the Denominator as long as you may upon a line, alwayes parting them with a right downe dash of your pen as you work, which may easily be done, if the numbers be even: as 2.468, or 10, but if they be od (though it be but one of them) then must you abbreviate them by 3, 5, 7, or 9, &c.

Note that to mediate any number is to divide by two.

And because examples doe most instruct, I have here set downe the manner of two or three, whose last number at the end of the line shew-

sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawne from it thus:

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator, and the Denominator end in in even numbers, I see this may be abbreviated by 2, or 4, or 6, &c. Therefore on the other side of the right downe dash toward the right hand, I first take the halfe of the Numerator: saying, the halfe of 2 is 1, the halfe of 8 is 4; and againe, the halfe of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right down dash as before.

Then do I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so have I 288 for a new Denominator.

Then beginning againe: saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or division by 2 untill you come to the last worke, as appeareth here in the example, where the same is reduced to $\frac{1}{2}$ which is equall to $\frac{288}{576}$.

So the second example $\frac{28}{112}$ first abbreviated by 2, and againe by 2, and last by 7, is reduced to $\frac{1}{4}$ which is equall to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Again,

Againe, $\frac{1465}{4395}$ abbreviated first by 5, then by 293.

$$\frac{1465}{4395} \Bigg| \frac{293}{879} \Bigg| \frac{1}{3}$$

Schollar. Sir I thanke you much, this is very easie and good for a yong learner.

Master. So it is, but yet notwithstanding, if you can without that division by memory, espy the greatest number that may divide exactly both termes of your fraction proponed, then need you not to use that division, as in this fraction $\frac{60}{96}$, I see that 12 is the greatest number that can divide them both: and therefore without any worke, by memory onely, I turn that into $\frac{5}{8}$, but this ability in knowledge is got by exercise.

Yet one other way of easie reduction in this kinde there is, when your fraction hath any cyphers in the first places of both termes, then may you by casting away the Cyphers, make a briefe reduction as thus: $\frac{300}{400}$ here take away the cyphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholar. And so if I have $\frac{400}{850}$, it will be $\frac{4}{85}$.

Master. You are deceived, for you take away more Cyphers from the Numerator then you doe from the Denominator, which you may not doe.

Scholar. I confesse my fault, which came of too much haste, I was more gladder of the

Rule,

Rule then wise in using it: but now I understand it I trust.

Master. Then may I go in hand with the fifth or last kinde of Reduction, which teacheth how to turne any fraction proponed into any other Denomination that you list, or into any part of common coynes, weights or measures, or such like.

The fifth kinde of Reduction.

To reduce fractions to a denomination appointed.

For declaration whereof, first you shall marke whether your fraction be a simple Fraction, either else a Fraction of sundry parts, I meane of more termes then two. And if your Fraction be a Fraction of Fractions, or otherwise compound, you must reduce it to one simple fraction. And then marke well the Denomination of that other fraction, into which you would turne this: For by that Denominator you must multiply the Numerator of your first Fraction, and the totall product thereof shall you divide by the Denominator of your first fraction, and that quotient shall be the Numerator of the Denominator proponed: as for example, I have this Fraction $\frac{3}{5}$, which I would turne into ten parts: therefore I multiply this 10 by 3, that is the Numerator of my Fraction, and there ariseth 30, which I divide by 5, and the quotient is 6, which must be the Numerator to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Schollar. This is easie enough to do.

Master. Then shall you see another example of the same fraction that is not so easie: as if I would turn $\frac{3}{5}$ into 8 parts, prove you that worke.

Schollar. I must multiply 8 by 3, and there amount-

amounteth 24, which I divide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this worke?

Schollar. I see that when 24 was divided by 5, there remained 4, which I did not passe of, because ye spake nothing of any remainer, but onely of the quotient.

Master. By likelihood you remember what I said to you in Division of whole numbers, that you should not passe of the remainer there but onely note it as a summe that could not be divided without knowledge of Fractions. Wherefore now marke this, that in all divisions of whole numbers, when there is any remainer, you shall set it over a line as a Numerator, and set the Divisor for the Denominator, and that fraction doth make the Division compleat, and is part of the quotient: as if I would divide 48 by 5, the quotient will be $9\frac{3}{5}$: so in your former work when 24 was divided by 5, the quotient should be $4\frac{4}{5}$, and so the new Fraction should be thus, $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, that is, $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may prove by example of some coyne.

Schollar. Then I take a crowne, whose $\frac{3}{5}$ is 3 s. Now if I would prove whether the 3 s be $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, I shall have a cumbrous worke to doe.

Master. Indeed for whole pence, your example is a little troublesome: yet turning the crown

crowne into halfe pence, it is easie enough.

Schollar. That will I try.

¶ First, I see that $\frac{3}{5}$ of a Crowne is 3 shillings, which is 36 pence, or 72 halfe pence. Now if I can finde that this fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall unto 3 shillings, then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a crowne, I turne the Crowne into halfe pence, as you willed me, which makes 120, which I divide by 8, my quotient is 15, which taken foure times, make 60 ob. Now resteth me to have $\frac{4}{5}$ of the $\frac{1}{8}$ part of a Crown, whereof $\frac{1}{8}$ part is 15 ob. the 15 being parted in 5 parts, the quotient is 3, which taken four times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{3}{5}$, my desire.

Master. I commend you for your diligence, you might have wrought it thus: either $\frac{4}{8}$ being abbreviated as before I taught, is $\frac{1}{2}$. Now halfe a Crowne is 2 shillings 6 pence. Now $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you do reduce into one entire fraction, as before you have learned, in saying, five times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abbreviated also make $\frac{1}{10}$. Now the tenth part of a crowne is 6 pence, which put to 2 shillings sixe pence make also 3 shillings, your desire

But now one example more for this Rule, and then wee shall end it. If I have $\frac{7}{15}$ of a soveraigne (accounting the Soveraigne 20 shil-

lings)

lings) how many shillings is that $\frac{7}{15}$?

Schollar. I must multiply 7 by 20, and that maketh 140, which I shall divide by 15, and the quotient will be $9\frac{5}{15}$, or in lesser terms $\frac{1}{3}$.

Master. That is 9 shillings, and one third part of a shilling, that is 4 pence, as by the same Rule you may prove. And this for this time shall suffice for Reduction. And now I will proceed to Addition.

Addition.

Whensoever you have any Fractions to be added, you must consider whether they be of one denomination or not, and if they bee of one denomination, then adde the Numerators together, and set that that amounteth for the Numerator over the common Denominator, and so have you done: The reason is because that such differ little in Addition *or* Subtraction *from the worke of vulgar denominations, where the denominators be of the number, as* 3 pence *and* 5 pence, *make* 8 pence, *where the denomination is not altered. But if the fractions be not of one Denomination, or any of them bee mixt of whole numbers and fractions, then must you first reduce them to one Denomination, and after adde them. And if they be many, then adde first two of them, and so the summe that doth amount of the Addition, and the third, and then the fourth, &c. if you have so many.* Addition of fractions of one denomination.

Schollar.

Scholar. This seemeth easie enough, now that I have already learned to reduce, without which I could never have wrought this. And therefore now I see good reason why you did place Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your understanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$, and because the Denominators are like (and so needeth no reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreviated $\frac{2}{3}$.

To adde fractions of divers denominations.

And if I have many numbers to be added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, first I must reduce them (because they have divers denominators) into one Denominations and then they will be thus, $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$ or in lesser termes, $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$, that is $2\ \frac{3}{40}$.

Master. Now may we go to Subtraction.

Sub-

Subtraction of Fractions.

Subtraction hath the same precepts that Addition had, for if the Denominators bee like, then must you subtract the one Numerator from the other, and the rest is to bee set over the common Denominator, and so your Subtraction is ended: but and if you have many fractions to be subtracted out of many, then must you reduce them to one Denomination, and into two severall fractions, that is, all that must be subtracted into one fraction, and the residue into another fraction, and then worke as I said before. Subtraction of fractions.

Scholar. For the first example I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$, and the rest will bee $\frac{2}{12}$ or $\frac{1}{6}$.

For another example, I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{24}{32}$ and $\frac{28}{32}$.

Then do I subtract 24 out of 28, and there resteth 4, which I set over the common Denominator for a Remainer, thus $\frac{4}{32}$: that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and because their Denominators bee divers, I doe reduce them into one denomination thus $\frac{1440}{1920}$ $\frac{1600}{1925}$ $\frac{[illegible]}{1920}$ $\frac{1728}{1920}$.

Then do I adde the two first, & they make $\frac{3040}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1920}$. Then doe I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smaller termes $\frac{23}{120}$. And thus have I done with Subtraction, except you have any more to teach me.

Master. Prove one example or more out of fractions of divers denominations.

Scholar. I take the two fractions $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced, will stand thus $\frac{168}{192}$ and $\frac{72}{192}$: Now would I subtract 168 out of 72, but I cannot.

168	72
$\frac{7}{8}$	$\frac{9}{24}$
192	

Master. Then may you perceive that you mistooke the fractions: for you can never subtract the greater out of the lesser, although you may adde, multiply, or divide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his terms lesser than $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser fraction: for generally if you multiply the Numerator and the Denominators of two fractions crosse-wayes, that fraction is the greatest of whose numerator commeth the greatest summe, as in this example: 7 multiplyed by 24 maketh 168, and 9 being multiplyed by 8 yeeldeth but 72, therefore is the first fraction 7 the greatest of these two, so can you not subtract it out of a lesser fraction.

The greatest of two fractions.

But if you should subtract a fraction out of a whole number, what should you doe?

Scholar.

Scholar. Marry I would reduce the whole number into a fraction of the same denomination that my fraction is, and then worke by Subtraction.

Master. So may you doe, but it is much easier, if your fraction be a proper fraction, that is to say, lesse than an unite, to take an unite from the whole number, and then turne it into an improper fraction, and so worke your Subtraction. As if I would subtract 3 $\frac{2}{5}$ from 4, I may take 1 from 4, and turne it into $\frac{5}{5}$, from which I bate 3 $\frac{2}{5}$, there will remaine $\frac{3}{5}$. And if the first fraction be an improper fraction, then may I take so many unites from the whole number, that they may make an improper fraction greater than that first, and then worke by Subtraction. As if there bee proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more than 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus $\frac{12}{3}$, then abate $\frac{10}{3}$, and from $\frac{12}{3}$ there resteth $\frac{2}{3}$: so the whole remainer is 2 $\frac{2}{3}$. Or else you may at your pleasure take 3 $\frac{1}{3}$, which is $\frac{10}{3}$: from 6 whole: then set 1 under 6, as thus $\frac{6}{1}$: And then to reduce those two fractions into one Denomination, as here appeareth $\frac{10}{3}$ from $\frac{6}{1}$: Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$, which maketh 2 $\frac{2}{3}$ your desire. And thus will I make an end of the worke of subtraction of fractions, and proceed to Multiplication.

Multiplication of Fractions.

Multiplication of fractions.

THerefore when any two fractions be propoued to be multiplied together, the Numerator of the one must be multiplied by the Numerator of the other: and the summe that amounteth thereof must be set for a new Numerator: likewise the Denominator of the one must bee multiplyed by the Denominator of the other, and that that amounteth shall be set for the Denominator, and this new third fraction expresseth the product of the multiplication of the two first fractions proponed, whereof take this example, $\frac{3}{5}$ multiplyed by $\frac{5}{12}$ doth make $\frac{15}{60}$.

$$\frac{15}{60} \quad \frac{3}{5} \quad \frac{5}{12}$$

Scholar. I perceive then that 3 being the Numerator of the first fraction, is multiplyed by 5, being the numerator of the second fraction, whereof amounteth 15, the numerator of the third fraction: And so likewise 5 being the denominator of the first fraction, is multiplyed by 12 the denominator of the second fraction, whereof amounteth 60 the new denominator: so that I perceive how the worke is done, but I doe not perceive how $\frac{15}{60}$ is greater than $\frac{3}{5}$, for if I shall use my former manner of examination by the parts of some coine, I see that $\frac{3}{5}$ of a Crowne is 36 pence, and $\frac{5}{12}$ of a Crowne is 25 pence, whereof the one multiplied by the other, doth make 900 pence, which is 15 Crownes, but by your multiplication

tiplication there amounteth $\frac{15}{80}$, which is but 15 pence, and that is much lesse than any other of both the first fractions.

Master. That difference is betweene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers, the sum that amounteth is greater than both the other whereof it came: but in fractions it is contrariwise: for the summe that amounteth is lesser than any of the other two fractions whereof it is produced.

Scholar. I desire much to understand the reason thereof.

Master. Although I purposed to reserve the reasons of works Arithmeticall for the perfect Booke of Arithmeticke, yet I will shew you this, because of the strangenesse of the work.

You see in whole numbers, that of two numbers being multiplied together, is made the third number, which third number doth beare the same proportion to the number multiplied, that the multiplier doth beare to an unite. And so in fractions, the third number which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an unite.

Scholar. Sir, I understand your words thus: when 40 is multiplied by 12, there doth amount 480, which 480 doth containe 40 so many times in it, as 12 doth containe Vnites, that is to say, twelve times. And so it

appeareth that 480 doth containe twelve so many times also as 40 doth containe unites, that is 40 times. But now I see not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you cannot see that thing which is not possible to bee seene of any man, how the third number in Multiplication of Fractions should be greater than any of the two former fractions: but yet this may you see (which I said) that the third number in Fractions so multiplyed, doth beare the same proportion to any of the two former fractions that the other of those two fractions doth beare to an unite, as in your example, $\frac{3}{5}$ being multiplyed by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now say I that $\frac{15}{60}$ doth beare the same proportion to $\frac{3}{5}$ that $\frac{5}{12}$ doth beare to an unite, as you may in your owne forme of examination by Coine, try it: for in an old Angel (which in times past was currant for 7 shillings six pence, are 180 halfe pence) which I set for the intire unite, whose parts (according to the fractions aforesaid) are these, for $\frac{15}{60}$ set 45 halfe pence, for $\frac{3}{5}$ take 108 halfe pence, and for $\frac{5}{12}$ put 75 halfe pence. Now doth 45 beare the same proportion to 108, that 75 doth beare to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserved till another time, when the knowledge of

pro-

proportions in due order shall be taught: yet in the meane season I will shew you how it commeth to passe, that in fraction the third summe must needs be lesser than any of the other two.

Consider this, that when a fraction is proponed, as in the former example $\frac{3}{5}$ if it be multiplyed by more than 1, it will make more than one entire number. As if I multiply $\frac{3}{5}$ by 5, that is to say, if I take it 5 times, it will make three entire unites. Example: in a Crown $\frac{3}{5}$ of it maketh 3 shillings which if I take five times, it will amount to 15 shillings, that is, three entire Crownes; so if I take the same $\frac{3}{5}$ but twice, it will yeeld 6 shillings, that is one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more than it was before, that is 5 shillings. And if I take it lesse than once, it cannot be so much as it was before, Then seeing that a Fraction is lesse than one, if I multiply a fraction by another fraction, it followeth that I doe take the first fraction lesse than once, and therefore the summe that amounteth, must needs be lesse than the first Fraction. Note.

Scholar. Sir, I thanke you much for this reason. And I trust I do perceive the thing, as by example of this same fraction $\frac{3}{5}$ I will expresse. If I take $\frac{3}{5}$ of a Crowne once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will be as it was before, but 3 shillings: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then

 will

will it be but half so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the first fraction.

And so to make an end: if I take but the twelfth part of one, that is, if I doe multiply it by $\frac{1}{12}$ it will yeeld but the twelfth part of the first fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make three pence, then $\frac{5}{12}$ must needs make five times so much, that is 15 pence which was the summe that hath given the occasion of all this doubt.

Master. Then I perceive you have sufficient understanding in this sort of multiplication for this time, wherefore I will proceed to the rest.

To multiply a whole number into a fraction.

In multiplication it happeneth sometime, that there be whole numbers to be multiplyed with Fractions; and may bee in two sorts: for either the whole number is severall from the fraction, and is the multiplyer, or else the whole number is joyned with one, or both of the Fractions, and so maketh a mixt number thereof. If it bee in the first sort, then needeth there no reduction, but onely multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the new Numerator.

Scholar. I understand you thus. If I have $\frac{6}{23}$ to be multiplyed by 16, then must I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator, keeping still 23

for

for the Denominator, and so the fraction will be $\frac{95}{23}$ that is $4\frac{3}{23}$.

Master. And in this sort of worke you may abridge the labour thus. If it happen the Denominator to be such a number as may evenly be divided by the said whole number proposed, then divide it thereby, and set the quotient of that division for the former Denominator, but reserve still the Numerator, and so is the multiplication ended.

Scholar. Then faine this example $\frac{7}{20}$ to be multiplyed by 5, and because 5 will justly divide 20, therefore I take the quotient of that division, which is 4, and set in stead of 20, and so the Fraction will bee $\frac{7}{4}$ that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$ that would have followed of the other sort of worke.

Scholar. I perceive it very well.

Master. *Now then for the other sort, where the number is mixt, take this way: first to reduce the said whole number and fraction into one improper fraction, (as I shewed you in* Reduction) *and then multiply them together, as if they were proper fractions.* How to multiply mixt numbers.

Scholar. $13\frac{3}{5}$ being set to be multiplyed by $\frac{5}{8}$ first I must reduce the mixt number, as in this example appeareth by multiplying 13 by 5, and that maketh 65, whereto I must adde the Numerator 3,

$13\frac{3}{5}$		340
68	by	5
5 40		8

and

and so the fraction will be $\frac{68}{3}$, which two Fractions now I shall multiply after the accustomed forme, and it will be $3\frac{10}{40}$, or $3\frac{1}{4}$.

Master. You have done well: and so may you see, that although most part of the formes of Multiplication may bee wrought without Reduction, yet some cannot, as namely, mixed numbers.

Duplation. *And yet one note more I will tell you of Multiplication before we leave it: That is, whensoever you would multiply any Fraction by 2, which commonly is called* Duplation, *you may doe it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be eaven.*

Scholar. Then if I would double $\frac{5}{12}$ I may chuse whether I will make it $\frac{10}{12}$ or else $\frac{5}{6}$. And indeed I see that is all one, but that the dividing of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall need the lesse Reduction.

Master. It is so: and now I shall not need to tell you that Multiplication is proved by Division, and Division likewise by multiplication: but the like worke that I shewed you in Multiplication, will I shew you in Division also.

Divisi-

Division of Fractions.

WHensoever two Fractions be proponed, that one should be divided by the other, I must set downe first the fraction that shall be divided (which is called the Dividend) and then after it the other which is the Divisor: Then shall I multiply the Numerator of the Dividend by the Denominator of the Divisor, and that which amounteth I must put for a new Numerator. Againe, I shall multiply the Denominator of the Dividend by the Numerator of the Divisor, and the number that amounteth thereof I must put for the new Denominator. And this third Fraction is the Quotient of the said Division. Division of Fractions.

Scholar. This seemeth easie in forme, as by example thus: If I would divide $\frac{5}{8}$ by $\frac{2}{6}$, first I multiply 5, (being the Numerator of the Dividend) by 6 which is the Denominator of the Divisor, and thereof riseth 30: then I multiply 8 (being the Denominator of the dividend) by 2, being the Numerator in the Divisor: and so riseth 16, the which I must make a third Fraction, thus $\frac{30}{16}$.

	30	
5		2
8		6
	16	

Master. Me seemeth you are quicker in understanding now, then you were when I taught

taught you the Art of whole numbers, but that is no marvell: for the more knowledge that a man getteth, the readier shall he finde his wit, and quicker in understanding: but yet of two things I will admonish you, which you might have observed here for the ease of work and lightnesse of understanding, the nature of the Quotient.

Whensoever you divide one Fraction by another, either they be both equall together, or else the one is greater than the other: if they be equall, their quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first fractions bee inequall, their quotient shall declare the same by the unequality of the Numerator and Denominator, as in these examples following shall appeare.

First, if equall fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equall together, and if the one be divided by the other the quotient will be $\frac{108}{108}$, as you may perceive by that Rule aforesaid.

Now in the unequall fractions, as $\frac{4}{9}$ and $\frac{3}{10}$, the quotient will be $\frac{40}{27}$, where the Numerator is greater than the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion betweene two numbers.

Master. The reason is this: When any Fraction is divided by another, the quotient declareth what proportion the Dividend beareth to the Divisor. So $\frac{1}{2}$ divided by $\frac{1}{4}$, maketh 2, which must be sounded, not 2, but twice,

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the quotient representeth the Dividend, and the Denominator representeth the Divisor. And this is alwayes true, whether the greater fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you have learned the Art of proportions: notwithstanding somewhat of it I have declared in the Rule of Reduction. But now for the easie remembrance of the quotient in division, as soone as you have set downe your two fractions the one against the other, then make a straight line for the quotient: and as soone as you have multiplyed the Numerator of the Dividend, by the Denominator of the Divisor, set the number that amounteth over the said line, and then multiply the other two numbers, and set their totall under the same line.

Scholar. I perceive you would not have me trust to memory till I were better expert, lest oftentimes I happen by misse-remembrance to be abused. This example I take for that declaration.

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other, (as here doth appeare) and then make another line for the quotient in some good distance,

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

distance, where I may set the numbers of the quotient, as soone as any of them is multiplyed. So then as soone as I have multiplyed 2 by 4, which maketh 8, I shall set that 8 over that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set under the same line, and then will the whole quotient appeare thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I perceive that?

$$\frac{8.}{9}$$

Master. Although you might better perceive it by the Rule of Reduction, yet this example may be declared in common coines, as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions do agree. And if you had taken this example before when you took the example of $\frac{5}{8}$ and $\frac{5}{9}$, your quotient would appeare (as this doth) more easie to understand; whereas that Quotient being $\frac{30}{18}$, is not an easie proportion for you to perceive, being yet little acquainted with proportions.

Scholar. If there be whole numbers to be divided by a Fraction, how shall I performe it?

To divide a whole number by a fraction.

Master. *When any whole number shall bee divided by a Fraction, you must multiply the said whole number with the* Denominator *of the Fraction, and set the totall thereof for the new* Numerator, *and for the* Denominator

set

set the Numerator *of the fraction.*

Scholar. Then 20 divided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

80

$\frac{20}{1}$ by $\frac{3}{4}$

3

Master. *Even so: but if you would divide the Fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator.*

To divide the fraction by the whole number.

Scholar. Then to divide $\frac{20}{23}$ by 4, it will bee $\frac{20}{92}$, as here appeareth $\frac{20}{23}$ by $\frac{4}{1}$ in this example $\frac{20}{92}$.

$$\begin{array}{ccc} & 20 & \\ \frac{20}{23} & \text{by} & \frac{4}{1} \\ & 92 & \end{array}$$

Master. You say well. And by the same example you give me cause to remember another briefe way to doe the same: for if you had divided the said Numerator by 4, and set the quotient for the numerator, keeping still the old denominator, it would have beene not onely as well done, but also in a fraction of lesser termes.

Another briefe way.

Scholar. I guesse it to be even so, by a like worke that you taught me in Multiplication: And for proofe thereof $\frac{20}{23}$ being the dividend, and 4 the divisor, I divide the Numerator 20 by 4, and the quotient is 5, which I set for 20 over 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these termes by 4, and so reducing them

to

to their least Denomination, I may easily prove: as appeareth by this example $\frac{20}{29}$ $\frac{5}{2?}$.

Master. You conceive it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction, & then work as you have learned,

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: if the Denominators be like then divide the numerators as it were in whole numbers, and the quotient, whether it be fraction, whole number, or mixt, is a good quotient for that division And generally, if one of the numerators may justly divide the other by that quotient, multiply the Denominator of the letter Numerator, and set it that doth amount in the roome of the same denominator, and then for a numerator to it, set the Denominator of the other fraction.

Scholar. Then if I would divide $\frac{3}{4}$ by $\frac{12}{17}$, I see that 3 will divide 12, and the quotient will be 4, by which I must multiply the other 4, that is the Denominator under 3, and then it is 16, which is set for the denominator 4, and over it in stead of 3 I must set 17 the other Denominator, and so it is thus $\frac{17}{16}$.

Master. And so is $\frac{17}{16}$ in stead of $\frac{51}{48}$, which would have risen by the common works, as here appeareth.

$$\frac{3}{4} \text{ by } \frac{12}{17} \quad \frac{51}{48}$$

And

And now for meditation(which is to divide by 2)marke this, if the Numerator *be an even number, set the halfe of it in his place without the* Divisor, *and so have you done: and if the* Numerator *be not even, then double the* Denominator.

Scholar. That is, if I would mediate $\frac{6}{11}$, I may make the quotient $\frac{3}{11}$, and if I would mediate $\frac{1}{11}$, I must make it $\frac{1}{22}$.

Master. And thus will I make an end of the workes of common fractions for this time, not doubting but you can apply them both to the Rule of Progression; and also to the Golden Rule, without any other teaching than you have learned before, which might seeme tedious to repeat, in regard you have sufficient knowledge in Reduction, Addition, Subtraction, Multiplication, & Division: And therefore will I goe in hand with the Rule of Proportion, or Golden Rule, which now will appeare easie enough.

The Golden Rule direct in Fractions.

Master.

Therefore as touching the Golden rule, for the placing of the 3 numbers proponed in the question whereby to finde the fourth, and for the forme of their worke, with other like notes, I refer you to that which you have already learned. The rule of proportion, in fractions.

But this easie forme of working by fractions shall you note, that if your three numbers be fractions, for an apt worke and certaine, multiply the numerator of the first number in the question, by the denominator of the second: And all that againe multiply by the Denominator of the third number, and the totall thereof shall you keepe for to be the Divisor. Then multiply the Denominator of the first number by the numerator of the second, and the whole thereof by the Numerator of the third, and the totall thereof shall be your Dividend.

Note this for a generall rule.

Now divide this Dividend by the Divisor which you found out before, and that number shall bee the fourth number of the question which you seeke for, as in this example.

A question of velvet.

If $\frac{3}{4}$ *of a yard of velvet cost* $\frac{2}{3}$ *of a Soveraigne, esteemed at* 20. shillings, *what shall* $\frac{5}{6}$ *cost?*

Scholar. If it please you to let me make the answer, I would first place these three numbers as I learned in whole numbers, thus:

$$\begin{array}{ll} \frac{3}{4} & \frac{2}{3} \\ \frac{5}{6} & \end{array}$$

And then according to your new rule, I must multiply 3, being Numerator, in the first number, by 3 the Denominator of the second: and thereto commeth 9, which I multiply againe by 6, the denominator of the third number, and so have I 54. which I keepe for the divisor. Then multiply I 4 the denominator of the first, by 2 the numerator of the second, and there ariseth 8, which againe I multiply

by

by 5, the Numerator of the third and it maketh 40. Then must I divide 40 by 54, and it will be $\frac{40}{54}$ that is, $\frac{20}{27}$ in lesser termes, and then the Figure will stand thus.

$$\begin{array}{ccc} \frac{3}{4} & & \frac{2}{3} \\ \frac{5}{6} & & \frac{20}{27} \end{array}$$

But what that is in money I cannot tell, except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done here to mingle divers works together, where we do not seeke the value of the thing in common money, but in apt number, which ye have well done. And therefore will I yet shew you another like way of easinesse in worke, how you may change your three fractions into three whole numbers, by which you shall worke, as if the question were proponed in whole numbers. The first number you shall find as I taught you: now to finde the Divisor of the second number, take the Numerator for the second fraction: and for the third number, take that that ariseth of the multiplication of the denominator of the first, by the numerator of the third, and then worke your question.

Scholar. *For example hereof, I put this question. If* $\frac{11}{12}$ *of* 1 pound *weight of silver be worth* $\frac{13}{4}$ *of a Soveraigne, what is* $\frac{1}{2}$ *of* 1 pound *weight worth?* A question of silver.

For the answer, first I place the fractions in order thus:

$$\begin{array}{ccc} \frac{11}{12} & & \frac{13}{4} \\ \frac{1}{2} & & \end{array}$$

Then to turne these fractions into whole numbers, I multiply 11, which is the numerator of the first by 4 (the denominator of the second) and there commeth 44, which I multiply by 2 the denominator of the third, and so amounteth 88, which I set for the divisor in the first place. Then in the second place I set 12, which is the numerator of the second fraction, & in the third place I set the sum that amounteth of 12, being the denominator of the first number, multiplied by one, being numerator in the third number, and so the figure will stand as here you see.

88 Z 12
12

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$ and then the figures will stand thus:

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{2}$ $1\frac{7}{11}$

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shall you marke for the proofe of this Rule, whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth: then multiply the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equall, then is your worke well done, else you have erred. And this shall suffice for ye former rule.

The proofe of the goldẽ Rule.

The

The Backer Rule, or Reverse Rule in Fractions.

BVt in the Backer Rule, this shall you note for ease of worke, that you multiply the numerator of the first by the numerator of the second, and the whole thereof by the denominator of the third, and that amounteth thereof, shall be the dividend. Then multiply the denominator of the first, by the denominator of the second, and that whole by the numerator of the third, and that that ariseth thereof, shall be the divisor. Example of this. The backer Rule in Fractions. Note this also for a generall Rule.

I did lend my friend $\frac{3}{4}$ *of a* Porteguise, *seven moneths upon promise that he should do as much for me again, and when I should borrow of him, hee could lend mee but* $\frac{5}{12}$ *of a* Porteguise: *now I demand how long time I must keepe his money in just recompence of my loane, accounting 13 moneths in the yeare?* A questiõ of Loane.

Scholar. The first number must be the first mony borrowed, that is $\frac{3}{4}$ of the Porteguise: the second number the 7 moneths, that is $\frac{7}{13}$ of a yeare: and the third number the money that was lent in recompence, that is $\frac{5}{12}$ of a Porteguise: then I set the numbers thus:

$$\frac{3}{4} \quad \frac{7}{13}$$
$$\frac{5}{12}$$

Then (as you taught mee) I multiply 3 (being numerator in the first number by 7, the numerator of the second number, and it maketh 21, which I multiply by 12 the denominator of the third, and so have I 252 for the dividend: then I multiply 4 the denominator of the first, by 13 the denominator of the second, and it yieldeth 52, which I multiply again by 5, the numerator of the third, and it will make 260, that is the divisor. Then must I divide 252 by 260, so it will be in the small fraction $\frac{63}{65}$ of a yeare.

Mast. And thus do you see some ease in working, better then to multiply and divide tediously so many Fractions.

Statute of Assise of Bread and Ale.

Another question yet will I propose, to the intent you may see thereby the reason of the Statute of Assise of Bread and Ale, which in all Statute Books, in Latine, French and English is much corrupted for want of knowledge in this Art, for the right understanding whereof, I propone this question.

Question of Bread.

When the price of a quarter of Wheate is 2 shillings, *the farthing white loafe shall weigh 68* shillings, *then I demand what shall such a loafe weigh, when a quarter of Wheat is sold for 3* shillings?

Scholar. This question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You seeme to say reasonably, howbeit in the Statute of Assise, the rate is made

by

by the proportion of parts in a pound weight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other monies doe: but this statute being well understood, is a continuall rule for ever, as I will anon declare by a New Table of Assize, converting the shillings into ounces, and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound weight, and so by a penny $\frac{1}{20}$ of an ounce: wherefore although you might worke this question proponed by whole numbers well enough, for that time when the Statute was made, yet to apply it to your time, and to make it serve for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings $\frac{68}{20}$, and so for three shillings $\frac{3}{20}$, and then will the Figure of the question stand thus.

$$\begin{array}{ccc} \frac{2}{20} & \text{——} & \frac{68}{20} \\ \frac{3}{20} & \text{——} & \end{array}$$

In which question because all the denominators bee like, you shall worke onely with the numerators.

Scholar. Then shall I multiply 68 by 2, whereof commeth 136, which if I divide by 3, the quotient will be 45 $\frac{1}{3}$: but how shall I make a fraction of that, to stand with the other?

Master. Have you so soone forgotten what was taught you so lately? this is his forme.

$$\frac{45\frac{1}{3}}{20}$$

Scholar. I remember it now, and then it signifieth 45 twenty parts, and the third deale of one twenty part.

Note what a shilling is. Master. So is it that maketh in shillings 45 shillings 4 pence, whereby you may note one great errour in the Statute Books; which have constantly 48 shillings in that Assise. And by this Rule, if you examine the Statute, you shall finde many summes false. Wherefore for the true understanding of that Statute, and such like as I have made mention of it, and somewhat recognized it, so do I wish that all Gentlemen and other Students of the Lawes would not neglect this Art of Arithmatick, as unneedfull to their studies. Wherefore to encourage them thereto, and to gratifie both them and all other in generall, I will exhibit a Table of that part of the Statutes in two columnes, and in a third columne I will adde the correction of those errours which have crept into it.

Here followeth the Table.

The

The price of a quarter of wheat.		The weight of a farthing white loafe, by the Statute bookes.			The Correction by just Assize.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	2	8	0	3	8	0
2	6	2	14	4½	2	14	4 4/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 4/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2½	1	7	2 2/11
5	6	1	4	8¼	1	4	8 1/11
6	0	0	2	8	1	2	8
6	6	0	16	11	1	0	11 1/13
7	0	0	19	1	0	19	5½
7	6	0	18	1½	0	18	1 2/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0¼	0	15	1 1/3
9	6	0	14	0¾	0	14	3 12/19
10	0	0	13	7½	0	13	7 2/7
10	6	0	12	11¾	0	12	11 1/8
11	0	0	12	4 3/5	0	12	4 4/11
11	6	0	11	10	0	11	9 21/23
12	0	0	11	4	0	11	4

In the common Bookes there is no further rate of Assise made, than into 12 s the quarter of Wheat, but in an ancient Copy of 200 yeares old (which I have) there is added the rate of Assise unto 20 s the quarter, but yet was that assise also either wrong cast at the first penning, or else corrupt since that time, for lack of just knowledge in the Rule of Proportion, which I will adde here also to gratifie such as be desirous to understand truth exactly.

The price of a quarter of Wheat.		The waight of a farthing white loafe by the Statute Bookes.			The Correction by just Assise.		
s	d	l	s	d	l	s	d
12	6	0	11	0	0	10	$10\frac{14}{25}$
13	0	0	11	$0\frac{1}{2}$	0	10	$5\frac{2}{13}$
13	6	0	10	$1\frac{1}{8}$	0	10	$0\frac{8}{9}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{2}$	0	9	$4\frac{14}{29}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{7}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{2}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	$3\frac{12}{11}$
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	$10\frac{2}{35}$
18	0	0	7	6	0	7	$6\frac{2}{3}$
18	6	0	7	3	0	7	$4\frac{2}{37}$
19	0	0	7	2	0	7	$1\frac{1}{3}\frac{7}{9}$
19	6	0	5	10	0	6	$11\frac{2}{23}$
20	0	0	5	6	0	6	$3\frac{1}{5}$

These

These two Tables I have set severall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seeme either superfluous, either diminute: but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but every good Subjects dutie with sobriety to correct. And for avoiding of offence, I have rather done it in this Private Book, than in any Book of the Statutes it self, trusting that all men will take it in good part.

Scholar. I would wish so, but I dare not so hope, sith never good man that would reforme errour, could escape the venemous tongues of envious detractors, which because they either cannot, or list not to do any good themselves, do delight to barke at the doings of other, but I beseech you to stay nothing for their perverse behaviour.

Master. I consider many things that some may object, whereunto I am not unprovided of just answers, but I will not seeme so hastie to make the answers, before I heare their objections, but as I trust that men are of a better nature, and more gratefull now then some have beene in times past, as I have done in the Statute of Assize for Bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applyed to all times: but I meane not by this to alter any word of the Statute,

Concerning the following Tables.

Statute, being so good an Ordinance, and of so great continuance) but onely to make it as a kinde of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood, and consequently better put in execution. And here you shall note, that I have accounted the shillings after the rate of 60 shillings to the pound weight, because I esteeme it the most apt for our time. Wherefore in the first Columne you finde the price of Wheat directly against it; in the second Columne, you may finde the weight of a Farthing white Loafe in this our time: and if you double the number (as I have done in the third Columne) then have you the weight of the Half-penny White-loafe; and so in the fourth Columne is set the weight of a penny white Loafe. It needeth not to tell that the sight doth testifie, how that every Columne is parted into three smaller pillars, whereof the first Columne hath these three titles; pounds, ounces, and penny weights. And as in the first Columne 12 pence make a shilling, and 20 shillings make a pound, so in the other three Columnes 20 pence weight maketh an ounce and 12 ounces do make a pound.

A pound weight.

Gentle

GEntle *Reader*, touching the understanding of the *Table* following, wherein according to our time, *Master Record* alloweth 60 *pence* to the *ounce*, and 3 *pound*, or 60 *shillings* to the *pound*, and thereupon after the rate of 60 *shillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *shillings the quarter*, till he comes to 40 *shillings* 6 *pence the quarter*. And this his *proportion* (for that he hath not set downe any one *Example* to continue the worke) hath beene hard for many to conceive or comprehend, and therefore the onely chiefe cause why I have written this *digression* for the better understanding of him therein.

The first thing therefore that is sought for in this *Table*, as in the other aforesaid, is a *Maxime* grounded upon the *Statute*, which is this. When the *quarter of Wheate* is sold for *two shillings*, then the *farthing white loafe* shall weigh 68 *shillings*, whereby a *shilling* is meant $\frac{1}{20}$ of a *pound*, and by a *penny* $\frac{1}{20}$ of an *ounce*. Now therefore for a *generall Rule*, to finde what weight the *Farthing white-loafe* shall weigh at 3 *shillings the quarter*, till you come to 40 *shillings* 6 *pence the quarter*, is thus to be wrought. Comming to the *first ground*, and working by the *Backer Rule*, *say*; if *two shillings the quarter* give, or allow the *Farthing white-loaf* to weigh 68 *shillings*, what weight ought the *Farthing-white-loafe* to weigh at 3 *shillings the quarter*? Worke, and you shall find 45 *shillings* 4 *pence*, as

as before in the correction of the *first Table* is noted. Then for the *second worke*, say by the *Rule of 3 direct*, if 20 *pence* give one *ounce*, what giveth 45 *shillings*, 4 *pence*? Multiply & divide, and you shall finde 544 *ounces*, which 544 *ounces* being multiplied by 3, for 3 *pounds*, or 60 *shillings*, yeeldeth 1632 *ounces*, which divided by 20, produceeth 81 *ounces*, and $\frac{12}{20}$, or rather $\frac{3}{5}$ of an *ounce*, equall unto 12 *penny weight*, which is *halfe an ounce*, and 2 *penny weight*, and so maketh in all 6 *pounds*, 9 $\frac{1}{2}$ *ounces*, and 2 *penny weight*. Now the next way to continue this *Table*, to know the weight of the *Halfe-penny white loafe*, is thus. Multiply 1632 *ounces* by 2, and it bringeth forth 3264 *ounces*, and divided by 20, it yeeldeth 163 *ounces* and $\frac{1}{5}$, which is equall to 13 *pounds*, 7 *ounces*, and 4 *penny-weight*, as *M. Reecord his Table noteth*.

Thirdly, for the weight of the Penny white-loafe, multiply 1632 *ounces* by 4, and divide by 20, and after by 12, as before, and you shall find 27 *pounds*, 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or else by doubling the *Farthing white loafe*; for the *weight of the Halfe-penny white loafe*, and so doubling the *halfe-penny white loafe*, for the *weight* of the *Penny White-loafe*, is the order to continue the *Table* to the end thereof.

The

The price of a quarter of Wheat.			The weight of a farthing white-loafe.		
po.	shi.	pe.	po.	ou.	penny-weight.
0	3	0	6	9	12
0	4	6	4	6	8
0	6	0	3	4	16
0	7	6	2	8	12 4/5
0	9	0	2	3	4
0	10	6	1	11	6 2/7
0	12	0	1	8	8
0	13	6	1	6	2 1/2
0	15	0	1	4	6 2/5
0	16	6	1	2	16 8/11
0	18	0	1	1	12
0	19	6	1	0	11 1/13
1	1	0	1	0	4 4/5
1	2	6	0	10	17 1/2
1	4	0	—	10	4
1	5	6	—	9	12
1	7	0	—	9	1 1/2
1	8	6	—	8	11 15/19
1	10	0	—	8	3 1/5
1	11	6	—	7	15 1/7
1	13	0	—	7	8 4/11
1	14	6	—	7	1 21/23
1	16	0	—	6	16
1	17	6	—	6	10 14/25
1	19	0	—	6	5 7/13
2	0	6	—	6	0 4/5

The price of a quarter of wheat. l.	s.	d.	The weight of a Halfe-penny white loafe. po.	ounces	penny waight.	The weight of a penny white-loafe. po.	ounces.	penny waight.
0	3	0	13	7	4	27	2	8
0	4	6	9	6	16	18	1	12
0	6	0	6	9	12	13	7	4
0	7	6	5	5	$5\frac{3}{5}$	10	10	$11\frac{1}{5}$
0	9	0	4	6	8	9	0	16
0	10	6	3	10	$12\frac{4}{7}$	7	9	$5\frac{1}{7}$
0	12	0	3	4	16	6	9	12
0	13	6	3	0	$1\frac{1}{3}$	6	0	$10\frac{2}{3}$
0	15	0	2	8	$12\frac{4}{3}$	4	5	$5\frac{3}{5}$
0	16	6	2	5	$13\frac{2}{11}$	4	11	$6\frac{10}{11}$
0	18	0	2	3	4	4	6	8
0	19	6	2	1	$2\frac{2}{13}$	4	2	$4\frac{4}{13}$
1	1	0	2	0	$9\frac{3}{5}$	3	0	$19\frac{1}{5}$
1	2	6	1	9	$15\frac{3}{5}$	3	7	$10\frac{2}{5}$
1	4	0	1	8	8	3	4	16
1	5	6	1	7	4	3	2	8
1	7	0	1	6	$2\frac{2}{3}$	2	0	$5\frac{1}{3}$
1	8	6	1	5	$3\frac{11}{19}$	2	10	$7\frac{3}{19}$
1	10	0	1	4	$6\frac{2}{5}$	2	8	$12\frac{4}{5}$
1	11	6	1	3	$10\frac{6}{7}$	2	7	$1\frac{3}{7}$
1	13	0	1	2	$16\frac{2}{11}$	2	5	$13\frac{2}{11}$
1	14	6	1	2	$3\frac{1}{2}\frac{2}{3}$	2	4	$7\frac{1}{2}\frac{5}{8}$
1	16	0	1	1	12	2	3	4
1	17	6	1	1	$1\frac{3}{25}$	2	2	$2\frac{6}{25}$
1	19	0	1	0	$11\frac{1}{13}$	2	1	$2\frac{2}{13}$
2	0	6	1	0	$0\frac{4}{45}$	2	0	$0\frac{8}{45}$

HAving *spoken before for the understanding of the* Table *placed by* M. Record, *a man indued with rare knowledge in* Arithmetical & Geometrical Proportions, *touching the Statute of* Coynage, and the Standard thereof, *as appeareth in his Epistle of this Book, dedicated to K.* Edward *the Sixt, insinuating unto his Highnesse that the* Standard of Coyne, *is much altered from the* 14 *yeere of King* Edward *the third (when this* Statute and Assise *was confirmed) to the* Standard of this our time. *For it appeareth that in K* Edw. *the thirds time, when the* Assise of Bread and Drink *was established, that a* sterling peny, round without clipping, *did then weigh* 32 cornes of wheat dry, *and taken out of the middle of the* eare, *and* 20 *of these* pence *made an* ounce, *&* 12 ounces *made a pound* Troy. *And so from the* weight of a peny *to* 20 shillings sterling, *which then weighed* 12 ounces, *took* Bread his weight *and proportion*. *And now finding* 60 pence is an ounce. *That onely cause (I perceive, for the zeale of a Common-wealth) moved him to set down the same* Table in this private Booke; *meaning not thereby to alter any word of the* Statute *being so good an ordnance, & of so long continuance, but as a kinde of exposition by the way that thereby the* Statute *may be better understood, and so consequently better put in execution. Which* Assise of his, *is three times greater then the* Statute *now alloweth. Therefore also (to gratifie such as are desirous of knowledge, according to these* prices of a quarter of wheat) *I have added to this* Author these three other new Tables following, *and reduced their* prices *into their just proportions of* sterling money, *and also reduced the* money *into knowne* weight Troy, *according to the* Statute. *And thereafter according to proportion in my other three* Tables, *have I noted the* just weight, *that a* Farthing, Halfe peny, and peny white Loafe *ought to weigh by the* Statute.

The price of a quarter of wheat.			The weight of a farthing white-loafe in Sterling money by Assise.			The weight of a farthing white-loafe in Troy weight by Assise.		
l	s	d	po.	ounces	penny waight.	po.	ounces.	penny waight.
0	3	0	2	5	4	2	3	4
0	4	6	1	10	$2\frac{2}{3}$	1	6	$2\frac{2}{3}$
0	6	0	1	2	8	1	1	12
0	7	6	0	18	$1\frac{1}{5}$	0	10	$17\frac{3}{5}$
0	9	0	0	15	$1\frac{1}{5}$	—	9	$1\frac{1}{3}$
0	10	6	0	12	$15\frac{3}{7}$	—	7	$15\frac{1}{7}$
0	12	0	0	11	4	—	6	16
0	13	6	0	10	$0\frac{8}{9}$	—	6	$0\frac{8}{9}$
0	15	0	0	9	$0\frac{4}{5}$	—	5	$8\frac{4}{3}$
0	16	6	0	8	$2\frac{10}{11}$	—	4	$18\frac{10}{11}$
0	18	0	0	7	$6\frac{2}{3}$	—	4	$10\frac{2}{3}$
0	19	6	0	6	$11\frac{9}{23}$	—	4	$3\frac{7}{13}$
1	1	[illegible]	2	6	$9\frac{2}{5}$	—	4	$1\frac{1}{2}$
1	2	6	0	6	$0\frac{8}{13}$	—	3	$12\frac{8}{15}$
1	4	0	0	5	8	—	3	8
1	5	6		5	4	—	3	4
1	7	0	0	5	$0\frac{4}{9}$	—	3	$0\frac{4}{9}$
1	8	6	0	4	$9\frac{5}{19}$	—	3	$17\frac{5}{19}$
1	10	0	0	4	$6\frac{2}{5}$	—	2	$14\frac{2}{5}$
1	11	6	0	4	$3\frac{17}{21}$	—	2	$11\frac{17}{21}$
1	13	0	0	4	$1\frac{1}{11}$	—	2	$9\frac{1}{2}$
1	14	6	0	3	$11\frac{8}{23}$	—	2	$7\frac{7}{23}$
1	16	0	0	3	$9\frac{1}{3}$	—	2	$5\frac{2}{3}$
1	17	6	0	3	$7\frac{13}{25}$	—	2	$2\frac{13}{25}$
1	19	0	0	3	$5\frac{11}{13}$	—	2	$1\frac{11}{13}$
2	0	6	0	3	$4\frac{8}{17}$		2	$0\frac{8}{17}$

The price of a quarter of Wheat.			The weight of the Half-peny white loafe in Troy weight by Assise.			The weight of the peny white loafe in Troy weight by Assise.		
l.	s	d	po.	ou.	peny weight	po.	ou.	peny weight
0	3	0	4	6	8	9	0	16
0	4	6	3	0	$5\frac{1}{3}$	6	0	$10\frac{2}{3}$
0	6	0	2	3	4	4	6	8
0	7	6	1	9	$15\frac{4}{5}$	3	7	$10\frac{2}{5}$
0	9	0	1	6	$2\frac{2}{3}$	3	0	$5\frac{1}{3}$
0	10	6	1	3	$10\frac{6}{7}$	2	7	$1\frac{5}{7}$
0	12	0	1	1	12	2	3	4
0	13	6	1	0	$1\frac{7}{9}$	2	0	$3\frac{1}{9}$
0	15	0	0	10	$17\frac{1}{6}$	1	9	$15\frac{1}{5}$
0	16	6	0	9	$17\frac{2}{11}$	1	7	$15\frac{7}{11}$
0	18	0	0	9	$1\frac{1}{3}$	1	6	$3\frac{2}{3}$
0	19	6	0	8	$7\frac{1}{13}$	1	4	14^{2}
1	1	0	0	8	$3\frac{1}{3}$	1	4	$0\frac{2}{5}$
1	2	6	0	7	$5\frac{1}{15}$	1	2	$10\frac{2}{15}$
1	4	0	0	6	16	1	1	12
1	5	6	0	6	8	1	0	16
1	7	0	0	6	$0\frac{8}{9}$	0	0	$1\frac{7}{9}$
1	8	6	0	5	$14\frac{12}{19}$	0	11	$9\frac{1}{19}$
1	10	0	0	5	$8\frac{4}{5}$	0	10	$17\frac{1}{5}$
1	11	6	0	5	$3\frac{14}{21}$	0	10	$7\frac{5}{21}$
1	13	0	0	4	19	0	9	18
1	14	6	0	4	$14\frac{14}{21}$	0	9	$9\frac{5}{23}$
1	16	0	0	4	$11\frac{1}{3}$	0	9	$2\frac{2}{3}$
1	17	6	0	4	$7\frac{1}{25}$	0	8	$14\frac{2}{26}$
1	19	0	0	4	$3\frac{2}{25}$	0	8	$7\frac{5}{23}$
1	0	6	0	4	$0\frac{16}{17}$	0	8	$1\frac{5}{29}$

Scholar. Sir I do thanke you most heartily for this, not onely in mine owne name, and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assie (I trust) shall bring restitution of the weight in Bread, which long time hath béen abused. And if you know any like things more, wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtain great thanks of all good hearted men that love the Common-wealth.

Master. I have sundry things to declare, but I have reserved them for a private booke by it selfe, yet notwithstanding because the Statute of the rate of measuring of ground is so common, that it toucheth all men, and yet no more common then néedfull, but so much corrupt, that is, too farre out of all good rate, not onely in the English books of Statutes, commonly printed, but also in the Latine books, and in the French also (for I have read of each sort, and conferred them diligently) I will give you a Table for the restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the use of that Statute, whereby you may perceive the order how to examine the whole Statute, and every parcell thereof, and the question is this.

A question of measuring of ground.

Whether the Acre of ground doth contain four Perches in breadth, then must it contain 40 Perches

ches

ches in length. Then doe I demand of you, how much shall the length of an acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declare unto you another Statute, which is the ground of the former Statute. And that Statute is this.

It is ordained that three Barly corns dry and round, shall make up the measure of an inch: 12 inches shall make a foot, and 3 foot shall make a yard, (the common English books have an Elne) five yards and an halfe make make a Perch, & 40 Perches in length, and 4 in breath, shall make an Acre. This is that Statute, whereby you may perceive, that the intent of the Statute is, that one Acre should contain 160 square Perches. Now let me heare you answer to the question.

Statute measure.

An Acre.

Scholar. As I perceive by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an acre, so could I make those numbers all in Fractions, and so worke the question: but seeing I may do it also in whole numbers, I take that forme for the most ease, therefore thus I set the question in forme. Then do I multiply 40 by 4, and it maketh 160, which I divide by 13, and the quotient is $12\frac{4}{13}$.

4	40
13	$12\frac{4}{13}$

Master. Now turn that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former Statute: howbeit it shall be best to take one of the least parts in Denominaton

for avoiding of much labour, as Feet, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to returne $\frac{4}{13}$ into Feet, I multiply $16\frac{1}{2}$ by 4. and it maketh 66, which I must divide by 13, and the quotient is $5\frac{1}{13}$.

Master. So I finde, that if the Acre hold in breadth 13 Perches, it shall contain in length 12 Perches. 5 Foot, and $\frac{1}{13}$ of a Foot, which is not fully an Inch, for the Inch is $\frac{1}{12}$ of a Foot. But here all the Statute books in Latine and English (that I have seene) do note it to be 13 Perches, 5 Foot, and one Inch, which maketh above 13 Perches too many in the Acre: so that I would have thought the errour to have crept into the Printed books, by the great negligence that Printers in our time do use, save that in written Copies of great antiquity, I do find the same: yet have I one French copy which hath 12 Perches $\frac{1}{4}$ and one Foot, and that misseth very little of the truth.

Note this errour.

Scholar. Then I see it is true that I have often heard say, that the truest Copies of the Statutes, be the French Copies.

Master. That is often true, but not generally, as I have by conference tried diversly: but in this Statute the French book is most corrupt: in all other places lightly.

But now to performe my promise, I will set forth the Table for measuring of an Acre of ground, onely by such parts as the Statute doth mention; because at this time I doe of pur-

purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table following, I have not done as in the other Statute before compared by restitution with the faults crept into the Statute, but onely have written that true measure, which the equity of the Statute doth pretend. For it were vile to judge of so noble Princes and worthy Counsellours, as have authorised and set forth this Statute, that they would make one Acre in any form greater then another, but every one to be just and equall with each other, which is the ground also of my worke: and hereby may all men perceive how needfull Arithmeticke is to the Students of Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it none obscurity that I should need to declare.

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feet.	Inches.	Parts of an Inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{2}{17}$
18	8	14	8	0
19	8	6	11	$\frac{2}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{2}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	7	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feet.	Inches.	Parts of an Inch.
28	5	11	9	$0\frac{1}{7}$
29	5	8	6	$0\frac{12}{19}$
30	5	5	6	0
31	5	2	7	$0\frac{22}{11}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$0\frac{11}{17}$
35	4	9	5	$0\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$0\frac{2}{37}$
38	4	3	5	$0\frac{11}{19}$
39	4	1	8	$0\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$0\frac{22}{41}$
42	3	13	4	$0\frac{1}{7}$
43	3	11	10	$0\frac{12}{43}$
44	3	10	6	0
45	3	9	2	0

Scho-

Scholar. Indeed, Sir, I understand the Table (as I think) by those other which you set forth before. For in the first columne is set the Perches of the breadth of an Acre and then in the two columnes following appeareth how many Perches, and how many foot that same Acre must have for his length.

Master. You take it well: howbeit to speak exactly of breadth and length, and the first columne doth somtime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remain, although the proportions of the numbers do change: for still by the first columne is expressed the measure of the one side, and by the two other Pillars in one columne, is set forth the measure of the other side. And this shall be sufficient now for the use of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certaine other Rules which for their severall names may seeme divers Rules, and distinct from this, but indeed they are but branches of it: yet because they have severall workings in apearance, but also pleasant in use, I will give you

you a taſte of each of them. As for the *Rule of Fellowſhip*, both ſingle and double, with time and without time, I ſhall need to ſay little more then I have already ſaid in teaching the works of whole numbers: yet an example or two will we have to refreſh the remembrance of the ſame; and to declare certain proper uſes and applications of it, as this for one.

The Rule of Fellowſhip without time.

Foure men got a booty or prize in time of war, the prize is in value of money, 8190 pound, and becauſe the men be not of like degree, therefore their ſhares may not be equall: but the chiefeſt perſon will have of the bootie the third part, and the tenth part over: the ſecond will have a quarter, and the tenth part over, the third will have the ſixth part: and ſo there is left for the fourth man a very ſmall portion, but ſuch is his lot (whether hee be pleaſed or wroth) hee muſt be content with one 20 part of the prey. Now I demand of you what ſhall every man have to his ſhare?

A queſtion of inequall ſociety.

Scholar. You muſt be faine to anſwer to your own queſtion, elſe it is not like to be anſwered at this time.

Maſter. The forme to underſtand the ſolution of this queſtion, and all ſuch like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of ſome other of them, for all ſuch parts you may overpaſſe, and take for them all thoſe numbers, whoſe parts they be: As in this example the ſhares be theſe $\frac{1}{3}\frac{1}{10}$, $\frac{1}{4}\frac{1}{10}$, $\frac{1}{6}\frac{1}{10}$, if

I

I multiply all the Denominators together, beginning with 3, and so go on unto 20, it will make 144000: but considering that 3 is a part of 6 I will omit that 3, and likewise ten, which is a part of 20, I may overpasse also, and then there is but 3 denominators to multiply, that is, 4. 6, & 20 which make 480, which summe I take for my worke, because all the Denominators will be found in it. Then I take such parts of it as the questiō importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 100, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$. which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must have $\frac{1}{6}$ I take 80. And for the fourth man there remayneth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had béene but 480 pound, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the bootie. And looke what proportion is betweene the first number and the second, the same proportion shall be betweene the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third

The reason of this Rule.

third

third place,one of the parts or ſhares,and then worke by the former Rule of proportion or Golden Rule. And becauſe I have foure ſeve-rall parts of the firſt number, by which I would finde out foure like parts of the ſecond number, therefore muſt I make foure ſeverall figures.

Scholar. Now I truſt I can anſwer to your queſtion,as by your favour I will prove.

And to try it, I ſet the foure figures thus, marked with A,B,C,D, to ſhew their order:

A		B	
480	8190	480	8190
208		160	
C		**D**	
480	8190	480	8190
80		24	

And then in each of them I multiply the ſe-cond number by the third, and divide their totall by the firſt,and ſo amounteth the fourth ſumme which I ſeek for: For if I do multiply 8190 by 208,it maketh 1733520, which being divided by 480, maketh in the quotient 3549 for the firſt mans portion.

And ſo working with the other three fi-gures, I finde for the ſecond man 2866 $\frac{1}{2}$, and for the third man 1365 ,and then for the fourth man 409 $\frac{1}{2}$, and ſo every mans ſhare is ſet forth in the figure here annexed.

A		B	
480	8190	480	8190
208	3549	168	2866$\frac{1}{2}$
C		**D**	
480	8190	480	8190
80	1365	24	409$\frac{1}{2}$.

And thus much I think I have done well.

The proof by Addition.

Master. If you misdoubt your working, and list to prove it, add all the shares together, and if they make the totall, then seemeth it well done.

Scholar. I may set them thus: and then by Addition the iust summe doth amount, that is, 8190, and therefore (as you say) it seemeth to be well wrought.

3549
2866$\frac{1}{2}$
1365
409$\frac{1}{2}$
8190

But I beseech you, is there any doubt in this triall, that you use that word, Seemeth?

Master. You may easily coniecture, that if you did assigne the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appeare all one, and therefore is not the proofe exact.

The just proof.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ and $\frac{1}{16}$ of the whole summe, and if it agree with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proofe faileth not. Now will I propound certain other questions, which have beene set

forth

forth by certain learned mee, albeit not without some oversight, which questions I protest heartily, I do not repeat to depraue those good men, whose labours and studies I much praise and greatly delight in. But onely according to my profession, to seek out truth in al things, and to remove all occasions of errour as much as in me lyeth: and for that cause I will onely name the questions without hurting the Authors name.

The first question is this.

Foure men did build an house, which cost them 3000 crowns, their shares were such that one man should pay $\frac{1}{2}$ of the summe, and six Crowns over: the second should pay $\frac{1}{3}$ and 12 Crownes over: the third man must lay out $\frac{1}{3}$ abating 8 Crowns: and the fourth man should pay $\frac{1}{4}$ and 20 Crownes more. Can you answer to this question? A question of building.

Scholar. No, I cannot Sir, and that you know best of any man, for I know no more then you have taught me.

Master. Then I dare say you cannot doe it, neither yet the best learned man that ever did propose it: for the question is impossible. For declaration whereof I will be bold to use first the representation of the numbers in their aptest forme (although I have not yet taught that manner of worke) because it may appeare plainly that the question is not possible. For here I have set the parts, and added them, and they make the whole summe, and $\frac{1}{4}$ and 30 more. Now, how is it possible An impossible question.

to

to diuide truly either gains, either charges, so that the particulars $1\frac{1}{4}$ shall be more then the totall?

$$\left\{\begin{array}{l} \frac{1}{2} + 6 \\ \frac{1}{3} + 12 \\ \frac{2}{3} - 8 \\ \frac{1}{4} + 20 \end{array}\right\}$$

$$1\tfrac{1}{4} + 30$$

Scholar. It is against the forme of proof by addition of parts.

Master. You say truth. And (because you shall perceiue it the better) I will try it after the vulgar forme, as in this figure you see where the $\frac{1}{2}$ with 6 over, is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would bee 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, doe make 5280, where the totall summe should be but 3000, which sum of 3000 if you diuide by $1\frac{1}{3}$ or $\frac{1}{4}$, you shall haue $\frac{1}{4}$ of it, that is 2250, and thereto adde 30 more, then will those 3 summes make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more then the whole summe by $\frac{1}{4}$, and thirty more, and therefore can that question not be accepted as a possible thing, but yet doe certaine learned men propound such questions, and answer to them:

$$\begin{array}{r} 1506 \\ 1012 \\ 1992 \\ 770 \\ \hline 5280 \end{array}$$

$$\begin{array}{r} 2000 \\ 2250 \\ \hline 5280 \end{array}$$

There-

Therefore somwhat to say to their excuse (rather of their good meaning, then for their doing) I will anon declare what may be said for their defence: but in the meane season, I will propound the Question as it may be wrought by good possibility.

As if foure men build a house together, and it cost them 3000 crowns; and then for the partition they agree thus: that as often as the first man doth pay 6 crowns, so often the second man shall pay 4, the third man 8, and the fourth man 3. Or else thus, that the first man shal pay double so much as the fourth, and the second man shall pay $\frac{2}{3}$ *of the first mans charge: the third man shall double so much as the second: (and these two wayes are to one end) but further for their agreement it is appointed also, that the first shall give 6 crowns overplus, and the second 12, and the fourth shall give 20: but the third man shal give no overplus, but shall have 8 crowns abated of his charge.*

The former question of building now possible.

Now is the question possible to be assoyled; and this is the way to doe it. Marke the proportion of the severall charges, and set out small numbers in that rate, by which you may reduce the work to the Golden Rule, as here in the first form, the numbers are already named, 6, 4, 8, 3: and in the second forme (although they be but plainly named, yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and again 8 is double to 4. Now adde these together, and they make 21, which 21 must be set for the first number in the Gol-

 den

den Rule: for if it with the overplus of each mans charge would make the totall summe of the charges, then were those severall summes the charges of each man, besides his overplus: but now it is not so.

The Rule. But yet this is true: (so excellent are conclusions Arithmiticall) that looke what proportion each of their severall sums doth beare to 21, the same proportion doth the just charges of every man (besides his overplus) beare to the totall of the charges, the overplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the Golden Rule, you must deduct the overplus, which is 6, 12, and 20 that is in the whole, 38: but then 8 must be restored for the abatement of the third man, and then remayneth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule, for the second summe: and for the third summe, I must put each of the small numbers before-mentioned, which although they be not severall charges, yet they represent them in proportion. And so making for every mans charge a severall question, the figures will be 4, which I mark with foure letters, A, B, C, D, thus.

A		B	
21	2970	21	2970
6	$848\frac{4}{7}$	4	$565\frac{5}{7}$

C		D	
21	2970	21	2970
8	$1131\frac{4}{7}$	3	$424\frac{2}{7}$

Where I have set for brefnesse the summe of every mans charge in the fourth place, presupposing that you can tel how to try out that fourth summe by so many Examples as yee have had.

Scholar: As I trust that I understand this forme, so I desire much to know what may bee said for them that mistooke this Question.

Master. You seeme so desirous to know this errour, that you have forgotten to examine, whether this work be without fault.

Scholar. Me seemeth this worke to be well done, because the Addition of the foure severall numbers doth make the totall summe of 2970, which was to be divided into such four parts.

Master. But then have you forgotten that the first man must pay six Crownes more besides his share, & the second man 12 Crownes more, the third man 8 Crownes lesse, and the fourth man 20 Crownes more: for without these your first totall of 3000 Crownes will not be made.

Scholar. Then must I adde to the first mans summe 6 more, and then it will be $854\frac{2}{4}$, and and to the seconds summe, I must adde 12, and it will be $577\frac{2}{7}$: from the thirds summe I must abate 8, and then will the summe be $1123\frac{1}{7}$: then adding unto the fourths summe

20, it will be $444\frac{2}{7}$, and these foure summes will make 3000, which is the whole change, as in this example it may appeare, where first I gather the $\frac{14}{7}$, that maketh 2, and so procéed I in the Addition to the end.

$854\frac{4}{7}$
$577\frac{2}{7}$
$1123\frac{1}{7}$
$444\frac{2}{7}$
3000

Master. Now have you well done, and this work in the same summes, is brought of other learned men for the true solution of the question, as it was first proponed, which (as I said) was impossible: and now examine by these severall summes, and sée whether it doth agrée with the summes in the question proponed.

8

The first man must pay $\frac{1}{2}$ and 6 over of the totall summe: how think you, is $854\frac{4}{7}$ the half, and 6 more of 3000?

Scholar. No that it is not, for it should be 1506: and for the second man 1012: and for the third man 1992, & for the fourth man 770: whereof not one summe agréeth to this work. But I marvell, that so wise men could be so much over-séen.

Master. It is commonly séene, that when men will receive things from elder Writers, and will not examine the thing, they séeme rather willing to erre with their Ancients for company, then to be bold to examine their Works or Writings. Which scrupulositie hath ingendred infinite errors in all kindes of

know-

knowledge, and in all civill administration, and so in every kind of Art. But these learned men did not mean any other thing by this question, then to finde such numbers as should beare the same proportion together, as those numbers in the question proportioned did beare one to another: which thing you shall perceive more plainly by another question of theirs, that is this.

A man lying upon his death-bed, bequeathed his goods (which were worth 3600 Crownes) in this sort. Because his Wife was great with child, and he yet uncertaine whether the Childe were male or female, he made his bequest conditionally, that if the Wife bare a Daughter, then should the Wife have halfe his goods, and the Daughter $\frac{1}{3}$; *but if she were delivered of a Sonne, then that Sonne should have* $\frac{1}{3}$ *of the goods, and his Wife but* $\frac{1}{3}$. *Now it chanced her to bring forth both a Sonne and a Daughter; the question is: How shall they part the goods agreeable to the Testator his Will?*

A question of a Testament.

Scholar. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite voide, and so the Man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it, and also it might have chanced that shee should have brought forth neither Sonne nor Daughter, as often hath bin seene: so is the Will insufficient to that point also.

Master. Such Scanners should seeme to cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken favourably for the aide of the Ligatories, when there ariseth such doubt. But let us try this worke, not by force of Law, but by proportion Geometrical, seeing the Testator did minde to provide for each sort of them.

Scholar. If the Sonne shall have $\frac{1}{2}$ by force of the Testament, so must the Mother have $\frac{1}{3}$. Again, because she hath a Daughter also, therefore ought she to have $\frac{1}{2}$, and the Daughter $\frac{1}{3}$, that is both wayes $\frac{1}{2}\frac{1}{3}$, and $\frac{1}{3}\frac{1}{2}$, which commeth to the whole goods, and $\frac{2}{3}$ more.

Wherefore it seemeth also impossible.

Master. In this matter, the minde of the Testator is so to be understood, that such proportion should be betweene the portion of the Wife, and the Sonne, as is betwéen $\frac{1}{1}$ & $\frac{1}{2}$, that is, the Sonne must have $\frac{3}{6}$ for $\frac{2}{6}$ to his Mother, so shall hee have 3 to 2 that is, as much as his Mother, and halfe as much more; and the Mother must have the like rate in comparison to her Daughter. Then must I finde out three numbers in such proportion, that the first may have as much as the second, and halfe as much more (that is) in proportion sesquialtera, and the second to the third, in that same proportion: such numbers be 9,6,4.

Scholar. I pray you Sir, how shall I finde out these numbers?

Master. That will I gladly tell you.

What

Whatsoever the proportion be of any three numbers, multiply the Tearms of that proportion together, and the number that amounteth, shall be the middle number of the three : then multiply that middle number by the lesser tearm, and divide that total by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreame, and divide the totall by the lesser extream, then will the greatest number of that Progression amount.

To finde 3 numbers in any proportion.

Scholar. Then in this example to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must divide (as you taught me in division) $\frac{1}{2}$ by $\frac{1}{3}$, and the quotient will be $\frac{3}{2}$, that is, $1\frac{1}{2}$, whereby I perceive that the proportion in this question, is as 3 to 2. Therefore as you taught mee even now, I multiply 3 by 2, and the summe is 6, which must be the middle number : then I multiply the middle number 6 by 2, which is the least tearme, and the summe is 12, that I doe divide by 3, being the greater tearme, and the quotient is 4 : so is 4 the least number of the three. Then I multiply 6 by 3, whereof commeth 18, and that I divide by 2, and so have I 9, which is the greatest number of the three.

To finde the proportion between two numbers.

Master. Another way yet may you finde the third number in any Progression, if you have two of them : for if the middle number be one of them which you have, then multi-

ply it by it selfe (as in their example, 6 by 6, maketh 36) and that totall divide by the other number which you have, and the third number will be the quotient.

Scholar. Then I divide 36 (which commeth of 6 multiplyed by it selfe) by 4 the quotient will be 9: and if I divide 36 by 9, the quotient will be 4. But what if I know the first number and the third, and would have the middle number?

Master. Multiply the two numbers together, and in their totall you must seeke the root of that number, and it shal be the middle number: but because as yet you have not learned to extract Roots, therefore use the first forme which I have taught you, till I teach you to extract Roots. And now go forwards with the answer of the same question.

Note.

Scholar. I perceive then, that the Sonne must not have $\frac{1}{3}$ of the goods, neither the Mother $\frac{1}{3}$, nor yet the Daughter $\frac{1}{3}$, but yet must the goods be divided into such proportion, that the Sonne shall have 9 Crownes for 6 to his Mother, and the Mother shall have 6 Crownes for every 4 to her Daughter. Then I apply it to the Golden Rule in three examples, as followeth.

Where the first number is the Addition of those three numbers 9, 6, 4: and the third is one of them severally: the second is the totall of the goods in that Testament: and then

by

by the worke of the Golden Rule, I finde out the fourth number in every work: that is for the Son $1705\frac{1}{19}$, for the Mother $1136\frac{16}{19}$, & for y^e Daughter $757\frac{17}{19}$: the which ſummes added together, doe make the ſumme of the whole goods, as may be ſeen by this Example.

19 — 3600
9
16 — 3600
6
19 — 3600
4

$$\begin{array}{r} 1705\frac{1}{19} \\ 1136\frac{16}{19} \\ 757\frac{17}{19} \\ \hline 3600 \end{array}$$

And this (mee thinketh) I doe perceive, becauſe in this caſe there is a neceſſary remedy deviſed againſt an urgent inconvenience: therefore thoſe learned men thought they might uſe the like liberty in that other queſtion.

Maſter. Your gheſſe is good, but they had ſo good reaſon for them in the one, as they have in the other: As in another example of theirs, it may better appeare, as in this.

A man left unto his three ſonnes 7851 crowns to be parted in ſuch ſort, that the firſt Son ſhould have $\frac{1}{2}$, the ſecond Son $\frac{1}{3}$, and the third Sonne $\frac{1}{4}$, which is not poſſible: for $\frac{1}{2}$, and $\frac{1}{3}$, and $\frac{1}{4}$ do make $\frac{26}{24}$, or $\frac{13}{12}$, that is, $1\frac{1}{12}$, ſo it is more than the whole, but reduce theſe Fractions into one denomination, the leaſt that they will come to, and they will Another queſtion of a Teſtament.

be

be $\frac{6}{13}, \frac{4}{13}, \frac{3}{13}$, and so may you part the goods into such proportion as these three numerators beare together, that is, the first to have 6 for every 4 to the second, & the second to have 4 as often as the third hath 3; and so their portions will be for the first, $3623\frac{7}{13}$, for the second $2415\frac{9}{13}$, and for the third $1811\frac{10}{13}$; and these three shares added together, will make the totall summe of the whole goods, as you may easily see in this example.

$$\begin{array}{r} 3623\frac{7}{13} \\ 2415\frac{9}{13} \\ 1811\frac{10}{13} \\ \hline 7851 \end{array}$$

Another Question is there proponed thus

There are 450 crownes to be divided between three men, so that the first man must have $\frac{1}{2}$ and $\frac{1}{3}$, the second man $\frac{1}{3}$ & $\frac{1}{4}$; the third man shall have $\frac{1}{4}$ and $\frac{1}{5}$.

Another like question.

Scholler. I marvaile that any man should be so overseen, to propone ye question as a thing possible, sith $\frac{1}{2}\frac{1}{3}, \frac{1}{3}\frac{1}{4}, \frac{1}{4}\frac{1}{5}$, doe make $1\frac{13}{15}$ that is almost double the whole summe.

☞ But I perceive it might be thus proponed: that as often as the first man did receive 50 Crownes, so often the second man should receive 35, and the third man 27: for $\frac{1}{2}$ and $\frac{1}{3}$ is equall to $\frac{50}{60}$: and so is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{35}{60}$, and $\frac{1}{4}$ & $\frac{1}{5}$ is $\frac{27}{60}$: and so working ye question the three figures will appeare in this form: whereby the first mans portion is found to be $200\frac{50}{56}$: the second mans part is $140\frac{35}{56}$

112	450	
50		$200\frac{50}{56}$
112	450	
35		$140\frac{35}{56}$
112	450	
27		$108\frac{27}{56}$

140$\frac{2}{3}\frac{1}{6}$: the third mans share 108$\frac{4}{5}\frac{7}{6}$: which in the whole doth make 450 Crownes to bee devided between them.

Master. And thus you are (I thinke sufficiently instructed in the Rule of Fellowship.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation ; which hath his name, for that by it there are divers parcells of sundry prices, and sundry quantities, alligate, bound, or mixed together : whereby also it may bee well called the Rule of Mixture ; and it hath great use in composition of Medicines , and also in mixtures of Mettalls, and some use it hath in mixtures of Wines : but I wish it were lesse used therein then it is now a dayes. The order of this rule is this.

The Rule of Mixture.

When any summes are proponed to bee mixed , set them in order one over another, & the common number (whereunto you will reduce them) set on the left hand ; then marke what summes bee lesser then that common number, and which be greater, and with a draught of your Penne evermore linke two numbers together, so that one be lesser than the

The reason of this Rule.

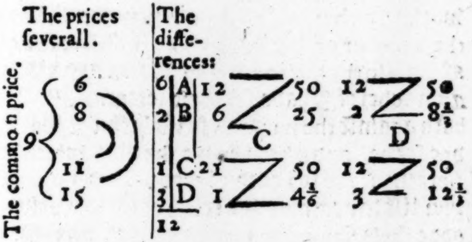
the common number, and the other greater than he: (for two greater, or two smaller cannot well be linked together) and the reason is this, than one greater and one smaller, may be so mixed, that they will make the meane or common number very well: but two lesse can never make so many as the common number, being taken orderly: no more can two summes greater than the mean, ever make the meane in due order, as it shall appear better to you hereafter. And as it is of necessity to link every smaller (once at the least) with one greater, and every greater with one smaller, so it is at liberty to linke them oftner than once, and so may there be to one question, many solutions. When you have so linked them, then marke how much each of the lesser numbers is smaller than the meane or common number, & that difference set against the greater numbers, which be linked with those smaller, each with his match still on the right hand, & likewise the excesse of the greater numbers above the meane, you shall set before the lesser numbers, which bee combined with them. Then shall you (by addition) bring all these differences into one summe, which shall be the first number in the golden Rule, & the second number shall be the whole masse that you will have all those particulars: the third summe shall be each difference by it selfe, and then by them shal be found the fourth number, declaring the just portion of every particular in that

that mixture: As now by these Examples I will make it plaine.

There are foure sorts of Wine of several prices, one of 6 pence *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence *the gallon. Of al these Wines would I have a mixture made to the summe of fifty gallons, and so the price of each gallon may be* 9 pence. *Now demand I, how much must be taken of every sort of Wine?* A questiõ of mixing of wines.

Scholar. If it shall please you to work the first example, that I may marke the applying of it to the rule: then I trust I shall be able, not only to doe the like, but also to see the reason in the order of the worke.

Master. Marke then this forme, and the placing of every kind of number in it.



Here you see I have set downe the severall

rall

rall prices, which be 6, 8, 11, 15, I have linked together 6 with 15, and 8 with 11. The common price 9, I have set on the left side, and the difference betweene it, and every particular price, I have set on the right hand, not against the summe (whose difference it is) but against the summe that is linked withall, so the difference of 15 above 9, is 6, which I have set, not against 15 but 6, that is linked with 15, and the difference between 6 and 9 (that is 3) I have set against 15. So likewise the difference between 8 & 9, is but 1, that I have set against 11, and the difference of 11 above 9 (which is 2) I have set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I should have, and the third summe is every particular difference. Now if you worke by the Golden Rule, you shall finde the number of Gallons that shall bee taken of each sort of Wine. For the better distinction wherof, I have set these letters, A, B, C, D both against the numbers for which the workes doe serve, and over the worke also, which severally serve for each of them. And now (if you list to examine the truth of these workes) adde these foure summes together, and they will make 50, that is the totall which I

The proofe of this Rule.

would

would have, as by this example you may easily perceive. And (for to prove how the prices do agree) doe this: multiply the totall summe 50, by the common price 9, & it will make 450: then keepe that summe by it selfe, and afterward multiply every severall summe of Gallons, by the price belonging to the same Gallons, and if that sum doe agree with this, which you have kept first, then is your worke well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: so againe $4\frac{1}{7}$ multiplyed by 11, doth make $45\frac{5}{6}$. And last of all, $12\frac{1}{2}$ multiplyed by 15, maketh $187\frac{1}{6}$: and these added together do make 450, as as in the Example annexed you may see, wherfore seeing it doth agree with ye former sum of 50, multiplyed by 9, I must iustly affirme this worke to be good, and well done.

25
$8\frac{1}{3}$
$4\frac{1}{6}$
$12\frac{1}{2}$
50

150
$66\frac{4}{6}$
$45\frac{5}{6}$
$187\frac{1}{6}$
450

And now to prove how you can doe the like, I propound the same question, onely willing you to use some other forme of combining or linking the summes. The variation of this question.

Scholar. That shall I prove with your favour, & therefore I combine 8 with 15, and 6 with 11, & then the form wil be as followeth,

6

			A		B	
	6	2 A	12	50	12	50
	8	6 B	2	$8\frac{1}{6}$	6	25
9	11	3 C	C		D	
	15	1 D	12	50	12	50
			3	$1\frac{1}{6}$	1	$4\frac{1}{6}$
		12				

Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I understand the reason by your former worke. And therefore here appeareth no strange thing, but that now I have $8\frac{1}{6}$ Gallons of 6 pence, and 25 gallons of 8 pence, and 12 gallons and $\frac{1}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{1}{6}$ by 6, it maketh 50. and then 25 multiplied by 8, maketh 200 likewise $12\frac{1}{6}$ multiplyed by 11 yeeld $137\frac{1}{6}$, and $4\frac{1}{6}$ multiplyed by 15 maketh $62\frac{1}{6}$, which 4 summes addec into one, will yeeld in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of gallons) be 9 the common or meane price.

50
200
$137\frac{1}{2}$
$92\frac{1}{3}$
450

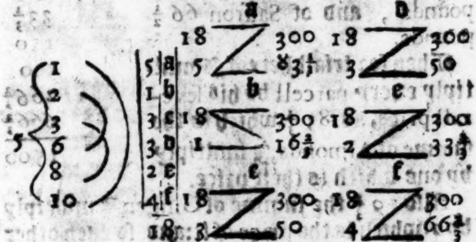
Master. Seeing you conceive this worke so well, I will propound another example unto you of more variety in the Alligations or combinings, as thus.

A Merchant being minded to make a bargain for Spices, in a mixt masse (that is to say) of Cloves, Nutmeggs, Saffron, Pepper, Ginger, and Almonds: the Cloves being at 6 shillings, Saffron at 10 shillings, Pepper at 3 shillings, Ginger at 2 shillings, and Almonds at 1 shilling. A question of Spices.

Now would he have of each sort some, to the value of 300 pound in the whole, and each pound one with another; to beare in price 5 shillings: How much shall hee have of each sort?

Scholar. That will I try thus.

First I set downe those six severall prices, and at the left hand I set the common price 5 shillings. Then I linke them thus, 1 with 10, 2 with 6, and 3 with 8: as in the example following.



Master. I had minded to have combined them in more variety: but I am content to see your own worke first, and then more varieties in combination may follow anon.

Scholar. Then to continue as I beganne, I seeke the difference between 1 and 5, (which is 4) and that I set against 10; then against 1 I set 5, which is the excesse of 10 above 5, so I gather the difference betweene 2 and 5, which is 3: and that I set ainst 6, because it is combined with 2: and likewise the difference of 6 above 5, (which is 1) I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 above 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden Rule, and so appeareth by those works, that of Almonds I must take $83\frac{1}{3}$ pound, of Ginger $16\frac{2}{3}$ pound, Pepper 50 pounds, of Cloves 50 pounds, of Nutmegs $33\frac{1}{3}$ pounds, and of Saffron $66\frac{1}{2}$ pounds.

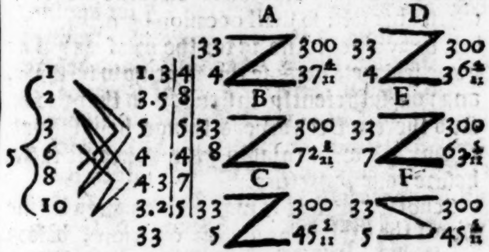
	$83\frac{1}{3}$
	$33\frac{1}{3}$
	150
	300
	$266\frac{2}{3}$
	$666\frac{2}{3}$
	1500

Then for triall hereof, I multiply every parcell by his severall price, as $83\frac{1}{3}$ which is the summe of Almonds, I multiply by one which is their price.

Also $19\frac{2}{3}$ the summe of Ginger, I multiply by 2, which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them altogether I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, wherefore

fore it appeareth well wrought.

Master. Now I will make the alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as by this Example you may see, where I haue



alligated 1 with 6 and 8, and therefore haue I set against 1 both their differences, that is 1 and 3: Likewise, because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I haue set onely 5, which is the difference of 10, with whom 3 is combined onely. Likewise 6 is onely alligate to 1, & therfore is the differences of 1 from 5, which is 4 onely set against it: 8 is linked with 1 & 2, and therefore hath set 4 against him, both their differences, 4 and 3: and 10 is ioyned with 2 and 3, therefore hath hee their differences, 3 and 2. And because of ease for you, in another columne I haue set the differences reduced into one number, for euery seuerall sort,

and have also added them together, whereby appeareth that they make 33, & so consequently you see the workes of the Golden Rule set forth. For the six Drudges I have added the letters A, B, C, &c. as before.

But I would not wish you to cleave still to these elementary aids, but accustome Memory to trust her self: so shall occasion of negligence best be avoided. And as for the proof try it at more leisure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth divers things behinde yet, of which I would gladly give you some taste, before your departure.

Note

Scholar. But if it may please you to let me see all the variations of this question, before you go from it, for me thinketh I could vary it two or three wayes more yet.

Master. I am content to see you make two or three variations: but I would be loath to stay to see all the variations: for it may be varied above 300 wayes, although many of them would not well serve to this purpose.

Scholar. I thought it impossible to make so so many variations.

Note.

Master. Marvell not thereat, for some questions of this Rule may be varied above 1000 wayes; but I would have you forget such fantasies till a time of more leisure. And now goe forward with some variation of this question.

Scholar. For the first variation, I linke th

the first number 1 with 8 and 10, and 2 I combine with 9 and 10: then ioyne I 3 with 6, 8, and 10, as in this forme.

And so doth there appeare the proportion of weight for every kinde of Drugge in this mixture. Now for the triall.

Master. Nay stay there: you shall not need to make triall in one example so often, or if you list to do it by your self, I am content. But now set forth (for declaration that you conceive the Rule) two or three examples of severall combinations, and then will wee passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties; in which, as the

combinations are seuerall, so doth it plainly appeare, that the differences by which the proportion of each seuerall kind is taken, are also seuerall. And yet I see in the three first of these five varieties, and in one other before, the totall summe of the differences to be one, that is to say, 18, whereby I perceiue that the variety of their mixture doth depend on the variety of their differences seuerall, and not of the variety of their totall summe.

Master. So is it. And seeing you conceiue it so well, I will make an end of this Rule, onely exhibiting vnto you one Question or two of the mixture of Metalls, that by it you may deuise others like, and exercise your selfe therein also, because the vse of it serueth often

in

in businesse of charge, not so much for Gold-smiths, as of coynage in Mints. First, I demand of you this question; *If a Mynt-Master haue Gold of 22 Karects, and some of 23 Karects, some of 24: Againe some 15 some 16, and some of 18 Karects and would mix them, so that he might haue 100 ounces of 20 Karects: How much must he take of each sort?*

Scholar. To know that, I answer in order thus.

Mast. You haue wrought the question well: but how chanced you made no doubt of that new name Kareet.

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much haste to end: and again in this case the proportion of the number, is sufficient for my purpose in this worke: trusting that another time you will instruct mee as well of this, as of sundry other things, which as I haue heard you talke of, so I haue a great desire to them.

Master. Your answer is reasonable, and your request and trust (with Gods helpe)

I intend to satisfie. And now to goe forward with this matter, let me see your examination of this last work.

Scholar. First for the one part I adde together all the particular summes, as they appeare in the worke, and they make 100, as here by their Addition doth appeare. And so it seemeth that the summes are well gathered: but for the further triall of them: I multiply first

	10
	15
	20
	25
	20
	10
	100

20 which is the common or meane summe of the Karects by 100, which is the summe of the whole Masse, which I would have, and it maketh 2000. Then I multiply every particular summe by the Karects that it doth containe, as 10 by 15, and that maketh 150.

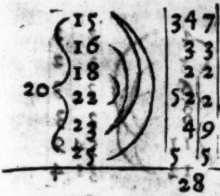
120
240
360
550
460
240
2000

Likewise I multiply 15 by 16, and it yeeldeth 240: so 20 by 18, maketh 360. And 25 by 22 yieldeth 550: likewise 20 by 23, bringeth forth 460; and last of all, 10 multiplyed by 24. yieldeth 240: which summes all joyned together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And now (if it please you) I would set forth some varieties of this question) to prove my wit.

Master. Go to let me see.

Scholar. Here be foure varieties.

And



And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessity.

But now for your exercise in this Rule, one other question I will propose. A Mint-master hath six Ingots of silver of sundry finenesse, some of foure ounces fine, and some of five ounces, some of six and other of eight, some of 11, and other of 12, and his desire is to mixe 500 pounds weight, so that in the whole masse every pound weight should beare nine ounces of fine silver: How much shall he take (say you) of every sort of silver? A question of mixing of silver.

Scholar.

Scholar. To finde out that, I set the numbers thus in order.

And gathering the differences, it will appeare, that, of the first sort there must be 43 $\frac{11}{23}$ of the second like much: of the third sort 65 $\frac{1}{23}$: and of the fourth sort as much: of the fift sort 195 $\frac{11}{23}$ and of the sixt sort 86 $\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of ỹ first sort 173 $\frac{21}{23}$, and of the second sort 217 $\frac{2}{23}$: of the third sort 391 $\frac{2}{23}$: of the fourth sort 521 $\frac{12}{23}$: of the fift sort 2152 $\frac{14}{23}$ and of the sixt sort 1045 $\frac{11}{23}$: which altogether do make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master. This is well done of you, therefore now make three or foure varieties, and so an end of this Rule.

Scholar. These 4 varieties I set for example.

Master.

Master. And by these it appeareth, that you can find out more, with which I will not now meddle, save onely (for to shew you an easie helpe drawing the lines of Combination) I will set forth two varieties here.

And this shall suffice now for the Rule of Alligation or mixture: for by these examples may you easily conjecture such other as do appertaine to it, as well for the due working, as for varietie of drawing the lines of Combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my musing at the many varieties that may fall in these Combinations,

and

and termed them phantasies, yet my phantasie giveth mee, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable. Therefore ye may well think, that at another time convenient I will request you to aide me herein.

Master. Truth it is that this consideration may fall in practice as well Politick as Philosophicall, and sundry ways in them be applied: Therefore when time shall call it, for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

NOw will I briefly also teach you somwhat of the Rule of Falshood, *which beareth his name, not for that it teacheth any fraud or Falshood, but for that by false numbers taken at all adventures, it teacheth how to finde those true numbers you seek for.*

Scholar. So might any other Rule be called the Rule of Falshood, for they work by wrong numbers, and by them finde out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship, and the Golden Rule partly.

Master. In the Golden Rule, the Rule of Fellowship, & the Rule of Alligation, although the numbers that you work by, be not the true numbers that you seek for, yet are they numbers in just proportion, and are found by orderly worke, whereas in this Rule the numbers are not taken in any proportion, nor found by orderly work, but taken at all adventures.

And therefore I sometimes being merry with my friends, and talking of such questions, to call unto them such Children or Idiots, as happned to be in the place, and to take their answer, declaring that I would make them solve those questions, that seemed so doubtfull.

And indeed I did answer to the question, and works the triall thereof also by those answers which they happened at all adventures to make: which numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false positions, and for briefnesse, the Rule of falshood: which Rule for readinesse of remembrance, I have comprised in the few verses following, in forme of an obscure Riddle.

Gesse at this worke as hap doth lead,
By chance to truth you may proceed,
And first work by the question,
Although no truth therein be done.
Such falshood is so good a ground,
That truth by it will soon be found.

From

From many bate too many mo,
From too few take too few also:
With too much joyn too few again:
To too few adde too many plain.
In crosse wise multiply contrary kinde,
And all truth by falshood for to finde.

The sense of these Verses, and the summe of this Rule is this.

When any *question is proponed* appertaining to this *Rule*, first imagine any *number* that you list, which you shall name the *first position*, and put it instead of the *true number*, and then work with it as the *question* importeth: and if you have missed, then is the *last number* of that worke either too *great* or too *little*: that shall you note as hereafter shall be taught you, and you shall call it the *first errour*.

The exposition of the Rule.

Then begin againe, and take another *number*, which shall be called the *second position*, and worke by the *question*: if you have missed againe, note the excesse or default as it is, and call that the *second errour*. Then multiply crosse-wise the *first position* by the *second error*, and againe the *second position* by the *first error*, and note their *totalls* severally by the names of *totalls*. Then marke whether the two *errours* were *both alike*, that is to say, both *too much* or both *too little*: or whether they be *unlike*, that is, the one too *much*, and the other too little:

little: for if they be like, then shall you *subtract* the one *totall* from the other (I mean the *lesser* from the *greater*) and the *remayner* shall be your *Dividend*: so must you abate the *lesser errour* out of the *greater*, and the residue shall be the *Divisor*. Now divide the *Dividend* by that *Divisor*, and the *quotient* will shew you the *true number that you seeke for*. But, and if the *errours be unlike*, then must you adde both those *totals*, (which you noted) together, and take that *whole number* for the *Dividend*, so shall you adde both *errours* together, and that *whole number* shall be the *Divisor*, and the *Quotient* of that Division shall give you the *true number* that the *question* seeketh for, and this is the whole Rule.

Scholar. **This Rule seemeth so unlike any other, that without some** example **I shall not easily understand it.**

Master. **With a good will: propose halfe a score sundry** questions **and** examples of varietie, **for the better understanding of the worke hereof: and for the** first, **take this example.** *A Mason was bound to build a wall in 40 days, and it was covenanted so with him, that every day that hee wrought, hee should have for his wages* 2 shillings 1 penny, *& every day that he wrought not, he should be amerced* 2 shillings *six pence, so that when the wall was made, and the reckoning taken of the dayes that he wrought, and of the other that he wrought not, the Mason had cleerly but five* shillings *five* pence *for the work. Now* A question of Masonry the first example.

doe

do I demand how many days did he work of those 40, and how many did he not work?

Scholar. I pray you expresse the order of the work, that I may partly by imitation, and partly by comparing it with the Rule, be able again to do the like.

Master. This order shall you kéepe in the worke of this rule: first take some number (as you list) at adventure, as for example, I say he played 12 days, and wrought 28 days. Now cast you the wages of every day, and sée whether it will agrée with the summe of 5 shillings 5 pence.

Scholar. The 28 days that he wrought after 25 pence the day, yieldeth 700 pence: Then 12 days that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should have had 340: therefore is this summe too much by 275, which summe I must set downe after this sort as you see here, where first I have made a crosse (commonly called S. Andrews crosse) and at the over corner on the left hand I have set the first position 12: and at the other corner under it I have set 275, which is the first errour, with this figure †, which betokeneth too much, as this line —— plaine without a crosse line betokeneth too little.

12

275†

On

On ẏ right hand of the crosse I have left two like roomes for the second position, and his error, Therefore to prosecute the worke, I suppose he played 16 dayes, and wrought 24.

Scholar. I was a while in doubt why you named the dayes of his working, seeing they bee not set in the figure: and I doubted how you knew them, or else whether that you did suppose them at all adventures, as you did the dayes that he played: but now I gather that seeing 40 daies is the whole time limited, then the dayes that he played being supposed, the rest of 40 must needs be the dayes that he wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceive why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle, but now examine you this second position.

Scholar. If he played 16 dayes, then abating 16 times 30 pence, the summe will be 480 pence, and for 24 daies that he wrought, every day yielding 25 pence, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that must be set on the right hand of the figure at the neather part and over it on the same side 16, which is the second position, thus.

12 16 And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

X

27† 55† Master. So were it. But this shall you marke, that, of what nature soever the two positions be, of the same nature is the quotient. Therefore when the positions in this question are 12 & 16, which both being numbers of the playing dayes, the quotient shall declare the true number of playing dayes: whereas if the positions had béene 28 and 24, which are supposed to be the working dayes, then would the Quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the worke of this question, and to finde the true number of playing dayes, I must multiply crosse-wise the first position by 55, that is the second errour, and the totall will be 660. Then I multiply 275 and 16, and it yéeldeth 4400. Now because the errors are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remayneth 3740, which is the dividend. Againe, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remayne 220, which will be the divisor: then dividing 3740 by 220, the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of dayes that the Mason played: and then it followeth that he wrought 23 dayes, and so is the question answered.

Now

Now for the order of triall of this worke, there needeth none other triall but onely this, to worke with this number according to the question, and if it agree, then appeareth the number to be it that you would have. The proofe of this rule.

And here now seeing hee wrought 23 dayes, and must have for every day 25 pence, the whole summe commeth to 575. Then again seeing he played 17 dayes, and must abate 30 pence for every day, the whole summe of the abatement will be 510: Therefore I subtract 510 out of 575, and there will remayne 65, which maketh 5 shillings, 5 pence, the cleere wages of the Mason, for his worke, according to the question.

Scholar. Now I trust I understand the worke and the Rule so well (and the better by this proofe) that I can be able to doe the like: And for a proofe, I take the same question all save the last number, where I wil suppose that he had 10 shillings for his wages cleere. And now to gesse at the number of the dayes hee wrought, I suppose, first, that hee wrought 20 dayes: then say I, if he wrought 20 dayes, his wages must be 500 d. then did hee play other 20 dayes, for which must be abated 600 d. and then he loseth 100 d. And so am I at a stay, for it is not like to your former work.

Master. You should have required of mee some question, and not have taken a question of your owne phantasying, untill you were more expert in this Art, for so might you as well

happen to an impossible question, as on a possible: but now to goe forward, consider that this number is too little by 220, seeing he should gaine by your supposition 120 pence, and in this position he loseth 100, those both make 220, which you shall set downe for the first errour with this signe —, betokening too little, as here in this forme following doth appeare.

20

X

220-

And now for the rest goe forward your selfe once againe.

Scholar. As my errour hath uttered my folly, so it hath procured me better understanding.

Now therefore considering this position not to solve the question, I take another, supposing that he wrought 30 dayes. Then for his wages hee must be allowed 750 pence, and for the 10 dayes which he wrought not, hee must abate 300 pence, and so remaineth cleere 450 pence, but it should be onely 120 pence, therefore it is too much by 330, which I set downe in the figure with the former position and his errour, and the figure appeareth thus:

20	30
X	
220-	330+

Now first I multiply in crosse wayes 20 by 330, and it will be 6600-

Then againe I multiply 30 by 220 and it will be also 6600. Wherefore if I shall subtract the one

out

out of the other, there will remaine nothing to be the Dividend.

Master. In this you forget your selfe again; for in as much as the signes in the errours bee unlike, therefore must you worke by Addition, adding together those two totalls to make the Dividend, and also adding the two errours to make the Divisor. And because you shall no more forget this part of that Rule, take this briefe remembrance.

Unlike require Addition,
And like desire Subtraction.

Scholar. You meane, that if the errors have like signes, then must the Dividend and ẏ Divisor be made by Subtraction, as is taught before: And if those signes be unlike (as in this last example they be) then must I by Addition gather the Dividend and the Divisor. Therefore must I adde 6600 to 6600, and it will be 13200, which will be the Dividend. Then againe I adde 220 to 330, and it will bee 550, which must be the Divisor: wherefore dividing 13200, by 550, the quotient will be 24, whereby I know that the Mason wrought 24 daies, and then it followeth that he played 16 dayes.

Master. Examine your worke, whether it be agreeable to the question or no.

Scholar. For 24 dayes worke, the wages must bee 600 pence, and for 16 dayes which the Mason wrought not, there must bee abated 480 pence, & then remaineth cleare to the

Mason 120 as the question importeth: wherefore it is evident that 24 is the true number of days that he wrought.

Master. Although you seeme now to understand this worke, yet to acquaint your minde the better with the new Trade of this Rule, I think it good to propone to you 5 or 6 examples more before I make an end of it.

Scholar. Sir, I thanke you that you doe so consider my commoditie and profit in knowledge, for undoubtedly it is practice and exercise that maketh men prompt and expert in every kinde of knowledge.

Master. You say well, so that they follow some certaine precepts to governe and rule their practice by, else may practice procure custome of errour and a repugnance to exactnesse of knowledge: namely, as long as the error is not plainly known to the vulgar sort. But to return to your work.

A question of wares, the second example.

There is a servant that hath bought of Velvet and Damask for his Master 40 *yards, the Velvet at* 20 shillings *a yard, and the Damaske at* 12 shillings, *and when he commeth home his Master demandeth of him how much hee hath bought of each sort: I cannot tell (saith he) exactly: but this I know, that I paid for Damask* 48 shillings *more then I paid for Velvet: now must you guesse how many yards there is of each sort.*

Scholar. Although the guesse seemeth difficult, yet I will prove what I can doe: for I remember your saying, that it forceth

b not

not how fond or false the guesse be, so it be somwhat to the question, and not an answer of a contrary matter.

Therfore first I imagine that he bought 20 yards of Damask, for which hee should pay after the former price 240 shillings: then must hee néeds haue of Velvet other 20 yards (to make up the 40 yards) & that would cost 400 shillings. So that the totall of the price of the Damaske is lesse then the summe paid for Velvet 160 shillings, and should be more by 48. Therefore the first errour is 208 too little: Then begin I againe, and suppose hee bought of Damask 30 yards, that cost 360 shillings, then had hee but 10 yards of Velvet, which cost 200 shillings: and now the price of the Damaske is greater then the price of the Velvet by 160 shillings, and should be but 48, therefore is the second errour 112 too much, which I set in forme of figure as here doth appeare. Then doe I multiply in crosse wayes 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signes of the errours be unlike, I know I must worke by Addition, therefore adde I these two totalls together, and they make 8480, which is the Dividend: then adde I also the two errours together, 208 and 212, and they make 320, which is the Divisor: wherefore

20		30
	X	
208-		112+

 divi-

dividing 8480 by 320, the quotient will bee $26\frac{1}{2}$, which is the true summe of yards of Damask that he bought, and in velvet 13 yards $\frac{1}{2}$, and that appeareth by examination, thus: $26\frac{1}{2}$ yards of Damaske at 12 shillings the yard, maketh 318 shillings: then in Velvet he had but 13 yards, and $\frac{1}{2}$, and cost 270 shillings, at 20 shillings the yard. Now subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the Damaske did cost more than the Velvet.

Master. Now shall you have a question of another kinde.

A question of debt, the third example.

There are three men that doe owe money to me, and I have forgotten what the totall summe is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace, you are too hasty, there is more helpe in it than yet you see; I have three severall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man & the third man did make 71 pound, and the second man his debt with the third, did rise to 88 pound. Now can you tell what every man did owe, and what was the whole summe?

Scholar. Nay, in good faith: but as I perceive that it must bee found by conjecture, so will I gesse at it, supposing that the first man did owe 20 pound, and the second man 30, and the third.

Master.

Master. Nay stay there, you are too farr gone already: you may not suppose a several summe for every man, for it is enough to suppose one summe for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to be 20 pound, the second man cannot owe 30 pound, for the declaration is, that their debts added together did make 47 pound, so must the second man his debt bee but 27 pound. Now the second debt with the third, must make 88: therefore subtract 27 out of 88, and there will remaine 61, as the third man his debt. Then saith the declaration, that the first and third mans debts do make 71: but by this supposition they make 81, that is 10 too much, which I must set for the first errour. Now worke you the second position.

Scholar. I suppose the first mans debt to be 24 pound: then must the second mans debt (by your declaration) be but 23 pound, seeing both they make but 47 pound. And the second man his debt with the third, doe make 88 pound, and the second man oweth but 23: therefore the third man must owe 65 pound. Now the third mans debt with the first, should make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound too much, and that is the second errour, which I set downe with the first, and their positions in this forme, and then I doe multiply in crosse wayes 20 by 18, & it is 360.

And

And 10 by 24 maketh 240. Also because the signes of the errours be like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Dividend: then doe I subtract 10 out of 18 by the same reason, and so is the Divisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15l. and then the second man must owe 32l. for those 2 doe make 47l. and the third mans debt is 56: for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88.

10	24
10†	18†

The fourth example.

Master. *For the fourth Example, take this easie question for the variety in work. Two men having severall summes, which I know not, doe thus talke together: the first saith to the second, if you give me* 2 shillings *of your money, then shall I have three times so much money as you. The second man answereth: It were more reason that our summes were made equall, and so will it be if you give me* 3 shillings *of your money. Now gesse what each of them had.*

Scholar. I imagine that the first had 9 s.

Note

Master. Consider evermore in your imagination that you take a likely summe, as in this question, take such a summe, that having 2 added unto it, may be divided into 3 parts even.

Scholar. Why? I remember you said before, it forceth not how fondly soever I gessed.

Master.

Master. As for the possibility of the solution, it is truth: but for easinesse in worke, the aptest numbers are most convenient.

Scholar. I thought no lesse, and therefore I tooke 9 as an apt number to be parted into three: but I perceive I should have considered the aptnesse of that partition after the addition of two unto it, and then 7 had beene more méet.

Master. That is truth, and then should the second man his summe be 5: for although hee have now but the third part of 9, that is 3, yet you must remember that he lent the first man 2, and so had he 5.

Scholar. Then to go forward: if the second man had three of the first man, then should hee have 8 and the first man but 4: so hath he double to the first man: yet he said in the question they should have equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that error with his supposition, & gesse again that he hath 10 shillings: whereunto I adde 2 shillings borrowed of the second man, & then he hath 12 shillings: so the second man hath remayning but 4, whereunto if I adde the 2 that hee lent to the first man, so had he but 6 shillings at the beginning.

Then take 3 s. from the first man, and give to the second, then hath the first man, but 7, and the second hath 9, which are not equall, but there are 2 to

7	10
4†	2†

many

many, wherefore I set downe both the positions with their errours as before you see, and multiply a-crosse, so commeth there 40 and 14: and because the signes be like, I take 14 out of 40, and so resteth 26 to bee divided: then likewise I take 2 out of 4, and there resteth 2, by which I divide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that 2 being added thereto, the summe will bee 15, so hath the second man but 5, and before hee had 7: then take 3 from the first, and put to his 7, and so have each of them 10, and that is equall as the question would.

The fifth example: A question of Lambes.

Master. *For the fifth example, take this question. One man said to another, I thinke you had this yeare two thousand Lambes: so had I, said the other; but what with paying the tythe of them, and then the severall losses, they are much abated: for at one time I lost halfe as many as I have now left, and at another time the third part of so many, and the third time $\frac{1}{4}$ so many. Now gesse you how many are left.*

Scholar. Because here is mention made of certaine parts, I must take a number that may have all these parts, that is to say, $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$ which will be 24, howbeit 12 hath the same parts. Therefore I take first 12 to bee the number that doth remaine, so hath he lost 6, 4, and 3, that is 13, and the whole 23, but it should be 2000.

Master. Ye are deceived yet still, you have for-

forgotten the 10 part, which muſt be defalked, that is 200, ſo there remaineth but 1800, and now goe on againe.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 too few, which I ſet for the errour. Then for the ſecond poſition I take 24, whoſe halfe is 12, the third part 8, and the quarter 6, whereby riſeth 50, which is too little by 1750, therefore I ſet downe both the poſitions with their errours thus:

And multiply in croſſe wayes 1775 by 24, whereof commeth 42600. Alſo I multiply 1750 by 12, and there ariſeth 21000. And because the ſignes are like, I doe ſubtract the one from the other, and ſo remaineth the Dividend 616000. Then doe I ſubtract 1750 out of 1775, and there reſteth 25, by which I divide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added together, will make 1800. And if you adde thereto the tenth which was abated before, then will the whole ſumme be 2000.

12	24
1775-	1750-

864
432
288
216
——
1800

And now doth there come a queſtion to my memory which was demanded of me, but I was not able to anſwer to it: and now methinketh I could ſolve it.

Maſter. Propone your queſtion.

Scholar.

A queſtiō of ſheepe and tillage, the ſixth example.

Scholar. *There is ſuppoſed a Law made, that (for furthering of tillage) every man that doth keep ſheep, ſhall for every ten ſheep eare and ſow one Acre of ground: and for his allowance in ſheep paſture, there is appointed for every foure ſheep one Acre of paſture. Now is there a rich Sheep-maſter which hath* 7000 *Acres of ground, and would gladly keep as many as hee might: by that Statute I demand how many ſheep ſhall hee keep?*

Maſter. Anſwer to the queſtion your ſelfe.

Scholar. Firſt, I ſuppoſe hee may keepe 500 ſheepe, and for them hee ſhall have in paſture after the rate of foure ſheepe to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I gueſſe again that he may keepe 1000 ſheepe, that is in paſture 250 Acres, and in tillage, 100 Acres, which make 350, that is too little by 6650. Both theſe errours with their poſitions, I ſet downe as you ſee, and multiply them croſſe 6825 by 1000, and it maketh 6825000 alſo I multiply 6650 by 500, and there commeth 3325000, which ſumme

500	1000
6825-	6650-

I ſubtract out of the former, and there remaineth 3500000 for the Dividend: likewiſe I ſubtract the leſſer errour out of the greater, and there reſteth 175, by which I divide 3500000 (the Dividend aforeſaid) and the

the quotient will be 20000, so that by this rate, he that hath 7000 Acres of ground may keepe 20000 sheep.

Master. You have done well, notwithstanding both this last question, and the next before might be wrought without the second position by the Rule of Proportion as this: When in this question you found in the first errour that for 500 sheep there must be 175 Acres, then might you reduce it to the Golden Rule, thus: Another way of working.

$$\begin{array}{ccc} 175 & & 500 \\ 7000 & & 2000 \end{array}$$

If 170 Acres will admit in allowance 500 sheep, then 7000 will have 2000. And so by one position, with the help of the Golden Rule may you answer that question.

Likewise for the question of Lambs, when you had found that 12 came of 25, you might have set the figure as followeth, and have said:

$$\begin{array}{ccc} 25 & & 12 \\ 1800 & & 864 \end{array}$$

If 25 doe leave but 12 what shall 1800 leave? and it would appear to be 864.

Scholar. Sir, I thanke you for this aide, for it doth much shorten the worke of this Rule.

Master. Yet againe, I will shew you an other way to answer to this last question without the Rule of false position, and that by the Rule of Fellowship, for it appeareth in the proponing of the question, that ten sheepe Another way yet.

must

muſt haue in paſture two Acres and $\frac{1}{2}$; and for them muſt there bee eared but one Acre; ſo it followeth, that for 2 Acres eared, there muſt be 5 ſet to paſture: and if you put them both into one ſumme, they will make 7. Therefore looke what proportion 7 being this totall, doth beare to 5 and to 2, ſuch proportion ſhall any totall in this queſtion beare to the paſture ground, and the eared ground.

Scholar. This ſerueth wondrous aptly. Therefore to proue it, I demand this by the former ſuppoſition: if a man haue 300 Acres how much ſhall he leaue in paſture, and how much ſhall he turne to tillage? You ſay, that as 7 is to 5, ſo ſhall 300 be to the Acres of paſture: and as 7 is to 2, ſo is 300 to the Acres of tillage, whereof for both I haue ſet examples here following, whereby appeareth that of Paſture, there ſhall bee 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$, which both ſummes added together, doe make 300.

7	5
300	214 $\frac{2}{7}$

7	2
300	85 $\frac{5}{7}$

Another queſtion, the ſeuenth example.

Maſter. *Now take another Example: A man hath three ſilver Cups with one Cover, the Cover weigheth* 18 *ounces, the ſecond Cup weigheth even halfe the weight of the firſt and the third. Now if the cover be put to the firſt Cup, they weigh iuſt as much as all the three Cups doe weigh: and if the cover be ioyned with the ſecond Cup, they weigh as much as the ſecond twice, and the third: & if the*

Cover

Cover be put to the Cup, they will make twice as much as the first and second Cup. Now try you what was the just weight of every Cup.

Scholar. I doe set the weight of the first Cup to be nine Ounces, then in as much as these two (that is to say, the cover and the first Cup) doe weigh the weight of the three Cups) I see that the three Cuppes must weigh 27 ounces, for so much is 18 and 9. Also because the first and the third doe weigh double so much as the second, therefore it is the third part of that weight, that is 9, and then would it follow, that the third Cup also should weigh 9 ounces; but then the question saith, that the Cover being joyned to the second Cup, they weigh as much as the second twice, and the third once, that should be 27, and so it doth; that being joyned with the third Cup, they should weigh twice as much as the first and the second, that should be 36, and they weigh but 27, so is that errour 9 too little. Then beginne I againe, and say, that the first Cup doth weigh twelve ounces which I joyne with the Cover, and they make thirty Ounces: then seeing the second is of that weight, it must needs weigh ten ounces, and the third must weigh 8 ounces, seeing the first and the third must weigh 20 ounces. Now put I the Cover to the second Cup, and they weigh 28 ounces, which should be even so: then joyn I the cover with the third Cup, and so should it wey twice the first, and the second,

9 12

9- 18-

that is 44 ounces, and they weigh but 26, that is 18 too little: those errours with their positions I set downe, and multiply in crosse wayes 9 by 12, whereof commeth 108: Also 9 by 18, and that yieldeth 162: and in as much as the signes be like, I abate the lesser out of the greater, and there doth remayne 54. Then doe I also abate the lesser errour from the greater, and so remayneth 9, by which I divide 54, and the quotient is 6, which I take for the true weight of the first Cup, which being ioyned with the Cover, must weigh as much as the three Cups, so doe they weigh but 24 ounces. Then seeing the second Cup is the third part of that weight, for the other two Cups (you say) must weigh double his weight, the weight of the second Cup is 8 ounces, & so the weight of the third Cup must be ten ounces. Now put the Cover to the second Cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is, twice 8, and once ten, and so is it. Again put the Cover to the third Cup of ten ounces, and they must weigh twice as much as the first and the second, that is, 28, and so is all agreeable.

Master. Then answer to this Question.

A question of water: the eighth example.

There is a Cisterne with foure Cocks, containing 72 barrels of water: and if the greatest Cocke be opened, the water will avoid clean in six houres; at the second Cock it will aske eight houres:

houres: at the third Cock it will avoid in no leſſe then nine houres: and at the ſmalleſt it will require twelve houres. Now I demand in what ſpace will it avoid, all the Cocks being ſet open?

Scholar. Firſt, I imagine it will avoid in two houres.

Maſter. Then muſt there avoid by the firſt Cock $\frac{1}{3}$ of the water, that is 24 Barrels, and by the ſecond Cock $\frac{1}{4}$, that is 18, and by the third Cock $\frac{2}{9}$, that is 16 Barrels, and by the ſmalleſt Cock $\frac{1}{6}$, that is 12 Barrels, all which ſummes put together, do make 70, as by their Addition it doth appeare, but it ſhould be $72\frac{1}{6}$ therefore the errour is too few.

24
18
16
12
70

Scholar. Then will I begin again by your favour, becauſe I thinke I underſtand the worke and put three houres for the due time: ſo ſhall there run out at the greateſt Cock $\frac{1}{2}$, that is, 36 Barrels, and at the ſecond hole $\frac{3}{8}$, that is 27, and at the third Cocke $\frac{1}{3}$, that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 Barrels which altogether doe make 105, and ſhould be but 72, ſo is it too much by 33: therefore doe I ſet the errours in order of the figure with their poſitions, and worke by Multiplication, in croſſe, ſaying, two times 3 is 6, and two times 33 maketh 66, and becauſe the ſignes are unlike,

2	3
2-	33+

I must adde these two totalls together, which make 72: also I adde the two errors, and they make 35, by which I divide 72, and the Quotient riseth $\frac{2}{35}$, whereby I see that all the Cocks being set open, the water will avoid in two houres, and $\frac{2}{35}$, of an houre.

Master. This exercise maketh you to grow expert in the Rule. Therefore I will inure you somewhat more with a question or two.

A questi-on of part-ners.

The ninth example.

There were two men that had beene partners, and had in account betweene them 300 *Duckets; whereof the one should have for his part* 180, *and the other* 120: *but in the parting of them, they fell at variance; so that each of them catched as many as he could: yet afterward being reconciled, they agreed that he which had gotten most part of them, should lay down* $\frac{1}{4}$ *of them again, and hee that had gotten lest, should lay down* $\frac{1}{3}$ *of those which he had taken, and then parting them into two equall p rts, each man to have halfe thereof, and so had they their just portions, as they ought: now I demand of you what each of them had gotten by the scambling?*

Scholar. I suppose hee that had least, got 108 Duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{1}{4}$, that is, 144, and so had hee left but 48. Also of the 108, there was laid downe 36, that is $\frac{1}{3}$, and so hee had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts even, and so commeth 90 to be given to each of them:

them: which summe put to 72 maketh 12 and joyned to 148, it maketh 238: and now I doubt how I shall go forward.

Master. You need not to take but one of them, which you list the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other summes, remembring that you make comparison to the same in the second worke: as for example of the first part. If you compare 138 with the lesser summe due, that is, 120, so is it 18 too much, and if you compare it with the greater summe, then is it 42 too little. Again, if you compare 162 to the greater summe, the errour will be 18, as it was in the other: but it will have a contrary signe; and if you compare it with the lesser summe, it will be 42 too much: so that the errour both wayes is either 18, or 42: and as for the signes it little forceth, for in them is nothing considered here, but likenesse and unlikenes, which in this case doth neither further or hinder: But now go on with the work. Note.

Scholar. If it be so, then am I out of my greatest doubt. Then I joyne that 90 (which I found as the half of the latter partition) unto 48, which is left with the one man, and so hath he, 138, which I may say) is 18 to many, for the least should be but 120, that errour do I note, and then make a new position, supposing the one man to have 204, and the other to have 96: wherefore of the 204, there

must be laid downe 153, and so remayneth with him 51. Also of the 96, there must be laid down $\frac{1}{3}$, that is 32, and so resteth with that man 64: Now of the 153 and 32, I make one summe, as 185, which I must divide into two equall parts, and so each man shall have 92 $\frac{1}{2}$, whereunto if I adde their former portions reserved, then the one shall have 156 $\frac{1}{2}$, and the other hath 143 $\frac{1}{2}$. Wherefore take the lesser summe now againe, as I did before, that is, 143 $\frac{1}{2}$, and finde that he hath to many by 23 $\frac{1}{2}$, for he should have but 120, and so have I for my two positiōs two errors, which I set down as here may be séene, each error under his position, and then by the Rule I doe multiply in crosse-wayes 108 by 23 $\frac{1}{2}$, and there riseth 2538, which I note, then againe I multiply 96 by 18, and thereof amounteth 1728.

108	96
18 †	23 $\frac{1}{2}$ †

Now because the signes are both like, that is, both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the Dividend 810: then for the Divisor I subtract 18 out of 23 $\frac{1}{2}$, and there remayneth 5 $\frac{1}{2}$, by which I divide 810, and the quotient will be 147 $\frac{3}{11}$, which is the just portion of him that had the least summe. And if I doe subtract it out of 300, being the totall summe, then will there remayn 152 $\frac{8}{11}$, as the portion that the other did get.

Master

Master. For the proofe of this worke, you may chuse whether you will examine those numbers according to the forme of the questions, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each work confirme other. Note.

Scholar. By your patience, I will prove both wayes, not only to seeke their agreement, but also to accustome my minde to those works, for I perceive it is exercise that must be the chiefe engraver of these Rules in my memory.

Master. You consider it well: then go to.

Scholar. First, I will by two other positions try to finde the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your worke the better, take the same positions that you did before, comparing them now to the greater, as you did before unto the lesser.

Scholar. Then I suppose that hee that had most, had 192, so had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144 and there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remayne to him 72: then joyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to each of those

two mens portions remaining with them,the one ſhall haue 138, and the other 162 , of which two I take the greater (that is 162) and ſee it to be 18 too few;for it ſhould be 180, that errour I note under this poſition. Then for the ſecond poſition I take(as I did before) 204 for the one, and ſo reſteth 96 for the o-ther : then (take I $\frac{1}{4}$ of 204 , and it will be 153, and there reſteth to him 51. Alſo of the 96 I take $\frac{1}{3}$ that is, 32, and there remayneth to him 64 ; now put I that 32 to 153, and it yeeldeth 185 : which being parted in equall values, maketh 92 $\frac{1}{2}$ to be added to each mans remaynder , and ſo the one hath 143$\frac{1}{2}$,and the other 156 $\frac{1}{2}$: wherefore I take the greateſt ſumme, and it is 23 $\frac{1}{2}$ too little, that do I note alſo, and ſet both theſe errors under their poſi-tions, as in this Example following doth ap-peare.

And then multiplying 102 by 23$\frac{1}{2}$, there doth ariſe, 4512.

102	204
18	23$\frac{1}{2}$

Again, I multiply 204 by 18,and it maketh 3672, which I doe ſubtract out of 4512, becauſe the ſignes be like, and there reſteth 840 for the dividend, then ſubtracting 18 out of 28$\frac{1}{2}$ there will remaine 5$\frac{1}{2}$, which I muſt take for the Diviſor. And ſo dividing 840 by 3$\frac{1}{2}$ the quotient will be 152 $\frac{8}{11}$, whereby I haue found an agreeable ſumme to that which I found by the former poſitions,for him that had

moſt

most which I doe subtract out of 300, that is the totall, there will rest $147\frac{3}{11}$, which was the portion of him that had the least part.

Master. So by diuers positions, you see, that one doth confirme the work of the other. Now examine those two numbers by the forme of the question, and so shall you proue your work good also.

Scholar. If that hee which gate most, had $152\frac{8}{11}$, then must hee lay downe $\frac{3}{4}$ of this sum. That is $114\frac{6}{11}$, and so shall remaine with him but onely $38\frac{2}{11}$. The other which had least, that is $147\frac{3}{11}$, must put down of his summe, $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doth there remayne with him yet $98\frac{2}{11}$. Then do I adde together $114\frac{6}{11}$, and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into equall parts, and that will be $81\frac{9}{11}$, to be giuen to each of them: putting $81\frac{9}{11}$ unto $38\frac{2}{11}$, there doth amount 120 iust, which is the true portion of him that should haue the lesser sum: and adding $81\frac{9}{11}$, $98\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the worke by this profe also tryed to be good. And this I mark by the way, that in their scambling, he got most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your study be to learne truth and iust Art of proportion, and to distribute e part according thereunto, as often as occasion shall be ministred. And here would I make an end of this Rule, saue that I remember one pleasant question which I cannot ouer-

passe,

passe which I will declare somewhat largely, because you shall as well understand some reason in the pleasant invention, as apt proceeding in the witty working thereof.

The tenth example of gold and silver.

Hiero *King of the Syracusans in Sicilia had caused to be made a Crown of Gold of a wonderfull weight, to be offered for his good successe in wars: in making whereof, the Goldsmith fraudulently took out a certain portion of Gold, and put in Silver for it, so that there was nothing abated of the full weight, although there was much of the value diminished.*

Which thing at length being uttered (as no evill can alwayes lie hid) the King was sore moved: and being desirous to try the truth without breaking of the Crown, proponed the doubt to Archimedes, unto whose wit nothing seemed unpossible, which although presently he could not answer unto, yet he had good hope to devise some policie for that invention, and so musing thereon, as he chanced to enter into a Baine full of water to wash him, hee observed, that as his body entred into the Bain, the water did runne over the Tub, whereby his ready wit of such small effects conjecturing greater works, conceived by and by a reason of solution to the Kings question, and therefore rejoycing exceedingly, more then if he had gotten the Crowne it selfe, forgat that hee was naked, and so ranne home, crying, as hee ranne, ἕυρηκα, ἕυρηκα, I have found, I have found. And thereupon caused two massie

pieces,

pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crowne was of: and considering that Gold is heavier of nature then Silver, and therefore Gold of light weight with Silver, must néeds occupie lesse rome by reason it is more compact and sound in substance, hee was assured that putting the masse of Gold into a vessell brim full of water, there would not so much water runne out, as when he should put in the silver masse of the like weight. Wherefore hee tried both, and noted not onely the quantities of the water at each time, but also the difference or excesse of the one above the other, whereby hee learned what proportion in quantity is betwéene Gold and Silver of equall weight. And then putting the Crowne it selfe into the vessell of water brim full (as before) marked, how much water did run out then, and comparing it with the water that ranne out when the Gold was put in, noted how much it did excéed that: & likewise comparing it to the water that ranne out of the Silver, marking how much it was lesse then that, and by those proportions found out the just quantity of Gold that was taken out of the Crowne, and how much Silver was put in stead of it: but séeing Vitruvius which writeth this History, doth not declare the particular worke of this tryall, it shall be no inconvenience to suppose an example for declarations sake, wherein although the true and just

proportion be not expressed, yet the forme of triall shall be truly set forth. And for an example, I suppose the weight of the Crowne to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ranne out two pound of water: and when the masse of silver was put in, I suppose there ranne out three pound $\frac{1}{2}$. Againe, when the Crowne was put in, there ran out two pound $\frac{1}{4}$: Now to know what quantitie of silver was in the Crowne, work by the Rule of false position, and imagine that there was two pound of silver, then must there be sixe pound of gold, then say thus by the Rule of Proportion. If eight pound of gold doe expell two pound of water, what shall six pound expell? and it will be 1 pound $\frac{1}{2}$. Againe, for the silver: if eight pound of silver expell three pound $\frac{1}{2}$ of water, what shall two pound of silver put out? it will be $\frac{7}{8}$, now adde those two weights of water together, and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{2}{4}$, so is it too much by $\frac{1}{8}$.

Scholar. Now do I understand the worke as I thinke, therefore I pray you let mee worke the rest of the question. And because this first supposition did erre, I note that position and his errour, and take a new position, esteeming the silver to be but one pound, so must there be in Gold seven pound. Then say I: if eight pound of Gold doe yeeld two

pound

pound of water, what ſhall ſeven pound yéeld? and it will be 1 pound $\frac{1}{4}$. Againe, if 8 pound of ſilver expell 3 pound $\frac{1}{2}$ of water, what ſhall 1 pound expell? and it will be $\frac{1}{18}$. Now muſt I adde thoſe two ſummes together, and they make two pound $\frac{1}{16}$, and they ſhould make 2 pound $\frac{1}{4}$, ſo is it too little, by $\frac{1}{16}$. Therefore I ſet the poſitions, with their errours in order as here followeth: And then I multiply in croſſe wayes 2 by $\frac{1}{16}$, and it maketh $\frac{1}{3}$: Likewiſe 1 multiply by $\frac{1}{8}$ maketh $\frac{1}{8}$. And becauſe the ſignes bee unlike, I muſt adde theſe two ſums which make $\frac{1}{4}$: and that is the dividend.

$$\begin{array}{ccc} 2 & & 1 \\ & \times & \\ \frac{1}{8}\dagger & & \frac{1}{16} \end{array}$$

Againe, I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{1}{16}$, that is the Diviſor. Now I ſhall divide $\frac{1}{4}$, by $\frac{1}{16}$, and the quotient will be $\frac{16}{12}$, that is, $1\frac{1}{3}$: whereby I know that there was put 1 pound and $\frac{1}{3}$ of ſilver into the Crowne, and ſo much Gold taken out for it.

Maſter. Prove it now by examination, according to the queſtion.

Scholar. If there were 1 pound $\frac{1}{3}$ of ſilver, then was there of Gold 6 pound $\frac{2}{3}$. Now ſay I by the Rule of Proportion: if 8 pound of Gold expell two pound of water, what ſhall 6 pound $\frac{2}{3}$ expell?

$$\begin{array}{ccc} 8 & & 2 \\ 6\frac{2}{3} & & 1\frac{2}{3} \end{array}$$

It

8 $3\frac{1}{2}$
$1\frac{1}{3}$ $\frac{7}{12}$

It will bee 1 pound $\frac{2}{3}$. Againe, if 8 pound of ſilver expell three pound $\frac{1}{2}$ of water, what ſhall $1\frac{1}{3}$ expell? It will be $\frac{7}{12}$, Now muſt I adde together 1 pound $\frac{2}{3}$ and $\frac{7}{12}$, and they will make 2 pound $\frac{9}{36}$, that is, 2 pound $\frac{1}{4}$, according to the ſuppoſition of the queſtion: whereby I perceive the work to be wel done. And I cannot but much reioyce of this excellent invention, ſo my deſire is kindled vehemently to be perfectly inſtructed in every part thereof, and namely in this point, whether the proportion betweene water and Gold be ſuch, that for 8 pound of Gold put into a veſſell full of water, there ſhall runne out two pound of water, and for as much ſilver, whether 3 pound $\frac{2}{3}$ of water would avoid.

Maſter. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the exact proportion betwéen Gold and Silver, and water, both in their weight and quantities, then could you eaſily finde out the mixtures of them, which thing I have reſerved for another worke that intreateth of ſuch matters epecially. And at this time you muſt conſider that you learne Arithmeticke, which intreateth of the manner to ſolve doubtfull queſtions touching number, without regard what matter is ſignified by that number: elſe were it neceſſary in Arithmetick, to teach all Arts, ſéeing in it may be moved queſtions of all Arts.

A queſtion of the proportion of gold ſilver, and quickſilver, unto water.

But ſeeing you are ſo deſirous to know theſe things, I will tell you in ſuch a ſort, that you ſhall practiſe your Art in finding it, and propound it in forme of a queſtion. Gold beareth a greater proportion to water, then ſilver doth, & their two proportions be in proportion together, as 48 to 25. But to helpe you ſomewhat in this Riddle, you ſhall note that the proportion of Quick-ſilver unto water, is the juſt middle number proportionall in progreſſion Geometricall between the proportion of gold and ſilver unto water.

And this proportion is $2\frac{20}{21}$. Now if you will know the juſt numbers of theſe three proportions, then muſt you finde out three numbers in Progreſſion Geometrical, whereof the middlemoſt muſt be $2\frac{20}{21}$, and the firſt muſt be unto the laſt, as 25 to 48. And thus I will leave you to finde thoſe numbers when you be at leiſure.

Scholar. Yet Sir, I thanke you heartily for thus much, for now I ſee the poſſibility to finde them out. Howbeit, becauſe this queſtion ſeemeth ſtrange, if it might pleaſe you to inſtruct me ſomwhat in the order of working for it, I ſhould the more eaſily finde the true working.

Maſter. You deſire too much if you will ſtudie for nothing: Therefore to occaſion you to ſtudy the better, I will leave this doubt wholly to your owne ſearch: But as touching the generality of the Rule, Archimedes, needed not to take two Maſſes of Gold

and

and Silver equall in weight with the Crown, for the proportion might as well be found in any other weight, yea, although the Masse of Gold were of one weight, and the Masse of Silver of another. As for example: if the Crowne were of 18 Pound weight as I did suppose, and I have not so much other fine Gold, but onely one pound, and trying that by water, and finding that it doth expell but $\frac{3}{4}$ of an ounce of water, yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewise of silver, whereof if I had but two pound, & find that it doth expell three ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is, one pound weight: and so is it good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmetick.

Scholar. Although I cannot sufficiently thank you for this, yet your promise made me to looke for the Art of Extraction of Roots, whereof hitherto I have learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within this three or foure moneths, if I perceive this my pains to be well taken in the meane season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the meane time, you shall take this Addition, not for the second Part of Arithmeticke which I promised, but for an aug-

augmentation of the firſt part, unto which I would have annexed the extraction of Roots ſquare and cubicke, namely, for examples of the Statute of Aſſiſe of Wood, but that in the ſecond Part I muſt write of divers other Roots, and thought it beſt to reſerve thoſe Rules alſo with their Examples unto the ſame ſecond Part.

Scholar. Sir, although I cannot recompence your goodneſſe, yet I ſhall allwayes doe mine endevour to occaſion you not to repent your benefit on me thus employed.

Maſter. That recompence is ſufficient for your part.

FINIS.

THE THIRD PART,

OR,

Additions to this Book,

Entreateth

Of brief Rules, called Rules of *Practice, of rare, pleasant and* commodious effect, abridged into a briefer Method then hitherto hath been published.

With divers other necessary Rules, Tables, *and* Questions, *not onely* profitable for *Merchants*, but also for *Gentlemen*, and all other *Occupyers* whatsoever, as by the Contents of this Book may appeare.

Set forth by JOHN MELLIS *Schoolmaster.*

Printed by *John Okes*, 1640.

The first Chapter of this Addition, entreateth of briefe Rules, called Rules of *Practice*, with divers necessary questions, profitable not *onely for Merchants, but also for all other Occupiers whatsoever.*

THe working of Multiplication in practice is no other thing then a certain manner of multiplying of one kinde by another: whereupon is brought forth the product of the proponed number, which is accomplished by the meanes of Division in taking the *half*, the *third*, the *fourth*, the *fifth*, or such other parts of the summe which is to be multiplyed.

And for the better understanding of such conversions, you shall understand that in the manner and use of these rules of practice, you ought first to know the even or aliquot parts of a shilling, which in this Table following doth appeare.

$$\text{Item}\left\{\begin{matrix}6\\4\\3\\2\\1\end{matrix}\right\}\textit{pence is the}\left\{\begin{matrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{6}\\\frac{1}{12}\end{matrix}\right\}\textit{of a shilling.}$$

Wherein as you see according to the order

of these Rules of Practice: at six pence the yard of any thing, you must take ½ of your number which is to be multiplyed, & the product that commeth thereof shal be shillings, if any unite do remayn, it is 6 pence.

For 4 d. take the ⅓ of the number that is to be multipled, and the product also produceth shillings, if any unites doe remayne, each one shall be worth in value 4 pence. The like is to be understood of the other 3, &c.

I *Example.*

At 6 d. the yard, what 379 yards?
189 s. —— 6 d.

II

At 4 d. the yard, what 104 yards?
34 —— 8 d.

III

At 3 d. the yard, what 5014 yards?
1253 s —— 6 d.

IV

At 2 d. the yard, what 532 yards?
88 s —— 8 d.

V

At 1 d. the yard, what 409 yards?
34 s —— 1 d.

Heere you may see in the first example, that 379 yards at 6 d. the yard, are worth 189 s. 6 d. in taking the ½ of 379. And in the second example the 104 yards at 4 pence the yard, are worth 34 s. 8 d. in taking the ⅓ of 104: Likewise in the third example, 5014 yards at 3 d. the

the yard, bringeth forth 1253 s. 6 d. in taking the $\frac{1}{4}$ of 5014. As also in the fourth example at 2 d. the yard, maketh 88 s. 8 d.

And lastly, in the fifth example: 409 yards at 1 d. the yard, amounteth to 34 s. 1 d. in taking the $\frac{1}{12}$ of 409: and so is to be done also of all other questions the like, when the number of the pence is any of the eve or aliquot parts of 12 d.

Item, to bring the products of these shillings, and all other the like into pounds is very easie in dividing of it in your mind by 20, for it is to be understood that as often as 20 is found in that product, so many pounds doth it contain: which with facility to perform, always strike off the figure towards your right hand, with a right down dash of your pen, for the o that appertaineth to the 20. And then begin at the left hand, in taking the half of the rest. And if that at the last any unite do remayne, the same shall be joyned with the figure that is cut off, which shall represent the odde shillings, contained in that work.

As for example, in your third question at 3 d. the yard, which amounteth to 1253 s. 6 d. the product whereof maketh 62 li. 13 s. 6 d. as here you may see, is easily performed by this example.

125|3 6
62 — 13 —— 6

Also for the working of one peny the yard, it is something harsh and hard to take the $\frac{1}{12}$ of some products: therefore to ease that hard work, you shall first bring your delivered

summe into *groats*, by taking ¼ part of the product, and if any unites remain of that ¼ part, as sometimes there may, they are pence: and must be signified with a line from the *groats* with their title of *pence*; and because that 60 *groats* maketh a pound or 20 shillings, *strike* off the first figure toward your right hand, for the 0 that appertaineth to 60 (as you did even now for the 0 that belongeth to 20;) Then in taking the ⅓ of that product, if there doe remaine any unites, the same shall you joyne with the figure that you cut off, esteeming thẽ as *groats*, which keep in your minde, and by taking the ⅓ part of them, you shall turn them into *shillings*, and so have you done: As *for example*, by a question or two hereafter proponed, shall more plainly by the work appeare.

At 1 d. the yard, what 54368 yards?

1359|2 *groats*.

⅓ li——226——10 s. 8 d.

Here in taking the ⅓ part of 1359, in comming to the last worke, the ⅓ part of 39 being taken, the *remainder* is 1, which joyned with the two that was cut off, maketh 32 *groats*, which converted into *shillings* by taking the ⅓ part, maketh as appeareth 10 *shillings* 8 d. Many other ways there are, but none more apt for a *young learner* to understand then this: wherefore this one way well impressed in memory, is better then 20 ways doubtfully understood.

At

At 1 penny the yard, what 4533 yards?

	113|3 *groat*	1 d.
li	18 -- 17 --	9 d.

At 1 penny the yard, what 64768 yards?

	1619|2 *groats*	
li	269 -- 17 --	4 d.

Now followeth also to be understood that if the number of pence be not an aliquot part of 12, you must reduce them into some aliquot part of 12: and after the aforesaid manner, you shall make of them two or three products, as need shall require, and adde them together into one sum. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken. 2 Rule.

for pence.			*and*		*or*				
5	*take*	3	*and*	2	*or*	4		*and*	1
7		4		3		6		&	1
8		4		4		6			2
9		6		3		4	4		1
10		6		4		4	4	*and*	2
11		6		4 & 1		4	4		3

Here in the first note of this *Table*, at 5 d. you shall first take for 3 d. the $\frac{1}{4}$ of the number that is to be multiplyed: and likewise for 2 d. the $\frac{1}{6}$ of the same number, adding together both the products: But if you will worke by 4 and 1, you must for 4 d. first take the $\frac{1}{3}$ of the number that is to be multiplyed: and for 1 d. take the $\frac{1}{12}$ of the whole summe, or rather, which is more better, for 1 peny you may take the $\frac{1}{4}$ of the product which did come of the 4 pence:

pence: because that 1 d. is the $\frac{1}{4}$ of 4 pence. The totall summes of these two numbers shall be the solution to the *question*. And in like manner is to be done of all others, as by these examples following shall appeare.

I

At 5 d. the yard, what	748 yards?	
3 d	187	0 d
2 d	124	8 d
shillings:	311	8 d

Otherwise.

At 5 d. the yard, what	758 yards?	
4 d	252	8 d
1 d	63	2 d
shillings	315	10 d

II

At 7 d the Ell, what	563 Ells?	
4 d	187	8 d
3 d	140	9 d
shillings	328	5 d

III.

At 8 d. the pound, what	112 pound?	
4 d	37	4 d
4 d	37	4 d
shillings	74	8 d

Otherwise.

At 8 pence the pound, what	112 pounds?	
6 d	56	0
2 d	18	8
shillings	74	8 d

IV.

IV

At 9 pence the Ell, what	356 Ells?
6 d	178---0
3 d	89---0
shillings	267---0 d

V

At 10 pence the piece, what	795 pieces?
6 d	397---6
4 d	265---0
shillings	662---6

VI

At 11 pence the pound, what	7576 pounds?
6 d	3788---0
4 d	2525---4
1 d	631---4
	6944---8 d
Pounds	347---4---8 d

I Here in this first example, where it is demanded (at 5 d. the yard) what will 758 cost? First, for 3 d. I take the $\frac{1}{4}$ of 758; and thereof commeth 189 s. 6 d. Then for 2 d. I take the $\frac{1}{6}$ of the same 758, which amounteth to 126 s. 4 d. these two sums added together, doe make 315 *shillings* 10 *pence*: and so much are the 758 *yards* worth at 5 d. the *yard*.

1 *Item*, for the same again: First, for 4 d. I take the $\frac{1}{3}$ of 758, and thereof commeth 252 s. 8 d. then for 1 peny I take the $\frac{1}{4}$ of the same 758, that is to say, of 252 s. 8 d. and it yieldeth me 63 s. 2 d. which both added together maketh 315 s.---10 d. as before.

2 *Item*,

2 *Item, for 7 d. there is taken the* $\frac{1}{3}$ *and the* $\frac{1}{4}$ of the whole sum which is to be multiplyed, and adde them together, that is to say, first, for 4 pence there is taken $\frac{1}{3}$ of 563: which comes to 187 s——8 d. as appeareth by the work, and for 3 d. there is taken the $\frac{1}{4}$ of the whole sum, which amounteth to 140 s—— 9 d. Both which products added together, do make 328 s—5 d. and so much comes 563 *els* to at 7 d. the *ell*.

3 *Item, for the first* 8 d. *there is taken for* 4 d. the $\frac{1}{3}$ of the whole summe, and another $\frac{1}{3}$ for the other 4 d. which added together, as in the example doth evidently appeare, amounteth to 74 s—8 d.

Againe, for the second work of 112 li. there is taken first the $\frac{1}{2}$ of the whole summe for 6 d. which comes to 56 s. then for that 2 d. you have to take $\frac{1}{6}$ of the whole summe, or if you will, the $\frac{1}{3}$ of the product that came of 6 d. either of which maketh 18 s—8 d. These two summes being added together, do make 74 s 8 d. as in the third example appeareth.

4 *Item, for* 9 d. *there is taken for 6 pence the* $\frac{1}{2}$ of the whole summe: and the $\frac{1}{4}$ of the whole summe for 3 d. or otherwise for the 3 d. you may take the $\frac{1}{2}$ of the product that came of 6 d. because 3 d. is the $\frac{1}{2}$ of 6 d. which added together as plainly appeareth in the *fourth example* amounteth to 267 s. 9 d.

Item, for 10 d. *first there is taken for* 6 d. the $\frac{1}{2}$ of the whole summe, which amounteth to 397 s—6 d. Then for 4 d. there is found 265 s. both

both which added together, make 663 ſhillings 6 d. appeareth in the *fift example*, it may alſo be wrought, *as appeareth by the ſecond note in the Table*, by 4 d. twice taken, and the $\frac{1}{2}$ of the product of 4 d. or elſe by the $\frac{1}{3}$ of the whole ſumme, &c.

Item, for 11 d. *there is firſt taken the* $\frac{1}{2}$ *for* 6 d. then the $\frac{1}{3}$ of the whole ſumme for 4 d. Laſtly, the $\frac{1}{4}$ of the laſt product for 1 d. All which 3 ſumms added together, maketh in ſhillings 6944 s---8 d. and in pounds 347—4—s—8 d.

Item, likewiſe by the ſame reaſon, when you will multiply (by ſhillings) any number that is under 20 s. *you ſhall have in the product pounds, if you know the even or aliquot parts of* 20, *which are here in this little Table ſet down to ſight.*

$$\text{Item, for}\left\{\begin{array}{c}10\\5\\4\\2\\1\end{array}\right\}\text{is the}\left\{\begin{array}{c}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{10}\\\frac{1}{20}\end{array}\right\}\text{of one pound.}$$

So that for 10 s. *which is the* $\frac{1}{2}$ *of a pound*, you may take the $\frac{1}{2}$ of the number which is to bee multiplyed, and you ſhall have in your product pounds: if an unite do remain, it ſhall be worth 10 ſhillings.

Likewiſe for 5 *ſhillings you muſt take the* $\frac{1}{4}$ of the number which is to be multiplyed, and if there doe remayne any *unites*, they ſhall bee fourth parts of a *pound*, every unite being in value five *ſhillings*.

For 4 *ſhillings take the* $\frac{1}{5}$ *of the number* which

is

is to be multiplied : and if there doe remayne any *unites*, they shall be fift parts of a pound, each unite being in value 4 shillings.

For 2 shillings you must take the $\frac{1}{10}$ *of the* number to be multiplyed, wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is neerest your right hand) from all the other figures with a small right down line or dash with a pen, and so have you done : for all the other figures which do remain toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a pound : and that figure so separated towards your right hand, shall be so many pieces of 2 s. the piece, the which figure you must double to make thereof the true number of shillings, as by the example shall appeare.

Finally, for 1 shilling needeth small work, for it is so many shillings as be proponed in the summe, which to bring into pounds, hath been already taught in the *first Rule*.

Example.

At 10 s. the piece, What	6543 pieces?
$\frac{1}{2}$ li.	3271 10 s.
At 5 s. the Ell, What	4373 Els?
$\frac{1}{4}$ li.	1093 5 s.
At 4 s. the yard, What	7839 yards?
$\frac{1}{5}$ li.	1567. 16 s.
At 2 s. the pound weight, What	752|7 pound?
$\frac{1}{10}$ li.	752. 14 s.

At

At 1 s. the piece, What		757|3 pieces ?
$\frac{1}{20}$	li.	378 13 s.

NExt followeth in order to be understood, that if the number of shillings, be not some even or aliquot part of 20, you must then convert the same number of shillings into the aliquot parts of 20. and thereof make two or three products as need shall require: which done, adde them together, and bring them into pounds. And here for thy furtherance I have set down a note of the order of their parts, as they are to be taken. 4 Rule.

s					s		
3	of	2 & 1	or		13	of	10. 2 & 1
6		4 & 2		5 & 1	14		10. & 4
7		5 & 2			15		10. & 5
8		4 & 4		5.2.1	16		10. 5.1
9		5 & 4		4.4.1	17		10. 5.2
11		10 & 1			18		10. 4.4
12		10 & 2			19		10. 5.4

For 3 s. according to the tenour that you see is expressed in the Table, you must first take for 2 shillings the $\frac{1}{10}$ of the number that is to be multiplyed. Then for one shilling you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part ; which two summes added together, there produceth the effect desired.

Item, for 6 shillings: according to the note set forth in the table first, for 4 s. I take the $\frac{1}{5}$ of the num-

number that is to be multiply: Then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s. and adde them together.

Or else as appeareth also in the Table, *for* 5 *shillings* you may take the $\frac{1}{4}$ and the $\frac{1}{3}$ part of the product that came of 5 *shillings*, and adde them together.

Item, for 7 s. *first for* 5 s. *take* $\frac{1}{4}$ *of the product* that is to be multipled, then for 2 s. take the $\frac{1}{10}$ of the number that is to be multiplyed, and adde them together, &c.

Item, for 8 s. *according to reason*, *and the* intent of the *Table*, for the first 4 s. take the $\frac{1}{5}$ of the product, and the same number again for the other 4 shillings, and adde them together.

Item, for 9 *shillings: first for* 5 *shillings*, take the $\frac{1}{4}$; then for foure shillings take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by the intent of the Table, work twice for 4 shillings, as was taught even now for 8; and then take the $\frac{1}{4}$ of the last product for the 1 shilling: but 5 and 4 is the shorter.

Item, for 11 s. *first dispatch* 10 *shillings*, for which you must take the $\frac{1}{2}$ of the product, then lastly, for 1 shilling take the $\frac{1}{10}$ part of the sum produced of the $\frac{1}{2}$ of the product, & adde them together.

Item, for 12 *shillings, where I will end with the first part of my Table.* First take the $\frac{1}{2}$ for ten *shillings*. And then for 2 *shillings*, take the $\frac{1}{5}$ of the summe that came of ten *shillings*, take and adde

adde them together, or elſe if you pleaſe for 2 ſhillings, you may take the $\frac{1}{10}$ of the whole given number.

To write more of the manner of taking the true parts, I omit. The deſirous practitioners will (no doubt) conceive it. Alſo the *Table* is ſome aide to help the unperfect whereupon by and by I will ſet downe three or foure of theſe notes in *Examples*, and the reſt I will leave to thine owne induſtry and practice, to labour upon.

This is the order moſt commonly uſed in practice, when the number of ſhillings is not an *aliquot* part of a pound. But (*loving Reader*) after I have touched the even or *aliquot* parts of a pound that falleth out in pence and ſhillings, I will deliver two new Rules that ſhall drowne this common order quite and cleane: wherein ſhal be comprehended in one line, or working both of even and odde parts of ſhillings under 20, without regard whether it be an *aliquot*, or not an *aliquot* part; which two Rules (when they come in place) I commit to thy friendly judgement in working.

Now follow the examples upon the notes beforeſaid.

At 6 ſhillings the yard, what	3215 yards?	
4 ſhillings	643	
2 ſhillings	321	10
li	964	10 s.

 Other-

Otherwise by Multiplication of 6.

		3215
6 shillings		1929|0
	li —— 964 ——	10 *shillings.*

At 7 shillings the Ell, what 4563 Ells?

5 shillings	1140	15
2 shillings	456	6
li	1597	1 shilling.

Otherwise by Multiplication of 7.

	4563	
7s	3194|1	
	1597	1

At 8 s. the piece, what 7563 pieces?

4 s	1512	12
4 s	1512	12
pounds	3025	4 s.

Otherwise by Multiplication.

	7563	
8 s.	6050|4	
pounds	3025	4 shillings.

At 13 s. the piece, what 401 pieces?

10 s	200	10
2 s	40	2
1 s	20	1
pounds	260	13

Other-

Otherwise by Multiplication.

	401	
13 s.	1203	
	401	
	521	3
Pounds	260	13 s.

These and such like questions of compound numbers, which I have here in this fourth rule for orders sake set downe, for that It hath been heretofore a common course of worke, I account but superfluous. For in the eighth and ninth Rules of this my simple Addition shall appeare, that the given price of any even or odde number of shillings, either under or above 20 shall be wrought at one or two workings at the most, how difficult soever the question be.

Item, *there resteth yet a kinde of practice, how to bring pence into pounds at the first working, whereupon you must understand that 240 pence maketh one pound, or 20 s. In consideration wherof I cut off the last figure or 0, and there remayneth but 24 (of which 24) 8 d. is the $\frac{1}{3}$ part thereof,* 6 d. *is the $\frac{1}{4}$ part,* 4 d. *is the $\frac{1}{6}$ part, and 2 pence is the $\frac{1}{12}$ part thereof.* 8 Rules To reduce pence into pounds at one operation.

Whereupon if it were demanded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in pricking or cutting off the first figure towards your right hand, for

the 0 that appertaineth to 240. There is remayning of the ſaid ſumme 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 li. and there reſteth 1, which 1 I put to the 6, that I pricke or cut off, and it maketh 16 pieces of 8 pence, which I double to make into groats, & they make 32, whereof the $\frac{1}{3}$ part maketh 10 s, and there remayneth $\frac{2}{3}$ s. which is 8 d. whereby it followeth, that the 1489 yards at 8 pence the yard maketh 49 li. 12 s. 8 d. as by the example ſhall appeare.

Item, for 6 d. take $\frac{1}{4}$ part of the number from the prickt figure; and if any unites remayne, they are ſo many ſix pences, whereof taking the $\frac{1}{2}$ they are ſhillings, if there doe remayne yet one, it is in value ſix pence.

Item, for 4 d. take the $\frac{1}{2}$ part of the number from the prickt figure; If any unites remayne, they are ſo many groats, which to convert into ſhillings, take the $\frac{1}{3}$ part. And if any yet remayne, they are thirds of ſhillings, each one in value being worth 4 pence.

Item, for 3 pence, take the $\frac{1}{8}$ part from the prickt figure, if any unites remayne, they are ſo many pieces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh ſhillings: If any thing yet remayne, they are the fourth parts of ſhillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth alſo by the Table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remayne, they are ſo many pieces of 2 pence, which by taking the

the $\frac{1}{6}$ part, you shall turn into shillings, and if any unites remayne, they are so many sixt parts of shillings, or pieces of two pence, whether you will.

If one cost 8 pence, what maketh pounds	1486 ?
	49---10---8 d.
If one cost 6 pence, what maketh pounds	7865 ?
	196--12--6 d.
At 4 pence the yard, what maketh pounds	8736 yards ?
	145---12--0 d.
If one cost 3 pence, what maketh pounds	9874 worth ?
	123----8---6 d.
At 2 d. the Ell, what maketh pounds	7894 Ells to ?
	65---15--8 d.

But if your number of pence be not an aliquot or even part of 24: then must you bring them into the aliquot parts of 24, and make thereof diverse products which must be added together, as by the question hereafter following shall appeare. 6 Rule.

Item, for 5 d. first take for 3 d. then for 2 d. and adde them together, according to the instruction of the *second Rule*: or else first take for 4 d. then for 1 d.

Item, for 7 d. first take for 4 d. then for 3 d. and adde them together.

Item, for 9 d. first take for 6 d. then for 3 d, and adde them together.

Item, for 10 d. first take for 6 d. then for 4 d, and adde them together.

Item, for 11 d. first take for 8 d. then for 3 d, and adde them together: as by these Examples.

Examples.

1 If one yard cost 5 d, what 759|6?

4 pence 126——11

1 peny 31——13

maketh pounds 158----5

Otherwise.

1 —— 5----759|6

3 pence 94——19

2 pence 63——6

maketh pounds 158——5 s.

2 If one cost 7 d. what 98|7 worth

4 pence 16—9

3 pence 12--6-9

maketh pounds 28--15--9 d

Otherwise.

1 ——7—— 98|7

6 pence 24——13--6

1 peny 4——2--3

maketh pounds 28——15--9 d.

3 If one cost 9 d. what 98|7 worth

6 pence 24--13--6

3 pence 12---6--9

maketh pounds 37---0--3 d

Other-

Otherwise.

1——9——9817?	
6 pence	24——13——6
3 pence	12——6——9
maketh pounds	37——00——3

4 If one cost 10 pence, what	9817?
6 pence	24——13——6
4 pence	16——9——0
maketh pounds	41——2——6

5 If one cost 11 pence, what	9817?
8 pence	32——18——0
3 pence	12——6——9
maketh pounds	45——6——9

But if you have any shillings and pence to be multiplyed together: Then are you to take for the shillings according to the instruction of the third Rule: and for the pence according to the first Rule before mentioned: unlesse you can spie the advantage thereof, and thereby help your self: as appeareth in this second examples, where first I work for 6 d. which is to be rebated out of the given number, and I have 719 li. 11 s. my desire.

At 19 s. 6 d. the yard, what 738 yards?

	738
10 s.	369----0
5 s.	184----10
4 s.	147----12
6 d.	18----9
pounds	719 —— 11 s.

Otherwise by Rebating.

	738
6 d	18--9 s.
li.	7--19--11 s.

The like again is done by rebating, as by these two examples appeareth.

At 18 s. the Ell, what	418 Ells?
2 s.	41——16
pounds	376——4 s.

At 16 s. the Ell, what	517 Ells?
4 s.	10——8
pounds	413——12 s.

7 Rule. *And now I will touch a little the even part of a pound, that falleth out in pence and shillings, whereof for those parts you shall take such like part out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound, which also for their better aid is heere set down.*

s.	d.			
1 s.	8 d	*is the*	$\frac{1}{12}$	*part of a pound.*
2	6		$\frac{1}{8}$	
3	4		$\frac{1}{6}$	
6	8		$\frac{1}{3}$	

Item,

Item, first for 1 shilling 8 pence take the $\frac{1}{12}$, part of the given number, and if any thing doe remayne, they are twelve parts of a pound, each one being in value 1 shilling 8 pence.

Item, for 2 shillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied. And if any thing doe remayne, they are eight parts of a pound, each one being in value 2 shillings 6 pence.

Item, for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ part of the given number, and if any thing do remayne, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item, for 6 shillings 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplied: And if any unites doe remaine, they are thirds of a pound, every one being worth 6 shillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a *pound*, as 16 shillings 8 pence maketh $\frac{5}{6}$: which 16 shillings 8 pence is easily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then set 60 the groats of a pound under 50: cutting off the two Ciphers as is here performed.

16---8
3
5|0
6|0

And then have you brought 16 *shillings* 8 pence into the knowne parts of a *pound*, which maketh

But

But yet gentle Reader, for thy further instruction, I have hereunto annexed in a *Table*, how pence and shillings beare proportion to a pound, which I commit to thy friendly benevolence; it will be some aid unto the ungrounded Practitioner: but I count him the best Workman that can presently reduce his given price into the known and proportionate parts of a pound.

A Table of the Aliquot parts of a pound, or 20 shillings.

s	d	p
0	2	1/120
0	3	1/80
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	20
3	4	1/6
3	9	3/16
4	0	1/5
5	0	1/4
6	0	3/10
6	3	5/16
6	8	1/3
7	0	7/20
7	6	3/8
48	0	2/3

s	d	p
8	4	5/12
8	9	7/16
9	0	9/20
10	0	1/2
11	0	11/20
11	3	9/16
12	8	2/3
12	0	3/5
13	0	13/20
13	4	2/3
13	9	11/16
14	0	7/10
15	0	3/4
16	0	4/5
16	8	5/6
17	0	17/20
17	6	7/8
18	0	9/10
18	4	11/12
18	9	15/16
19	0	19/20

Here follow foure Examples upon the foure Notes delivered.

At 1 s. 8 d. the yard, what maketh pounds	3884 yards? 323--13--4 d.
At 2 s. 6 d. the yard, what maketh pounds	4563 yards? 570--7--6 d.
At 6 s. 8 d. the Ell, what maketh pounds	7562 Ells? 2520--13--4 d.

Now by custome you are able to worke by all sorts of summes being delivered in shillings and pence, as one shilling one peny, two shillings two pence, three shillings three pence, and so of all other: wishing you to have some consideration of of your questions, when they are set downe, for there are many subtile abreviations, and great advantages to be gotten, & easily to be perceived.

As of 3 s ———— 8 d. of 2 s. and 1 s 8 d, Of 4 s. ———— 2 d. of 3 s. ———— 4 d. & 10 d. which 10 d. is $\frac{1}{4}$ of 3 s. ———— 4 d. Of 5 s. ———— 8 d. of 4 s. 1 s——8 d. Of 5 s. 10 d. of 5 s. and 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this mean when you have taken one product, you may oftentimes upon the same take

take another more briefly then upon the sum which is to be multiplied, &c.

Now (gentle Reader) that you have seene the vertue of the even or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise hereafter followeth a briefe and easier method for any even number of shillings, either under or above 20, then ever yet hath been published; Notwithstanding M. Humphrey Baker, *whose travell is worthy commendation, and whom for knowledge sake I reverence, hath in some part touched this first part, though not in this method. The work of the Rule both pleasant, ready, and briefe, as by the variety of the examples delivered thereupon shall appeare. And first I will set forth one question, thereby the better to expresse or teach you the order thereof: which is this.* 8 Rule. Note a generall rule.

If one cost 6 s. what 8574?

I	6	8574
maketh pounds		2572 — 4 s.

To the understanding of this example, after you have set down your given number in form of the Rule of 3, with a line drawn under it, you shall presently set a prick under your first figure 4. towards your right hand, drawing from the prick, as heretofore hath beene practised, a little short line, thereto set downe the Mr. *Iohn Mellis* his first Rule.

the ſhillings anon, which done, multiply the firſt figure 4 by 6, the value of your price; (which here you ſee ſtandeth in ſight above the line) it maketh 24, which is one pound foure ſhillings. The one pound keep to carry to the next place, and the foure ſhillings ſet down at the end of the preſcribed line towards your right hand. Thus have you done now with 6 above the line, and alſo with 4 in the firſt place (for the prick under 4 doth ſignifie that 4 hath done his office.) Then ſecondarily for a generall Rule take but the ½ of the given price, which here is 3, which 3 is the number that ſhall now continue the reſt of the multiplication, and end the work, whereupon I multiply 3 into 7, ſtanding in the ſecond place it maketh 21, and with the one pound I kept in 22; ſet down 2, and keep 2 in minde, working according to the Rule of Multiplication, delivering the tenths in minde in their due place, which done, the product from the pricke to your left hand repreſenteth the pounds, and the other at the end of the ſhilling, as appeareth by the examples.

Note a generall rule.

If one yard coſt 2 s. what 7536?

7536 yards

maketh pounds 753 — 12 s.

If one yard coſt 4 s. what 8792?

8792

maketh pounds 1758 — 8 s.

If

If one piece cost 6 s. what 9537 ?
1 6 9537
maketh pounds. 2861 ——— 2 s.

If one piece cost 8 s. what 7509 ?
1 8 7509
maketh pounds 300 ——— 12 s.

If one cost 12 s. what 5794 ?
1 12 5794
maketh pounds 3476 ——— 8 s.

If one cost 14 s. what 3705 ?
1 14 3705
maketh pounds 2593 ——— 10 s.

If one cost 18 s. what 5703 ?
1 18 5703
maketh pounds 5132 ——— 14 s.

If one cost 22 s. what 953 ?
1 22 953
maketh pounds 1048 ——— 6 s.

Let these suffice (gentle Reader) for an entrance into even numbers. And now I will shew the like rule for any odde or uneven part of a pound.

To helpe you to the understanding of these other Questions that hereafter follow: where in my first Example the given number is 6487

at 3 s. the yard. I multiply 3 above the line into 7, it maketh 21. The one shilling is set downe, and the 1 pound I keepe. Now am I to take the ½ of three, which because it is an odde number I cannot.

Mr. *Iohn Mellis* his second Rule.

Therefore I shall keep and continue my multiplication by three still, and worke by the ½ of the rest of the given figures or number, to wit, 648. And first the ½ of 8 which is 4 multiplied into 3, maketh 12, thereto joyn the 1 li. in minde, it maketh 13 set downe 3, keep one. Then again multiply by two the ½ of foure, it maketh six, and with one in minde it maketh 7. Then lastly, take the ½ of six, which is 3, saying, 3 times 3 is 9, which 9 set down, and so is the question answered, as appeareth by the practice, and examples following.

At 3 s. the yard, what 6487?

	1	3	6487
maketh pounds			973 —— 1 s.

If one yard cost 5 s. what 4269

	1	5	4269
maketh pounds			1067 —— 5 s.

At 7 s. the Ell, what 6489?

	1	7	6489
maketh pounds			2271 —— 3 s.

If one Ell cost 9 s. what 2807?

	1	9	2807
maketh pounds			1263 —— 3 s.

If

At 11 s. the Pistolet, what	8263 ?
1 11	8263
maketh pounds	4544 —— 13 s.
If one piece cost 13 s. what	4629 ?
1 13	4629
maketh pounds	3008 —— 17 s.

But now note (gentle Reader) when the given price falleth upon an odde number, as 3, 5, 7, 11, 13, &c. then it is to be presupposed that the given summe to be multiplyed, must be a summe made of even numbers, 2, 4, 6, 8, 10, &c. else cannot that question be wrought at one line or working.

Providing always that it may beare an odde figure in the first place towards your right hand, as appeareth in these six examples, which last were wrought, and such like, &c. which may beare an odde number for the price, and be done at one line, or working very well.

But if the given price be an odde number, and the summe to be multiplyed, odde numbers also: then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall Rule. First, work for the even number, contained in that question, or given price, according as you have learned, and then afterwards for the one odde shilling, take the $\frac{1}{20}$ of the

A generall Rule.

the summe given to be mulliplyed, omitting the first prickt place, as was taught for the working of one shilling in my first Rule of Practice, and adde those two together, and you shall have your desire.

Examples.

At 3 s. the yard, what	7539 yards?
2 s.	753 —— 18
1 s.	376 —— 19
maketh pounds	1130 —— 17 s.

At 7 s the Ell, what 7539?	
5 s	7539
2 s	2261 —— 14
1 s	376 —— 19
maketh pounds	2638 —— 13

At 13 s. the yard, what	7534?
10 s.	3767 —— 0
2 s	751 —— 8
1 s.	376 —— 14
maketh pounds	4897 —— 2

Note this well. *And thus have I abridged into these two rules how to bring any number of shillings, whatsoever they be into pounds, with a briefer Method, than ever yet hath been published, which I commend unto thy friendly censure and judgement in the use and practice thereof.*

If

If one cost 6 s. 5 d. what	1231 ?
6 s.	369——6
4 d.	20--10--4
1 d.	5--2--7
maketh pounds	394--18--11

At 14 s. 2 d. what	2825 ?
14 s.	1977——10
2 d.	23--10--10
maketh pounds	2001--0--10

At 16 s. 4 d. what	2531 ?
16 s.	2024--16
4 d.	24--3--8 d.
maketh pounds	2066--19——8

At 3 s. the Pistolet, what	8325 ?
maketh pounds	1248——15 s.

At 7 s. the Crown, what	6529 ?
	2285——3 s.

At 9 s. the piece, what	6567 ?
maketh pounds	2955——3 s.

These three last questions may seeme something harder, yet they are easie enough, if you mark them well: if I should explaine them, then are they too easie. Therefore I leave them to whet the mindes of the desirous.

10 Rule. *Item, when any one of the summes, which is to be multiplyed, is composed of many denominations, and the given number but of one figure alone, then shall you multiply all the denominations of the other summe by the same one figure, beginning first with that summe which is least in value toward your right hand, and bring the product of those pence into shillings, and the product of the shillings into pounds, as by this example appeareth.*

At 3 li. 7. s. 4 d. a yard, what are 9 worth?	
maketh pounds	30--6 s--0 d.

11 Rule. *But if in any of the summes that are to be multiplyed, there be a broken number, First work for the whole according to the instructions that you have learned, and then take such part of the given price, as that broken number beareth in proportion to the price; as in the examples following. After you have wrought for 3 s. and for 6 d. then are you to take the* ½ *of* 3 s. 6. d. *for the* ½ *yard, and adde that to the summe: So adding all the 3 products together, which make 43 li. 2 s. 9 d. the just price of 245* ½ *Ells, and thus must you doe of all other.*

At 3 s. 6 d. the Ell, what	246 ½ ?
3 s.	36--18
6 d.	6--3
½	1--9
maketh	43--2-9

At

At 16 s. 4 d. the piece, what 14 $\frac{1}{4}$?

16 s.	11 — 4
4 d.	0 — 4 — 8
$\frac{1}{4}$	12 — 3
maketh pounds	12 — 0 — 11

If one piece cost 4 li. 3 s. 6 $\frac{1}{2}$ d. what 12 pieces?

4 li.	48 —
3 s	1 — 16 —
6 d.	6 — 6
$\frac{1}{2}$	6
maketh pounds	50 — 2 — 7

The proofe.

If 12 pieces cost 50 li. 2 s. 6 d. what one piece?

maketh pounds 4 — 3 — 6 $\frac{1}{2}$

Item, *touching the manner how to understand the order of this question, and others the like, first seeke how many times 12 is contained in 50, which is 4 times, and so resteth 2 pound, which 2 pound converted into shillings, and joyned with the other 2 shillings, maketh 42 shillings: wherein is found 12, three times, resteth 6 shillings: which turned into pence, putting thereto the 6 pence in the first place, it maketh 78, wherein 12 is found 6 times, resteth 6 pence: which containeth 12, but $\frac{1}{2}$ a time, put that $\frac{1}{2}$ to the 6 pence, and then the solution is 4 li. 3 s. 6 $\frac{1}{2}$ d. as appeareth by the practice thereof.* 14 Rule.

13 Rule. Item, *the like is to be done of any thing that is bought or sold after five score to the hundred, or the Quintall. As for example.*

If 100 pound cost 27 li. 13 s. 4 d. what one pound?

27 li —— 13 s —— 4 d.
20
s. 5 | 53
| 12
1 | 10
5 | 3
d. 6 | 4 | 0
| 10 | 0. or $\frac{2}{5}$

Maketh 5 s. 6 $\frac{2}{5}$ d.

I have wrought this at length for the aid of the yong learner, because he should understand how all the Multiplication is set down.

But to work it more neatly, it is by a little understanding ended thus.

27 li. — 13 s. — 4 d.
20
s 5 | 53
| 12
d 6 | 40
| 100

Maketh 5 s. 6 $\frac{2}{5}$ d.

Item, *to the understanding of this and such like questions, the right downe line is all the guide, which is pulled down close by 20, as you see in the example, where 27 pound 13 shillings is reduced all into shillings and maketh 553 shillings.*

The 5 towards the left hand being separated with the hanging or right down line, is the just *num-*

number of shillings, that answereth to the question. *Nextly, 53 shillings is multiplyed by 12, to reduce them to pence, putting to the 4 d. it yieldeth for the multiplication of the first figure two* 110: *the one beyond the line towards the left hand, is 1 peny towards the rest of the price: then 53 also multiplyed by 1 yieldeth 53: but the 5 behinde the line toward the left hand, is also 5 pence more, towards the price, which 1 and 5 I adde together under the line, it maketh 6* d. *So is there found now, as appeareth by the titles of shillings and pence, 5 shillings 6 pence.*

Finally, I come now on this side the line towards the right hand, and under 12 I finde first 10, *and then 3, which added together, maketh* 40, *under which* 40, *you must put the* 100, *and it maketh* $\frac{40}{100}$, *which abreviated, commeth to* $\frac{2}{5}$. *So the just price of one pound after 5 score to the hundred, maketh* 5 s. *6* $\frac{2}{5}$ d. One example more, and so I will leave this Rule.

If 100 cost 10 $\frac{1}{4}$ d. what 9874?

6 d	246	17	
4 d	164	11 ^{1}	4
$\frac{1}{4}$ d	20	11	5
$\frac{1}{4}$ d	10	5	8 $\frac{1}{2}$

Maketh
li. 44 2 | 5 | 5 $\frac{1}{2}$
20
s. 8 45
12
d. 5 $\frac{45 \frac{1}{2}}{100}$ $\frac{91}{100}$ parts of a peny,

 Also

Also the like may be done of the usuall weights here in *England*, (which is 112, for every hundred weight) in case you know the aliquot parts of a hundred weight, which are these, 56 li. 28 li. 14 li. and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li 28 li. is the $\frac{1}{4}$ of 112 li. 14 li. is the $\frac{1}{8}$ and 7 li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the summe of the money that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the summe of money that 112 li. weight is worth.

For 14 li. take the $\frac{1}{8}$ of the summe that 112 li. is worth.

And for 7 li. the $\frac{1}{16}$ of the summe of money that 112. li. is worth.

As for example, at 17 li. 19 s. the hundreth pounds weight, that is to say, the 112 li. what shall 3 quarters and 7 pound cost?

1. C.	17 li.	19 s.	3 q;	7 li.
2 quarternes	8	19	6	
1 quarterne	4	9	9	
7 pounds	1	2	$5\frac{1}{4}$	
Maketh pounds	14	11	$8\frac{1}{4}$	

The

The second Chapter intreateth of the Reduction of divers measures to others value by Rules of Practice.

NOw will I shew a few examples of Practice in reducing of measures, as Ells, Yards, Braces, Pawns of Genee, &c. Much more would I have touched, but that I feare the Book will rise to too great a Volume. 18 Rule.

In 864 Ells of *Antwerpe*, how many yards of *London?*

864	864
432	216
216	648

maketh 648 yards of *London.*

ITem, *in these and such like questions of* Flemmish *measure, to be brought into yards* English, *first take the* $\frac{1}{2}$ *of the given number, as appeareth in the first example towards your left hand. Then take halfe of that product, or the* $\frac{1}{4}$ *of the given number, and adde those 2 products together, as they shall be yards* English; *as by the example you may perceive.*

The

The second example toward your right hand is yet briefer than the first, whose worke is this; Take the ¼ of the delivered number, and that product subtract out of the given number, and the rest sheweth your desire. Of these two wayes use which you thinke best.

The proof.

In 648 yards *London*,
How many Ells of *Antwerpe?*

	648	
	216	
maketh	864	Ells of *Antwerpe*.

15 Rule: ITem, *for the understanding of this worke, first take the ⅓ part of the yards of* London, *which found, adde that ⅓ part and the yards together, as appeareth by the practice, and the product sheweth the Ells of* Antwerpe.

Item, in 320 yards of *London*,
How many Ells of *Antwerpe?*

maketh	426 ⅔ Ells.	
	320 yards	Proof.
⅓	106 ⅔	426 ⅔ Ells.
	426 ⅔ Ells.	⅓ 106 ⅔
		320 yards.

Other Reductions.

16 Rule. ITem, *you shall understand, that forasmuch as six Braces of* Millain, *make five Ells of* Antwerpe, *whereupon according to the* Rules of Practice, *you may reduce the one into the other, by the like reasons aforesaid, in taking the ⅙ part,* and

and then subtract the same, to make Els of Antwerpe. *And again, by the contrary, taking the* $\frac{1}{3}$ *part with adding the given number, to turne the Els to Braces. As for example.*

In 876 Braces, how many Els of *Antwerpe?*

876.
146
Els 730 *Antwerpe.*

The contrary.
730 Els *Flemmish.*
146 Braces.
816 Braces.

Els 730 *Antwerpe.*
$182\frac{1}{2}$.
Yards $547\frac{1}{2}$ *English.*

Thus appeareth, that 876 Braces by Practice, make 730 Els *Flemmish*, which Els *Flemmish* reduce into *English* yards.

So again upon the same first question of Braces, I would know how many yards *English* they make.

After the rate that 100 Braces are
worth $62\frac{1}{2}$ yards.
876 Braces.
438
$109\frac{1}{2}$
I answer, $547\frac{1}{2}$ yards.

Item, *to the understanding of this worke, and such like, first take the* $\frac{1}{2}$ *of the given Braces, and after take the* $\frac{1}{4}$ *of that halfe, or the* $\frac{1}{8}$ *of the given number, and adde them together, and the products are also yards* English. 17 Rule.

Item,

Item, *three Ells of* Rochell *make 5 Els at* Lisbone. *So likewise three Els at* Lions *make 5 Els at* Antwerpe.

To work these and such like, double the Ells of Lions, *and the Els of* Rochell, *and from their products subtract the* $\frac{1}{6}$, *and the rest shall be the Els of* Antwerpe, *on the Els of* Lisbone.

I *Example.*

In 63 Ells of *Lions* how many Ells of *Antwerpe?*	In 100 Ells of *Rochell*, how many Ells of *Lisbone?*
63	
63	100
126	200
$\frac{1}{6}$) 21	$\frac{1}{6}$) 33 $\frac{1}{3}$.
Ans. 105 Ells *Ant.*	Ans. 166 $\frac{1}{3}$ Ells of *Lisb.*

Touching the proofe or returne of these and such like questions, for a generall Rule, you shall first take the $\frac{1}{5}$ of the given number: and adde that $\frac{1}{5}$ and the given number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Ells of *Antwerpe*, how many Ells of *Lions?*	In 166 $\frac{2}{3}$ Els of *Lisbone*, how many Ells of *Rochell?*
105	166 $\frac{2}{3}$
$\frac{1}{5}$) 21	$\frac{1}{5}$) 33 $\frac{1}{3}$
$\frac{1}{2}$) 126	$\frac{1}{2}$) 200
Ans. 63 Ells of *Lions.*	Ans. 100 of *Roch.*

Que-

Questions of Factoridge and Interest, briefe and truly resolved by the Rule of Practice without Time.

1 Question.

AT 5 shillings *per Centum*, what comes 8860 li. 15 s. 4 d. unto?

Answer. Note that 5 s. is the fourth of 20 s. I take the ¼ part of 8860 li. 15 s. 4 d. which makes 2215 li. 3 s. 10 d. Now the Roote is 100, which you should divide by, so cutting the two last figures away of the pounds, I 22 li. then multiply 15 li. by 20 s. so adde the 3 unto , you shall have 303 s. cut away the two last figures, there resteth 3 s. Lastly, there remaines 3 s. which I multiply by 12 to bring into pence, and so I finde o d. and $\frac{44}{100}$ remayning, which being abreviated, makes $\frac{22}{50}$ parts of a peny, so I finde that there is gained 22 li. 3 s. o d. $\frac{22}{50}$ parts of a peny.

li. 22 15.03.10
20
s. 303
12
06
o d. 4
4 | 22
100 | 50

2. *Quest.* At ten s. *per centum*, what comes 1448 li. 16 s. 8 d. unto

li.	7	24.8.4.
		20.
s.	4.	88
		12
		180
		88
d.	10	6 | 0 | 3
		10 | 9 | 5

Answer. Note that ten s. is the $\frac{1}{2}$ of 20 s. I take the $\frac{1}{2}$ of 1448 li. 16 s. 8 d. which makes 724 l. 8 s. 4 d. cut off the two last figures, & there resteth 7 li. then multiply the 24 li. by 20 s. and adde the 8 s. and it maketh 488 s. cut the two last figures off, and there resteth 4 s. then multiply 88 s. by 12 d. and take in 4 d. and there resteth 1060 d. cut off the two last figures, and there resteth ten d. and $\frac{60}{100}$ which is $\frac{3}{5}$ of a peny, so the whole sum is 7 li. 4 s. 10 $\frac{3}{5}$ which is the answer to the question.

3. *Quest.* At 15 s. *per centum*, what comes 1008 l. 12 s. 0 d. unto

	504.6	0
	252.3	0
7	56.9	0
	20	
11	29	
	12	
	58	
	29	
	348	24|12
	100	50|25

Answer. Note 15 s. that is $\frac{1}{2}$ and $\frac{1}{4}$ of 20 s. take the $\frac{1}{2}$ of 1008 li. 12 s. there resteth 504 l. 6 s. the take the $\frac{1}{4}$ adde the together the totall will be 756 li. 9 s. cut off the two last figures resteth 7 li. then multiply by 20 s. and take in your 9 s. it maketh 1129 s. cut off the two last figures, there resteth 11 s. then multiply by 12 d. there

there commmeth 348 d. cut off the last two figures, there resteth 3 d. and $\frac{48}{100}$ which being abreviated maketh $\frac{12}{25}$ parts of a peny, so shall you finde 7 li. 11 s. 3 d. $\frac{12}{25}$, which is the answer to the question.

4 *Quest.* At 1 li. *per centum*, what comes 868 li. 13 s. 4 d. unto

Answer. Cut away the two last figures, and multiply by 20, and 12, and take in your shillings and pence. And you shall finde 8 li. 13 s. 8 d. $\frac{4}{5}$ as doth appear by this work.

li 8|68 li. 13 s. 4 d.
20
s. 13|73
12
1 50
73
d. 8|8|0|4
100 5

5. *Quest.* At 2 li. *per centum*, what comes 5608 li. 6 s. 8 d. unto

Answer. Multiply the whole summe by 2 li. thus, then cut off the two last figures of your pounds, as you did before, and you shall find 112 li. then multiply by 20 and by 12, taking in your shillings & pence and you shall find 112 l. 3 s. 4 d. which is either for Factor or Broker, &c.

5608 li. 6 s. 8 d.
2
112|16 13 4
20
3|33
12
70
33
4|00

6. *Quest.*

6 *Question.* At 3 li. *per centum* what comes 800 li. 18 s. 2 d. unto

Answer. Multiply the summe by 3 li. thus then cut off the two last figures, and you shall find 24 l. then multiply by 20, and by 12. taking your shillings & pence and you shall finde 0 s. 6 d. $\frac{27}{50}$ parts of a peny, which is something above a half-peny.

```
800 li. 18 s. 2 d.
                3
-------------------
  24|02. 14. 6.
    |20
  --------
   0|54
    |12
  --------
   114
    54
   6|54|27
    -------
    100|50
```

7 *Question.* At foure *per centum* what comes 298 li. 15 s. 9 d. unto

Answer. Multiply by 4 li. thus, cut of the two last figures, Multiply by 20, and by 12 taking in your shillings and pence, and you shall finde 11 li. 19 s. 0 d. $\frac{2}{25}$ parts of a peny, which is something above a farthing.

```
298 li. 15 s. 9 d.
                4
-------------------
  11|95. 3. 0
    |20
  --------
  19|03
    |12
  --------
    06
    03
   0|36|18|9
    ---------
   100|50|25
```

8. *Queſt.* At 5 li. $\frac{1}{4}$, *per centum*, what comes

Anſwer. Multiply by 5 li. thus then take the $\frac{1}{2}$ of the whole ſumme and place the figures even, then take the $\frac{1}{2}$ of that $\frac{1}{2}$, and add all three ſummes together, cut off the two laſt figures, then multiply by 20 & by 12, taking in your ſhillings and pence, and you ſhall find 210 l. 7 s. 7 d. $\frac{2}{10}$ parts of a peny, which is the anſwer to the queſtion.

3658 li. 16 s. 8 d.
unto 5

18294 03 4
1829 08 4
914 14 2

210|38 05 10
20

7|65
12

130
66

7|190
|10|0

9 *Queſt.* At 6 l. $\frac{1}{2}$ *per centum*, what comes

Anſwer. Multiply by 6 li. and then take $\frac{1}{2}$ of the whole ſumme, adde them both together, then multiply by 20, and by 12, taking in your odd ſhillings & pence, and you ſhall finde 369 li. 10 s. 10 d. $\frac{3}{20}$ parts of a peny, which is the anſwer to your queſtion.

5684 li. 12 s. 6 d.
unto 6

34107 15 0
2842 06 3

369|50 01 3
20

05
01

15 |3
100 20

10 *Quest.* At 7 li. $\frac{1}{2}$ *per centum*, what comes 3868 l. 13 s. 4 d. unto

Answer. Multiply by 7 li. then take the $\frac{1}{2}$, adde them together, cut off the two last figures, then multiply by 20, you shall find 290 li. 3 s. the answer to the question.

3868 l. 13 s. 4 d. unto
7
27080.13.4
1934.06.8
290|1500.0
20
3|00

11 *Question.* At 8 li. *per centum*, what comes 2560 l. 17 s. 9 d. unto

Answer. Multiply 8 li. cut off the two last figures, multiply by 20, and by 12, and you shal finde 204 li. 17 s. 5 d. $\frac{2}{25}$ parts of a peny.

2560 l. 17 s. 9 d. unto
8
204|87.02.0
20
17|42
12
84
42
5|04 | $\frac{2}{50}$ | $\frac{2}{25}$
100

Qu-

Questions of Interest with Time, wrought by Practice.

I *Question.*

AT 6 *per Centum*, what comes unto 468 li. 16 s. 8 d. for 1 month.

```
468 li. 16 s. 8 d.
               6
------------------
2813.  00    0
------------------
li. 2|34. 08. 4
     |20
     ----
s. 6 |88
     |12
     ----
   180
    88
------------
d.10|5  0 |3
    -----------
     10|0 |5
```

Answer. Multiply by 6 li. there commeth 2813 li. 00 s. 0 d. then take for 1 moneth the $\frac{1}{12}$ of the Totall, and you ſhall finde 234 li. 8 s. 4 d. of the two laſt figures of the pounds, Multiply by 20 and by 12, taking in your odde money, and you ſhall finde 2 li. 6 s. 10 d. $\frac{4}{5}$ parts of a peny, which is the anſwer to the queſtion.

2 *Quest.* At 7 li. $\frac{1}{2}$ *per centum*, what comes unto for 2 moneths.

3800 li. 12 s. 8 d.
7

26604	08	8
1900	06	4
28504	15	0
47 | 50	15	10

20
10 \| 15
12
30
1 6
1 9 \| 0 \|
10 \| 0 \|

Answer. Multiply by 7 li. then take $\frac{1}{2}$, adde them two together, thē for your two moneths take the $\frac{1}{6}$ of the Totall, multiply by 20 and 12, taking in your odde shillings and pence, and you shall find 47 l. 10 s. 1 d. $\frac{2}{10}$ parts of a peny, which is the answer to the question.

3 *Question.* At 8 li. *per centum*, what comes unto for 3 moneths.

9864 li. 16 s. 4 d.
8

78918	10	8
197 | 29	12	8

20
5 \| 92
12
192
93
11 \| 12 \| 6 \| 3
100 \| 50 \| 25

Answer. Multiply by 8 li. then take for your 3 moneths take $\frac{1}{4}$ of the Totall, multiply by 20, and by 12 adding in your odde shillings and pence, and you shall finde 197 li. 5 s. 11 d. $\frac{2}{25}$ parts of a peny, your demand.

4 *Quest*

4 *Quest*. At 6 li. $\frac{1}{2}$ *per centum*, what comes unto for 4 moneths.

Answer. Multiply by 6 l. then take $\frac{1}{2}$ adde both together, then for your 4 moneths take $\frac{1}{3}$ part of the whole, cut away your two last figures, multiply by 20, and by 12: adde in your odde shillings & pence, and you shall finde 131 li. 14 s. 10 d. $\frac{42}{55}$ parts of a peny, your demand.

```
6080 li. 13 s. 0 d.
               8
36483    18    0
 3040    06    6
39524    04    6
131|74   11    2
   |20
 14|91
   |12
  184
   91
 1094|47
     |100|50
```

5 *Question*. At 8 *per centum*, what comes unto for 5 moneths.

Answer. Multiply by 8 li. then for 5 moneths take $\frac{1}{4}$ of the Totall, cut off the two last figures of your pounds, Multiply by 20 and by 12, adde in your odde shillings and pence, and you shall finde 100 li. 13 s. 4 d. your demand.

```
3020 li. 00 s. 00 d.
                8
24160    00    00
 6040    00    00
 4026    13    04
100|66   13    04
   |20
 13|33
   |12
   70
   33
   4|00
```

6 *Quest.* At 8 *per centum*, what comes unto for 6 moneths.

8060 li. 12 s. 0 d.
8
64484. 16 s.
322 | 42. 08 0
20
8 | 48
12
96
48
5 | 76 | 38 | 19
100 | 50 | 25

Answer. Multiply by 8 li. then for your 6 moneths take the $\frac{1}{2}$ of the Totall, cut off the two last figures of your pounds, Multiply by 12, taking in your odde shillings and pence, and you shall finde 322 li. 8 s. 5 d. $\frac{12}{21}$ parts of a peny, your desire.

7 *Question.* At 8 li. *per centum* what comes unto for 7 moneths.

5896.00.0 d.
8
7168.13.4
11792.00.0
275 | 14.13.4
20
2 | 93
12
190
93
11 | 2 | 0 | 1
10 | 0 | 5

Answer. Multiply by 8 li. then for your 7 moneths take $\frac{1}{3}$ and $\frac{1}{4}$ of the Totall, cut off the two last figures of your pounds, then multiply by 20 and 12, taking in your odde money, and you shall finde 275 li. 2 s. 11 d. $\frac{1}{5}$ your desire.

8 *Queſtion.* At 8 *per centum*, what comes unto for 8 moneths.

Anſwer. Multiply by 8 li. then for 8 moneths take $\frac{2}{3}$ of the Totall, cut off the two laſt figures of your pounds, then multiply by 20, and by 12 adde in your odde money, and you ſhall finde 116 li. 5 s. 9 d. $\frac{2}{25}$ your deſire.

3680 li. 08 s. 03 d.
8
29443. 04 0
9814. 08 0
9814. 08 0
116|28. 16 0
20
5|76
12
152
76
912 6|3
|100 50 25

9 *Queſt.* 8 li. *per centum*, what comes unto for 9 moneths

Anſwer. Multiply by 8 li. then for your 9 moneths take $\frac{1}{2}$ and $\frac{1}{4}$ of the whole ſumme, cut off the two laſt figures of the pounds, then multiply by 29, and by 12: taking in your odde ſhillings & pence, and you ſhall finde 221 li. 1 s. 11 d. $\frac{7}{25}$, which is ſomething above a farthing.

3684 li. 19 s. 0 d.
8
29479. 12 0
14739. 16 9
7369. 18 0
221|09. 14 0
20
1|94
12
188
94
11|28 |14|1
100|50|25

10. *Quest.* At $6\frac{3}{4}$ *per centum*, what comes unto for 10 moneths.

Answer. Multiply by 6 li. then take the $\frac{1}{2}$ & $\frac{1}{4}$ of 100 li. adde all 3 summes together, then for the 10 months take $\frac{1}{2}$ and $\frac{1}{3}$ of the Totall, adde them together, off the two last figures of the pounds, multiply by 20, and 12, adding in your shillings and pence, cutting off the last figures of your shillings and pence, you shall finde 5 li. 12 s. 6 pence your desire.

	li.	s.	d.
	100 li.	0 s.	0 d.
			6
	600.	0	0
	50.	0	0
	25.	0	0
	675.	0	0
	337.	10	0
	225.	00	0
5	62.	10	0
	20		
12	50		
	12		
	100		
	50		
6	00		

11 *Quest.*

11 *Queſt.* At 8 li. *per centum*, what comes unto for 11 moneths.

Anſwer. Multiply by 8 li. then for 11 months take $\frac{2}{3}$ and $\frac{1}{4}$, from the Totall adde all three ſummes together; cut off the two laſt figures of your pounds. Multiply by 20 and by 12, adding in of your ſhillings & pence, cutting off the two laſt figures of your ſhillings and pence, and you ſhall finde 65 li. 0 s. 7 d. $\frac{17}{25}$ parts of a peny, your deſire.

886 li 16 s. 0 d.
8
7094 08 0
2364 16 0
2364 16 0
1773 12 0
65|03 04 0
|20
0|64
|12
128
64
7|68 |34|17
100|50|25

12. *Queſt.* At 8 li. *per centum*, what comes unto for 12 moneths.

Anſwer. Multiply by 8 l. cut off the two laſt figures of the pounds, Multiply by 20, and by 12, adding in your ſhillings and pence, cut off the two laſt figures of your ſhilling, and the two laſt of yonr pence, and you ſhal find 726 l. 8 s. 11 d. $\frac{17}{25}$ parts of a peny, your deſire.

9080 li. 12 s. 2 d.
8
726|44. 17 4
|20
8|97
|12
11|68 |34|17
100|50|25

FINIS.

The

The third Chapter teacheth of the Order and worke of the Rule of three in broken numbers after the Trade of *Merchants*, digressing something from Master *Records* which is comprehended in three Rules.

Ow, that I have somewhat intreated of the Rules of Practice, *I will give a few instructions, after my simple order, for the working of the* Rule of three *in broken numbers, wherein I shall need to say the lesse, because I hope the studious learner, that hath travelled any thing in the* Grounds of Arts, *is not unfurnished of knowledge capable to understand me.*

But before I deliver any instructions for broken numbers, I will propone a question which shall be wrought three sundry wayes, thereby to shew, as it were, three degrees of Comparison: how farre the Rule of three in broken, for more speed of worke, differeth from the whole, which I rather set down for a view, that the studious herein may be more desirous to attaine broken, leaving any more to discourse in Dialogue forme, but onely to give instructions where need is: and in the rest to put forth the questions with their answers.

My

My first Question is thus.

If one yard cost 6 s. 8 d. what are 789 worth at that rate ?

1 ——— 6 s ——— 8 d ——— 789 The first way.

12 80

80 63 120 d.

Here the product of the summe are pence, according to the nature of the middle number.

11
1370 1
63120 (5260 (263
12222 2220
111

I answer ——— 263 li.

1 ——— $6\frac{2}{3}$ s ——— 789 The second way.

——————————————— 20

3 20 ——————————

15780 s.

Here the product of the summe are s. according to the nature of the middle number.

1
15780 (5260 (263
3333 2220
li.

1 ——— $\frac{1}{3}$ ——— 789 The third way.

3 1 1

789

Here

Here the product is pounds, according to the title of the ſecond number.

$\cancel{1}$

$\cancel{6}\cancel{8}\cancel{9}$ (263

$\cancel{3}\cancel{3}\cancel{3}$

I anſwer, 263 li.

Now that you have ſeene the three former vertues of the Rule of three, whoſe products have firſt brought forth d. next s. and laſtly li. I will deliver three notes in order following: and with them a dozen queſtions that ſhall ſhew the work of the Rule of three in broken numbers or Fractions.

Note theſe three Rules. 1 The firſt foure ſhall be ſundry queſtions of a Fraction comming in the ſecond place.

2 The ſecond foure ſhall be of two Fractions comming in the ſecond or third place.

3 The third foure of Fractions in all three places.

Notes upon the firſt Rule for a Fraction comming in the ſecond place.

My firſt Queſtion is this.

1 Rule. *If one yard coſt mee* 3 s. 4 d. *what are* 756 *worth at that price?*

The firſt variety. In ſetting down the queſtion to perform the work, I turn four pence into the part of a ſhilling, which is $\frac{1}{3}$ and then the queſtion ſtandeth thus:

1 ——— 63$\frac{1}{3}$ ——— 756.

To

To the ready working of this question, and all ſuch other like: my firſt note is this, which take for a generall Rule; that when any one Fraction ſhall come, either in the ſecond or third place, that the Denominator of that Fraction or Fractions, muſt alwayes be brought unto the Number, or Numerator of the firſt place; and thereby multiply the one into the other. A generall Rule.

And this benefit is always gotten by the vertue of bringing the Denominator of the ſecond Numbers Fractions unto the firſt place: For the Fraction in the middle number is now releaſed: and the product that commeth of the multiplication, is of the nature and like denomination of the whole number in the ſecond place which here are ſhillings. Note this.

Whereupon now to worke the Queſtion, I bring 3, the Denominator of the Fraction in the ſecond place, unto my firſt Number 1, with a line ſet under thus $\underline{1}$, and the 3 under it thus, $\frac{1}{3}$: ſaying once 3, is 3 my Diviſor: that done, reduce 3 $\frac{1}{3}$ ſaying, 3 times 3 is 9, and the other 1 over 3 make 10: my ſecond number in the *Rule of three*, by which 10 I do multiply my laſt number 756, as appeareth by the worke thereof, and it yieldeth 7560 ſhillings my Dividend.

Then dividing 7560 by 3 my Diviſor, it yieldeth in quotient 2520 ſhillings, which maketh 126 pounds, as appeareth here moſt plainly, both by the example and the worke.

At

At 3 s. 4 d. the yard, what 756 yards?

$\frac{1}{3}$ —— $3\frac{1}{3}$ —— 756
10 10
7560 s.

~~1~~	~~1~~	
~~7560~~	~~2520~~	(126.
~~3333~~	~~2220~~	

I answer 126 li.

Yet otherwise upon the same question, altering the price now into the proportion it beareth to a pound, for the 3 s. 4 d. is $\frac{1}{6}$ part of a pound: which example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

The second varietie.

1 —— $\frac{1}{6}$ —— 756 : 6   1 —— $\frac{1}{6}$ —— 756
1 1
756 pounds.

As soon as I have carried 6 the denominator of my middle number unto my first place, as before hath been taught, I pull down 1, the numerator of 6, with a line under 6, thus, $\frac{6}{1}$, and that one in custome I pull down in sight; being the figure that I will multiply my third or last number by, according to the tenour of the *Rule of three.* And because one can neither multiply nor yet divide (though here it is set downe in forme of Multiplication, the rather for your understanding) the product of the Multiplication according to the declaration of

of this my first Rule or note, is converted into the title of my second number, which here are pounds. Now followeth the division performed in my Divisor 6, to make an end of that question.

13
756(126. *which maketh* 126 li. *as before.*
666

And thus much for the variety in working that question.

And now followeth another.

My second Question.

If one yard of Cotton cost $8\frac{1}{4}$ d. what 895 ?

1 ———	$8\frac{1}{4}$ ———	859
4	33	33
	2577	
	2577	
	28347	

1
1|321 1|1 li. s. d.
28347 (7086(590 (29——10——$6\frac{1}{4}$
4444 1222 220

This question was also wrought like the first, and bringeth forth 29 li, 10 s. $6\frac{1}{4}$ d. *the price of* 859 *yards.*

My

My third Question.

If seven pounds of any thing cost 3 li.--10 s. what comes 987 pounds to ?

li.

7 —— $3\frac{1}{2}$ —— 987
2 —— 7 | 7
14 | 6909

00
$\cancel{1}\cancel{4}\cancel{1}$
$\cancel{2}\cancel{3}\cancel{4}$7
$\cancel{6}\cancel{9}\cancel{0}\cancel{9}$ ($493\frac{7}{14}\frac{1}{2}$
$\cancel{1}$444
$\cancel{1}\cancel{1}$

I answer, 493 li.--10 s

Notes upon my second Rule for two Fractions comming in the second and third place.

My first Question is this.

IF one Ell cost 13 s —— 4 d. what half a quarter or $\frac{1}{8}$ of an Ell ?

Answer. *First bring* 13 s--4 d. *into the parts of a pound, which is* $\frac{2}{3}$, *and then will the question stand thus.*

1 —— $\frac{2}{3}$ li. —— $\frac{1}{8}$.

I*Tem*, for the performance of this work, doe as before was taught in the first Rule: first

bring

bring 3 *the Denominator of the second Fraction unto your first number* 1, setting a line under it thus : 1, Saying once 3 is 3, that done, bring 8 the *Denominator of the third Fraction*, setting it under 3, and multiply them together, saying, 3 times 8 maketh 24, which 24 is your *Divisor*. (Now have you done with the *Denominator* 8) Therefore you shall put a line under, thus, 3. And the like line also under 8, setting or pulling downe under them their own *Numerators*, that is, 2 under 3, and also 1 under 8, as appeareth in the example, which *numerators* for a generall rule evermore to be pulled down of custome in sight. to multiply the one by the other, according to the tenour of the *Rule of Three*. Then I multiply the one by the other, saying, once 2 is two, which signifieth 2 li. being of the nature and like denomination of the middle number, which 2 li. is to be reduced into shillings, otherwise it cannot be divided by my first number 24.

Then dividing 40 by 24, the quotient bringeth forth 1 $\frac{2}{3}$. So much is $\frac{1}{8}$ of an Ell worth after that rate Otherwise although 2 pound could not be divided by 24, yet it might have beene abreviated to $\frac{1}{12}$ of a pound : which is worth 1 s. 8 d. as before.

li.

$$\frac{1}{3}\text{———}\frac{2}{3}\text{———}\frac{1}{2}\;(1$$

3	2	1 2	(6
8	20	40	$(1\frac{2}{3}$ s.
24	40	24	

Second question.

IF *one pound of any weight cost* 13 *shillings* 4 *pence, what are* $\frac{7}{8}$ *of the pound worth after that rate?*

Answer. Reduce the 13 shillings 4 pence into the parts of a pound: which is $\frac{2}{3}$, and then will the question stand thus.

li.

$$1\text{———}\frac{2}{3}\text{———}\frac{7}{8}$$

ITem, *for the understanding of this, if you mark well the last example, this and the rest lyeth open, and needs small instruction. For as you did last, so again, bring the* Denominator *of the second & third* Fraction, *unto the first figure* 1, *multiplying the one into the other, which maketh also* 24, *your* Divisor.

Note. Then making a line under 3, thus, $\underline{3}$ and a line under 8, thus, $\underline{8}$ and pulling downe their *Numerators* under each figure, that is 2 under 3, and 7 under 8, which as I said before for a generall rule I pull downe of custome in sight,

ſight, to be the two numbers; that of duty ought to be multiplyed together, which done, I bring 2, being the leſſer figure under 7, multipling them together, it maketh 14, which are of the nature of the middle number: that is to wit, pounds, which 14 cannot aptly be divided among 24: therefore are reduced into *ſhillings*, as is plainly to be ſeen in the example: then 280 *ſhillings* parted among 24, yieldeth for his *quotient* 11 s. 8 d. your deſire, and the juſt price of $\frac{7}{8}$ of an *Ell*. Otherwiſe 14, though it could not be divided by 24, might by *mediation* or *diviſion* in broken *numbers* have been divided or abreviated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11 s. 8 d. as before. But my good will and meaning is to aid young beginners: therefore have I reduced the 14 *pound* into *ſhillings*, which is the eaſier way.

Now followeth the Example.

$\frac{1}{3}$ ——	$\frac{2}{3}$ ——	$\frac{7}{8}$	1
			~~2~~
$\frac{8}{24}$	2	7	~~4~~ (6
		$\frac{2}{14}$	~~280~~ (11$\frac{2}{3}$ s.
		20	~~224~~
			~~2~~

280 s. I anſwer, 11$\frac{2}{3}$ s.

The third Example.

If one yard cost mee 2 s. — 6 d. what $345\frac{1}{4}$ yards?

Answer. First put 6 d. into the parts of a shilling, and then the question standeth thus:

$$1 — 2\frac{1}{2} — 345\frac{1}{4}.$$

Item, to the ready understanding of this, and all such like, according as before hath been declared, bring the *Denominators* of the second and third *Fractions* unto the first place, multiplying them the one into the other, all which make 8 for the common *Divisor*. Then next reduce your second number: saying, two times 2 is 4, and 1 is 5; as was taught in the example aforesaid. Lastly, reduce your third number $345\frac{1}{4}$ all into fourths, and they make 1381, which 1381 is to be multiplyed by 5, according to the tenour of the *Rule of three*: which done, maketh 6905 s. and divided by 8, your Divisor yieldeth in Quotient $863\frac{1}{8}$ s. which maketh in pounds 43 li. 3 s. $1\frac{1}{2}$: and so much are the $345\frac{1}{4}$ yards worth at that price.

The same question wrought again by two shillings 6 pence, is now converted into the parts of a pound, and standeth thus:

$$1 — \frac{1}{8} — 345\frac{1}{4}.$$

Item, After I have brought here my second and

and third *Denominator* unto my first place, and found 32 to be my *divisor*; having thus finished my first place with al things unto him belonging (wch is meant of bringing and multiplying the *Denominators* of the second and third *Fractions* into him) I then go in hand to see what is to do in my second place, where presently of custome I pul down my Numerator 1 under 8, being the figure in sight that shall multiply my third number.

Then lastly, I reduce 345 $\frac{1}{4}$ all into fourths as afore was practised, which maketh 1381, the which 1381 I am to multiply by 1 my second number, they are nothing increased, but by the *Metamorphosis* of my work they are now 1381 pound, being of the nature of the middle number, as I have often shewed you, which divided by 32 my Divisor yieldeth 43 pound, and $\frac{5}{32}$, which $\frac{5}{32}$ of a pound reduced into knowne numbers, make 3 shillings 1 $\frac{1}{2}$ d. as before.

Example.

```
                          1
1 —— 1/8 —— 345 1/4     105
32    1                 1381  (43   5/32
            1381        522
                          3
```

NOw follow foure other questions, which are in all three places broken numbers: or whole and broken together.

Item, First, for the finding out of your *Diviſor*, you ſhall take this for a moſt certaine, and generall rule: That you muſt multiply the *Numerator* of the firſt *number* in the queſtion, by the *Denominator* of the ſecond: And that *Product* againe by the *Denominator* of the third: And the totall thereof ſhall be your *Diviſor*.

Secondly, for a generall rule to find out your *Dividend*, *multiply* the *Denominator* of the firſt number by the *Numerator* of the ſecond, and the whole thereof by the *Numerator* of the third. And the totall thereof ſhall evermore be your *Dividend*.

Now for an example, I propone this queſtion, thereby to make my meaning more plain, and to ſhew you, as I have done in the reſt, the manner and order of the worke.

If $\frac{2}{3}$ *of any weight or meaſure coſt* $\frac{1}{6}$ *of a pound, or 20s. what are* $\frac{1}{8}$ *of the like weight or meaſure worth after that rate?*

Example.

$$\frac{2}{3} \text{———} \frac{1}{6} \text{———} \frac{1}{8}.$$

ITem, for the more plainer underſtanding hereof, and all other the like, in broken Numbers: Firſt, you ſhall pull down two, the *Numerator* of the firſt Nnmber or Fraction, with a line under, thus, $3\frac{1}{2}$: that done, according as you have learned before, bring 6, the *Denominator* of the ſecond Fraction, and ſet it under

under two, multiplying the one into the other, which maketh 12. Then lastly, bring 8, the *Denominator* of the third *Fraction*, and set it under 12, multiplying that 12 by 8, which amounteth to *96*, or else for more briefe, multiply *6* by 8, saying, six times 8, makes 48, which 48 set under 2, and multiply the one into the other, it maketh *96*, as before. And this *96* is the first *number* in the *Rule of three*. That shall alwayes for a most generall Rule be your *Divisor*.

Secondly, to worke for your *Dividend*, you shall, (as it hath been sufficiently declared before) pull down 5, the *Numerator* of your second *Fraction*, and set it under *6*, with a line under, thus 6/5

That done (as you know) you are to pull downe 3, the *Numerator* of the third *Fraction*, and set it under 8, with a line under it, thus, 8/3 multiplying the one into the other, according to the tenour of the *Rule of three*; which maketh 15. Then according to my note, forget not to bring the *Denominator* of the first *Fraction*, which is 3, under 15, and multiply them together, which maketh 45, which 45 is your *Dividend*, and are of the nature of *Denomination* of the middle *number*, as I have taught you before: And therefore are 45 li. which aptly cannot be divided by *96*. Therefore you shall reduce the 45 li. into s. as you see performed in the Example, which amounteth to 900 s. which divided by *96* your Divisor,

visor, it yieldeth 9 s. and $\frac{36}{96}$ of a shilling, which in lesser terms is $\frac{3}{8}$: which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$ d: and so much will the aforesaid $\frac{4}{5}$ cost, as by the work following shall appeare.

The Example.

$$\frac{2}{3}\quad\frac{5}{6}\quad\frac{3}{8}$$

2	5	3	
3	6	8	
2	5	3	
6		5	3
12		15	96
8		3	9000 (9 $\frac{36}{96}$ | $\frac{3}{8}$ s.
96		45	96
		20	
		900	

Otherwise though 45 could not be divided by 96, yet by Division in broken numbers it might have beene abreviated to $\frac{15}{32}$ of a pound, which reduced into knowne parts, will make 9 s. $4\frac{1}{2}$ d. as before.

Now my second example shall be the proof of this question.

If $\frac{3}{8}$ yards cost $\frac{15}{32}$ of a pound, or 20 shillings, what shall $\frac{4}{5}$ cost?

Answer. Worke as was taught you before, and you shall have your desire.

Here

$\frac{1}{2}$ ——— $\frac{1}{5}\frac{1}{2}$ ——— 8

3	15	2
32	2	
96	30	
3	8	
288	240	

Here as appeareth by the work, the multiplication being ended, 240 is to be divided by 288, which to some perchance may seem hard, yet notwithstanding is the worke good. Therefore abreviate 240 by 288, as you see here is practised: and the end of your abreviation shal come to $\frac{5}{6}$ your desire, $\frac{240}{288}\Big|\frac{10}{12}\Big|\frac{5}{6}$.

Otherwise, $\frac{240}{288}\Big|\frac{120}{144}\Big|\frac{60}{72}\Big|\frac{30}{36}\Big|\frac{5}{6}$

Otherwise, $\frac{240}{288}\Big|\frac{40}{48}\Big|\frac{5}{6}$

The third Question.

If $\frac{3}{4}$ *Ells cost* 13 s.—4 d. *what* 156 $\frac{1}{2}$ *Ells?*

Answer. To work this question the shortest way: reduce 13 s. 4 d. into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you have set down the question, the *Numerator* of the first *Fraction* 3 is pulled down under 4, and *Denominators* of the other, two *fractions* multiplied into

into him, which maketh 18, your *Divisor*.

Then the *Numerators* of the second *fraction* is pulled downe, under 3 of custome now in sight, ready to *multiply* my third number, by which is performed as soon as the last numbers 156 $\frac{1}{2}$ is reduced into halfs.

Then lastly, I multiply that product by 4, the *Denominator* of the *fraction*: it yieldeth 1504, which I divide by 18, and my quotient is 139 l. and $\frac{1}{2}$ of a pound remayning, which is worth 2 s.--2$\frac{2}{3}$ d. And so much will 156$\frac{1}{9}$ Ells cost, as by the work following doth appeare.

3	2	156 $\frac{1}{2}$	1	
4	3	313	47	
3	2	2	1762	li.
6		626	2504	(139$\frac{1}{8}$
18		4	1888	
		2504	11	

The fourth Question.

If 2$\frac{1}{2}$ *Els cost* 1$\frac{2}{3}$ *pounds, what commeth* 29$\frac{1}{4}$ *Els to?*

Item, to the workmanship of this question, first reduce your second *number* in saying, three times 1 is 3, and 2 is 5. Then bring the *multiplication* of the *Denominators* of the second and third *Fractions* which maketh 12: and *multiply* that 2 by 5 your first *Numerator*, and it maketh 60, which is your *Divisor*.

Then the *Reduction* of the second number, which

which is 5, *multiplyed* by 117 the product of the last *numbers* reduction, make 585, which 585 yet resteth to be *multiplyed* by 2, the *denominator* of the Fraction in the first place, yieldeth 1170, which divided by your *Divisor*, 60 yieldeth 19 pound, 10 s. as appeareth the work thereof.

Thus having now touched the 12 questions whereof I first pretended, which with diligence and oft practice, I trust are sufficient to aid the desirous unto the working of any broken numbers, I will now intreat of divers necessary Rules incident unto traffick, as hereafter followeth.

The fourth Chapter treateth of losse, and gain in the trade of Merchandise.

IF one yard cost 6 s---8 d. and the same is sold againe for 8 s---6 d. the question is, what is gained in 100 pounds laying out on such commodities?

A*nswer*. The *Rule of three* direct, applyed two manner of wayes to doe the same: the one is to say, If $6\frac{2}{3}$ give $8\frac{1}{2}$, what giveth 100? *Multiply* and divide, and looke what your quotient bringeth forth above your laying out, is the neat gaines and solution

to

to your question: If you follow the work, your solution will bring forth 127 li.—10 s. which is 27 li.—10 s, more than your principall, and so much is gained in the 100 pounds laying out.

Item, to work it the other way, which I take the neerest, seeke the difference betwixt the just price and the other price, which is one shilling ten pence; then say by the *rule of three*. If 6⅔ s. gain 1⅚ s. what shall 100 pound gain? *Multiply* and divide, and you shall finde 27 li. 10 s. and so much is gained in 100 li. laying out.

You may use which of these two wayes you think good.

The proofe.

If a yard of cloth be delivered for 8 s. 6 d. whereupon was gained after the rate of 27 li. 10 s. in 100 pounds laying out: The question is, what the yard cost at the first hand?

Answer. Put your gaine 27 li.—10 s. to 100 pounds, all maketh 127 li—10 s. Then say, If 127 li. 10 s. give but a 100 pounds, what giveth 8½ s Worke, and you shall find 6 s. 8 d. the true solution to your question.

Yet another Example or Proofe upon the first Question.

If one yard cost 6 s.—8 d. the question is, what price the same is to be sold again, for to

gain

gain 27 li. 10 s. in 100 pounds laying out?

Answer. *Say by the Rule of three, if* 100 li. *gain* 127 li. 10 s. *what giveth* $6\frac{2}{3}$ s? *Multiply and divide, and you shall finde* 8 s. 6 d. *your true solution.*

If one Ell cost 7 s. 8 d. and be sold again for 8 s. 6 d. *The question is*, What is gained in 20 pounds laying out in such commodities?

Answer. *Seek the difference betwixt the just price, and the other price which is ten pence, and then apply the* Rule of three, *as before is taught, saying, If* $7\frac{2}{3}$ s. *give* $\frac{5}{6}$ *shillings, what giveth* 20 li? M*ultiply and divide, and you shall finde* 2 li. $3\frac{11}{23}$ s. *and so much is gained in* 20 li. *laying out.*

The proof also by an example of losse.

A Merchant *hath bought Holland cloth at* 8 s. 6 d. *the Ell, which proveth not to his expectation, whereupon he is content to lose* 2 li. $3\frac{11}{23}$ *in* 20 *pounds laying out, The question is, what price ought to be made of the Cloth, abating this losse?*

Answer. Do as before in Gaines hath beene taught, putting 2 li. $3\frac{11}{23}$ s. to your 20 pound, all together, maketh 22 li. 3 $\frac{11}{23}$. Then say by *the Rule of three.* If 22 li. 3 $\frac{11}{23}$ s. give but 20 l. what shall come of $8\frac{1}{2}$ s? work, and you shall finde 7 s. 8 d. the just price that the Ell ought to be sold for after the rate of this losse.

Thus

Thus it appeareth evidently, as in company the *Rule* is applyable as well to gain as loſſe.

If 20 $\frac{1}{4}$ *yards coſt* 36 li. $\frac{1}{2}$ 10 s. *how ſhall I ſell the ſame again* $\frac{2}{3}$ *of the principall, or to make of* 3,4: *which is all one.*

Anſwer. By the *Rule of three*, if 3 do give 4, what will 36 $\frac{1}{2}$ give? Multiply and divide, and you ſhall finde 48 $\frac{2}{3}$ li. Then ſay againe, if 20 $\frac{1}{4}$ yards do give 48 $\frac{2}{3}$ pounds, as well principall as gain, what will one yard be worth at that price? Multiply and divide, and you ſhall finde 2 li. 8 $\frac{16}{243}$ s.

If one Ell of Cloth coſt mee 8 s. 8 d. *and afterwards I ſell* 10 $\frac{1}{2}$ *Ells thereof for* 5 li. 13 s. 4 d. *I would know, whether I winne or loſe: and how much upon the* 100 *pounds of money.*

Anſwer. See firſt at 8 s. 8 d. the Ell, what 10 $\frac{1}{2}$ Ells comes to, and you ſhall finde 4 li. 11 s. and I ſold the ſame for 5 li--13 s--4 d. ſo that I did gain upon the 10 $\frac{1}{2}$ Ells 22 ſhillings 4 d. Then if you would know how much is gained in 100 pounds, I ſay by the *Rule of three*, If 4 li--11 s. did gain 22 s--4 d. what will 100 pounds gain? Multiply and divide, and you ſhall finde 24 li--10 s--10 d. $\frac{10}{91}$ and ſo much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ *yards coſt me* 11 *pound five ſhillings, and I ſell the yard againe for* 16 *ſhillings, the queſtion is whether I doe winne or loſe, and how much in or upon the pound of money?*

An-

Answer. Look what the 12 $\frac{1}{2}$ yards come to at 16 s. the yard, and you shall finde ten pound. But they cost 11 pound 5 shillings. So there is lost upon the whole 1 pound 5 s. Then to know how much is lost in the pound, say by the *Rule of three*, if 11 $\frac{1}{4}$ pound do lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and divide, and you shall finde 2 s. 2 d $\frac{2}{3}$, and so much is lost in the pound of money.

If I sell the 100 *weight of any commodity for* 4 *pound, whereupon I doe lose after ten pound in the* 100 *pound, I demand how much I shall lose or gain in the* 100 li? *if in case I had sold the same for foure pound ten shillings.*

Answer. Say, if 90 pound yield 100, how much will 4 give? multiply and divide, and you shall finde 4 $\frac{4}{9}$. Then say again, if 4 $\frac{4}{9}$ give me 4 $\frac{1}{2}$ what will 100 come to? Multiply and divide, and you shall finde 101 pound $\frac{1}{4}$ which is more then 100 pound by 1 pound 5 shillings: and so much is gained in the 100 pound.

A Merchant hath sold Currants for the sum of 430 *pound, and hee hath gained therein after ten pound in the* 100 *pound. The question is to know how much he gained in all.*

Answer. Say by the *Rule of three*. If 100 pound doe gaine ten pound, what will 430 pound gaine? Multiply and divide, and you shall finde 43, and so much hath hee gained in all.

If

If one yard be worth 28 $\frac{1}{3}$ s. *for how much shall* 10 *yards be sold to gain after* 8 li. 6 s. 8 d. *in the* 100 *pound?*

Answer. First, adde 8 li.--6 s.--8 d. to 100. Then say, if 100 li. do give 108 $\frac{1}{3}$ for principall and gain, what will 28 $\frac{1}{3}$ s. principall yield? Multiply and divide, and you shall find 30 $\frac{2}{3}$ s. Then say, again, by the *Rule of Three*, if 1 yard do give 30 $\frac{2}{3}$ s. (which is aswell the principall as the gain) what shall ten yards give? Multiply and divide, and you shall finde 15 l. 8 s. 9 d. And for the same price shall the ten yards be sold, for to gain after the rate of 8 li.--6 s.--8 d. upon the 100.

A branch or proofe out of this Question.

A Merchant hath sold clothes for 15 li.--8 s--9 d. *and he hath gained in the whole, the summe of* 1 li.--3 s--9 d. *The question is, to know how much he hath gained in the* 100 *pound?*

Answer. To know this, first rebate the gains from the price, and there will remaine 14 li. 5 s. 0 d. Then say by the *Rule of three* direct, if 14 li. $\frac{1}{4}$ give mee 1 li. 3 $\frac{3}{4}$. what will 100 li. give? Multiply and divide, and you shall finde 8 li. 6 s. 8 d. the effect desired, the proofe is apparant in the question before.

Yet

Yet another branch or proofe of the first Question.

If ten yards be delivered for 15 li. 8 s. 9 d. *whereupon was gained after the rate of* 8 li. 6 s. 8 d. *upon the* 100 *pound, the question is, what the yard did cost at the first hand?*

Answer. First, say by the *Rule of Three,* if ten with principall and gain yield 15 li. 8 $\frac{1}{2}$ shillings, what shall 1 yield? Multiply and divide, and you shall find 30 $\frac{7}{8}$ s. Then say again by the *Rule of three,* if 108 $\frac{1}{3}$ principall and gain give but 100, what shall 30 $\frac{7}{8}$ s. of principall and gain yield? Worke, and you shall finde 21 $\frac{1}{2}$ s. And so much did the yard cost at the first peny.

If one yard cost 36 s. *how much shall* 12 *yards be sold for to gain after the rate of ten* li. *in the* 100?

Answer. First, say, If 100 give 110 li. principall and gain, what will 36 s. give? Multiply and divide, and you shall finde 39$\frac{3}{5}$ s. Then say again by the *Rule of three,* If one yard of principall and gain yield 39$\frac{3}{5}$ shillings, what shall 12 yards gain? Multiply and divide, and you shall finde 23 li--15 $\frac{1}{5}$ s. which $\frac{1}{5}$ s. in known number, is 2$\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gain after the rate of 10 in the 100.

The Proofe.

If 12 *yards be sold for* 23 li--15 2 $\frac{2}{5}$ d. *whereupon*

upon is gained after 10 li. *in the* 100. *The question is, what the yard cost at the first peny?*

Answer. First say, If 12 give 23 li. 15 $\frac{2}{3}$ s. what one yard? Multiply and divide, and you shall finde 39 $\frac{2}{3}$ s. Then say againe by the *Rule of three*, if 110 pounds give but 100, what shal 39 $\frac{2}{3}$ s. give? Work, and you shall finde 36 s: the just price of the yard at the first hand.

Item, *When one Merchant selleth wares to another, and he giveth to the buyer* 1 li. 6 s. 8 d. *upon the score, or* 20 li. *The question is, How much shall the buyer gain upon the* 100 li. *after that rate.*

Answer. First adde 1 li---6 s---8 d. unto 20 li. and they are 21 $\frac{1}{3}$. Then say, if 20 pound give 21 $\frac{1}{3}$, what shall 100 give? Multiply and divide, and you shall finde 106 $\frac{2}{3}$. So the buyer getteth after the rate of 6 $\frac{2}{3}$ li. upon the 100 li.

Gentle Reader, other necessary questions appertaining to Losse and Gain, you shall have in the eight Chapter of this Treatise.

The

The fifth Chapter entreateth of Loſſe and Gain upon time, wrought by *the double Rule of Three, or by the Rule compoſed:* which is contained in foure ſpeciall ſelected branches or queſtions of divers formes, *each one of them ſpringing from the firſt queſtion,* and each one of them alſo being a proofe, to other, &c.

IF one yard coſt me 2--8 d. *readie money, and after I ſell the ſame again for* 2 s--10 d. *to be paid for it at the end of three moneth: the queſtion is, what I gain upon the* 100 li. *in* 12 *moneths?*

Anſwer. Firſt ſay, if $2\frac{2}{3}$ gaine $\frac{1}{6}$, what ſhall 100 li. gain? Multiply and divide, and you ſhal find $6\frac{1}{4}$ li. Then ſay again, by the *Rule of three,* if three moneths gain $6\frac{1}{4}$ pound, what ſhall 12 moneth gain? Work, and you ſhall finde 25 li. and ſo much ſhall I gaine in 12 moneths after that rate.

Item, you may alſo work it all at one working by the firſt part of the *Rule of three* compoſed, ſaying, if $2\frac{2}{3}$ d. in three moneths do gain $\frac{1}{6}$ of a ſhilling (which is 2 d.) what will 100 li. gain in 12 moneth: which for thy further encouragement, the work of this one example I

have here put down, to verifie that I affirme in the first part of this *Ground of Arts*, that this Rule, and so all others, more rejoceth in *Broken* then in *Whole*.

s	moneths		s	li.	mo.
$2\frac{1}{3}$ ———	3 ———		$\frac{1}{8}$ ———	100 ———	12
8			1	20	
8	2			2000	
24	72000		1	3	
6	14444	(5000	(25	6300	
144	144	220		12	
	1			72000	

Where the multiplication and the division being ended, maketh 25 li. your desire.

If a yard be delivered for 2 s--10 d. *to be paid at* 3 *moneths, whereupon was gained after the rate of* 25 li. *in the* 100 *for* 12 *moneths, the question is now what the yard cost at the first hand?*

Answer. First say, if 12 moneths gain 25 li. what shall three moneths gain? Work and you shall finde $6\frac{1}{4}$ li: Then say againe the second time, if $106\frac{1}{4}$ li. give but 100, what shall $2\frac{1}{6}$ s. give? Work, and you shall finde 2 s. 8 d. which is the just price that the yard cost at the first hand.

If one yard of cloth cost mee 2 s.---8 d. *readie money, for what terme shall I sell the same again for* 2 s. 10 d. *so that I might gain after the rate of*

of 25 *pound upon the* 100 *pound in* 12 *moneths?*

Answer. First say, if $2\frac{2}{3}$ gaine $\frac{1}{6}$ what shall 100 pound gain? Multiply and divide, and you shall find $6\frac{1}{4}$ pound. Then say again for the second worke, if 25 pound be come of 12 moneths, what shall come of $6\frac{1}{4}$? Worke, and you shall finde three moneths, the just terme of time that the Cloth ought to be delivered at 22 s. 10 d. to gain 25 pound upon the 100 li. in 12 moneths.

If one yard cost me 2 s. 8 d. *ready money, for what price shall I sell the same again to be paid at the end of three moneths, so that I may gain after the rate of* 25 *pound in the* 100 *pound for* 12 *moneths.*

Answer. First say, if 12 gain 25 li. what shall 3 moneths gain: Multiply and divide, and you shall finde $6\frac{1}{4}$ li. Then say for the second worke, if 100 li. give $106\frac{1}{4}$, what giveth $3\frac{2}{3}$ s.? Work, and you shall finde 2 s——10 d. and for that price must the yard be sold to gain after 25 pound in the 100 pound for twelve moneths.

Many other of these questions I might here have delivered, but for feare the Book would rise to too thick a volume, and so to make the price so much the dearer, whereby it might not be so portable to my Countrymen as I wish it. But these 4 I have of purpose framed in this order, having relation one to another, assuring you that what question soever may be proposed within the compasse of this Rule

you shall finde by one of these 4, to make a solution. And moreover, divers others are yet to be delivered, where the Creditor giveth divers dayes of payment, which can never be well wrought, nor yet understood, unlesse you can first find by art the just times that all those payments how different soever they be, ought to be paid at once; whereupon first I thinke good here to give some instructions into such a Rule, for it is the onely aid for the finishing of such question as hereafter shall follow.

The sixth Chapter intreateth of Rules of payment, which is a right necessary Rule, and on of the chiefest handmaids that attendeth upon buying and selling, &c.

Example.

A Merchant doth owe a summe of money, whereof, the $\frac{1}{3}$ is to be paid at 6 moneths, and the $\frac{1}{2}$ at 8 moneths, and the rest at a yeare. If he would pay all at one payment, the question is, what time ought to be given him.

Answer. I have omitted the quantity of the summe, for you shall understand, the Rule is ap-

applyable, and yieldeth a true solution to what summe soever shall be proposed : But now for order sake in teaching, I doe imagine the sum to be 60 pounds, whereupon the manner of this work is to multiply the proportionate part of the money by the time, as in company. Then 20 being the first payment, and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplyed in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the example in the operation, maketh two moneths: next 30 which is the $\frac{1}{2}$ multiplied by his terme 8, yields 4 moneths, then the rest which is 10 li,

$\frac{1}{3}$ by $\frac{6}{1}$	2 Moneths.
$\frac{6}{3}$	
$\frac{1}{2}$ by $\frac{8}{1}$	4 Moneths.
$\frac{8}{2}$	
$\frac{1}{6}$ by $\frac{12}{1}$	2 Moneths.
$\frac{12}{6}$	

must needs be abreviated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 moneth, produceth $\frac{12}{6}$ maketh two moneths, All which added together, as appeareth in the operation, maketh eight moneths, which is the just time that all those payments ought to be paid at once.

A Merchant hath 800 li. *to pay, the* $\frac{1}{8}$ *thereof ready money, the* $\frac{1}{4}$ *at two moneths, the* $\frac{1}{2}$ *at* 4 *moneths, and the rest at a yeare. The question is, if hee would pay all at one payment, what time ought to be given him ?*

Answer.

Answer. The ready mony is never multiplyed, then $\frac{1}{4}$ multiplyed by two moneths as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 moneths, as appeareth here in the operation: But now for the rest of the mony you cannot multiply it untill you have sought what proportion it beareth to 800 pounds. Therefore you must subtract the ready money, the $\frac{1}{4}$ and $\frac{1}{2}$ out of the principall. The rest will be $66\frac{1}{3}$ li. which you must looke what part it beareth to the principall, which you shall finde to be $\frac{1}{12}$, the same you must also multiply by his time 12 moneths, and it yieldeth one moneth, so all make $3\frac{1}{2}$ moneths, as appeareth in the operation.

$\frac{1}{4}$	$\frac{1}{2}$	
$\frac{1}{4}$		$0\frac{1}{2}$
$\frac{1}{2}$	$\frac{4}{1}$	
$\frac{4}{2}$		2
$\frac{1}{12}$	$\frac{12}{1}$	
$\frac{1}{1}$	$\frac{2}{2}$	1

A Merchant is to pay 1200 li. *in three terms, that is to wit*, 400 li. *at two weeks: and* 600 li. *at foure moneths: lastly*, 200 li. *at five moneths: The question is, in what time they ought to be paid at once.*

Answer. Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600 you shall finde $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you shall have $\frac{2}{3}$ weeks, more eight weeks, and lastly $3\frac{1}{3}$ weeks, which together maketh 12 weeks, or three moneths your desire.

A Merchant is to pay 600 *pound in three termes, whereof* 100 *pound is payed present, more*

300 *pound at twenty days, and the rest at five moneths, accounting thirty dayes to a moneth. The question is, what time ought these payments to be payed at once?*

Answer. Worke, and you shall finde two moneths.

The seventh Chapter intreateth of buying and selling in the Trade of Merchandize, wherein is taken part readie money, and divers dayes of payment given for the rest, and what is wonne or lost in the 100 pound forbearance for 12 moneths more or lesse, according to the quantitie of money, or proportion of time, &c.

A Merchant hath bought satins which cost eight shillings the yard ready mony. And he selleth the same again to another man for ten s. *the yard, but he giveth two days for the payment, that is to say, three moneths for the one halfe, and five moneths for the other halfe. The question is to know how much the seller doth gain upon the* 100 li. *in 12 moneths after that rate.*

Answer. Seeke first by the Rules of payment, at what time those two payments ought to be payed at once, and you shall finde foure

moneths,

moneths, at which time the ſecond Merchant ought to have paid the whole entire payment: And therefore ſay by the firſt part of the *Rule of three* composed: If 8 ſhilling in 4 moneths doe gain 2 s. what will 100 li. gain in 12 moneths?

s — m — s — li — m.
8 — 4 — 1 — 10 — 12

4 | 20
32 | 2000
 | 2
 | 4000
 | 12
 | 4800

1
16 — 15
48000 (1500 (75
322 — 620
3

Multiply and divide, and you ſhall finde 75 pounds, as appeareth in the example, and ſo much doth the firſt Merchant gaine upon 100 pounds in 12 moneths.

A Merchant hath ſold 50 *Clothes, at* $9\frac{1}{2}$ li. *the piece, to be paid the one* $\frac{1}{2}$ *at foure moneths, the* $\frac{1}{3}$ *at five moneths, and the* $\frac{1}{6}$ *at ſeven moneths, and the ſellers minde is to take no more but after eight pounds in the* 100 *for* 12 *moneths. The queſtion is now, what the firſt Merchant gaineth in the ſale of theſe Clothes after that rate.*

Anſwer. Firſt, looke what the 50 Clothes come to at that price, and you ſhall finde 475 pounds. Then ſecondly, according to your direction in the Rules of payment, ſeek at what time all the payments are to be performed at once. And you ſhall finde $4\frac{1}{6}$ moneths. Then thirdly ſay, by the firſt part of the *Rule of three* com-

compoſed. If 100 li. in 12 moneths gain 8 li. what will 475 li. gain in 4 $\frac{5}{6}$ moneths? Work, and you ſhall finde 15 li. and $\frac{11}{36}$ of a pound, which is the neat gains that the firſt Merchant hath after the rate aforeſaid.

A Merchant hath bought Holland at 7 s. 3 d. *the Ell ready money, and hee ſelleth the ſame again for* 8 s. 4 d. *the Ell to be payed* $\frac{1}{4}$ *part in ready money, more* $\frac{1}{3}$ *part at 2 moneths, and the reſt at 4 moneths. The queſtion is now to know how much the firſt Merchant doth gain upon the* 100 *pounds in* 12 *moneths after that rate?*

Anſwer. According to the direction delivered you in the Rule of payment, the ready money is not to be multiplied. Then working for the other two payments to finde out the true proportion at what time they ought to be paid at once, you ſhall finde for $\frac{1}{3}$ at two moneths, $\frac{2}{3}$ of a moneth. And the reſt of the money which is $\frac{5}{12}$ multiplied by his term 4 moneths; yieldeth 1 $\frac{2}{3}$ moneths, both which added together make 2 $\frac{1}{3}$ moneths, the juſt time that both the payments ought to be performed at once. And therefore ſay by the firſt part of the *Rule of three* compoſed, if 74 $\frac{1}{4}$ in 2 $\frac{1}{3}$ moneths do gain $\frac{11}{146}$ uf a pound, what ſhall 100 pounds gain in 12 moneths after that rate? work and you ſhal finde 78 $\frac{17}{20}\frac{2}{3}$ pounds. And ſo much doth hee gain upon 100 pounds in 12 moneths.

A Merchant hath bought 30 *Clothes at* 6 *pounds the piece for ready money. Afterward hee ſelleth ten of them for* 7 *pound the piece, for three moneths*

monethꝭ terme. And the other twenty hee ſelleth for 8 *pound the piece for* 4 *moneths terme. The queſtion is now what he gaineth upon* 100 *pounds in* 12 *moneths?*

Anſwer. Firſt, finde the value of the thirtie clothes, which amount to 180 pounds. Secondly, ſeeke what the ten pieces come to at 7 pounds, and what the twenty pieces come to at 80 pounds; the one comes to 70, and the other to 160, both which together make 230, which is 50 pounds more then they coſt. Thirdly, as I have taught you in the Rule of payment, proportionate the firſt and ſecond prices unto the proportion they beare unto 230, the product of their two prices, and you ſhall finde $\frac{7}{23}$ for the firſt, and $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe parts by their times, and you ſhall have $\frac{21}{23}$ and $\frac{64}{23}$ both which together maketh three whole months, and $\frac{16}{23}$ of a moneth, which is the juſt time that both thoſe paymeuts ought to be paid at once.

Then ſay by the firſt part of the Rule of three compoſed. If 180 pounds in $3\frac{16}{23}$ moneths doe gain 50 pounds, what ſhall 100 gain in twelve moneths? Multiply and divide, and you ſhall finde $90\frac{10}{51}$ pound, and ſo much doth hee gaine upon 100 pounds in twelve moneths.

A *Merchant hath bought Cinamon which coſt him* 9 *ſhillings the pound ready money. The queſtion is now at what price he ought to ſell thee* 100 *weight. To wit,* 112 *pounds to be paid the* $\frac{1}{4}$ *at two moneths, and the reſidue at the end of*

three

three moneths, so that he may gain after the rate of ten pounds upon 100 *pounds for twelve moneths.*

Answer. Seek first by the Rules of payment at what terme both the payments ought to be paid at once, where the $\frac{2}{4}$ multiplyed by his terme two moneths, maketh $\frac{4}{4}$ moneths.

Likewise the next payment, which is $\frac{2}{4}$ multiplyed by his terme three moneths, maketh $2\frac{2}{4}$ moneths, both which added together, maketh $2\frac{2}{4}$ moneths, which is the time, that both the payments ought to be paid at once. Then say by the Rule of three, if 12 moneths doe give me ten pounds, what will $2\frac{2}{4}$ moneths give? Multiply and divide, and you shall finde $2\frac{2}{24}$ pounds. Then say again by the Rule of three, If one pound cost me 9 s. what will 112 pounds cost? Multiply and divide, and you shall finde 50 li–8 s. Then say once againe: If 100 pound doe give $102\frac{2}{24}$, what will $50\frac{2}{5}$ pounds give? Multiply and divide, and you shall finde 51 li. 11 s—$1\frac{1}{3}$ d. and for that price ought I to sell 112 pound of Cinamon to be paid at the two severall payments aforesaid, to gaine thereby after the rate of ten pounds upon the hundred pound in twelve moneths.

Brief Rules for our hundred weight here at *London*, which is after 112 pound for the 160.

ITem, *who that multiplyeth the pence that one pound weight is worth by 7, and divideth the product by 15, shall finde how many pounds in money 112 pound weight is worth.*

And contrariwise, hee that multiplieth the pounds that 112 pounds weight is worth by 15, and divideth the product by 7, shall finde how many pence in money the one pounds weight is worth.

Example.

At 10 pence the pound weight, what is 112 pounds weight worth?

Answer. Multiply 10 by 7, and thereof commeth 70, the which divided by 15, and you shall finde $4\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li. 13 s. 4 d, after the rate of 10 pence the pound aforesaid.

At 6 pounds the 112 pounds weight, what is one pound worth?

Answer. Multiply 6 by 15, and thereof commeth 90, the which divide by 7, and you shall finde $12\frac{6}{7}$ d. So much is one pound worth, when the 112 pounds did cost 6 pounds.

The

The eight Chapter intreateth of Tares and allowances of Merchandize sold by weight, and of losses and gains therein, &c.

AT 16 pound the 100 Suttle, what shall 895 pound Suttle be worth in giving 4 pound weight upon every 100 for Treat?

Answer. *Adde 4 unto 100, and you shall have 104. Then say by the Rule of 3: If 104 be worth 16 pounds, what are 895 pounds worth? Multiply and divide, and you shall finde* 237 li. 13 s. 10 $\frac{2}{13}$ d. *and so much shall the 859 pounds weight be worth.*

Item, at 3 s. 4 d. the pound weight shall 754 $\frac{1}{2}$ pound be worth, in giving 4 pounds weight upon every hundred for Treat?

Answer. See first by the *Rule of three* what the 100 pound is worth, saying: If one cost 8 $\frac{1}{3}$ s. what 100? Multiply and divide, and you shall finde 16 $\frac{2}{3}$ pounds. Then adde 4 unto one 100 and they are 104. Then say againe by the *Rule of three*, if 104 be sold for 16 $\frac{2}{3}$ pounds, for how much shal 754 $\frac{1}{2}$ be sold for? Multiply and divide, and you shal finde 120 li. 18 s. 3 $\frac{1}{13}$ d. And for so much shall the 754 $\frac{1}{2}$ pound be

sold

ſold for at 3 s. 4 d. the pound, in giving 4 upon the 100.

Other neceſſary briefe Rules there are for the finding of Treats, or caſting up of Cheſts of Sugar, &c. which for that it is a myſtery, I omit: if any lack instruction that way, they ſhall find me ready to pleaſure them.

Item, if 100 pounds be worth 36 s. 8 d. what ſhall 860 pounds be worth in rebating foure pounds upon every hundred for tare and cloff?

Anſwer. Multiply 860 by 4, and thereof commeth 3440, the which divide by 100, and you ſhall have 34 $\frac{2}{5}$ pounds, abate 84$\frac{2}{5}$ from 860, and there will remayne 825 $\frac{3}{5}$ pounds. Then, ſay by the Rule of three. If 100 pound coſt 36$\frac{2}{3}$ s. what will 825 $\frac{3}{5}$ coſt after that rate? Multiply and divide, and you ſhall find 15 li. 2 s. 8$\frac{16}{25}$ d. And ſo much ſhall the 860 coſt in rebating foure pounds upon every hundred, for tare and cloffe.

Item, *whether doth hee loſe more that giveth* 4 *pounds upon the* 100, *or he that rebateth* 4 *pounds upon the* 100?

Anſwer. Firſt note, that hee that giveth 4 pounds on 100, giveth 104 for 100. And hee which rebateth 4 pounds upon the 100, giveth the 100 for 96. Therefore ſay by the *Rule of three,*

three, If 104 be delivered for 100, for how much shall the 100 be delivered? Multiply and divide, and you shall finde 96 $\frac{2}{13}$. and hee which rebateth 4 in the 100, maketh but 96 pounds of 100, so that hee loseth 4 pounds in the 100, and the other which giveth 4 pounds upon the 100 loseth but 3 $\frac{11}{13}$ pounds upon the 100. Thus you may see, that he which rebateth 4 pounds in the 100, loseth more by $\frac{2}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100, for tare and closse.

If 100 pounds of any thing cost me 23 s. 4 d. the question is, how I shall sell the pound, to gain after the rate of ten pounds, upon the 100 pound.

Answer. Say by the rule of three, if 100 pounds give 110 pounds, what shal 23 $\frac{1}{3}$ s. give? Multiply and divide, and you shall finde 1 $\frac{17}{60}$ pounds. Then say again, If 100 pound be worth 1 $\frac{17}{60}$ pounds, what is one pound worth? Multiply and divide, and you shall finde 3 d. $\frac{4}{25}$. And so much is the pound worth in gaining ten pounds upon the 100.

Item, *A Grocer hath bought C. weight of commodity for* 6 li. 10 s. *The question is now to know how many pounds thereof he shall sell for* 33 s. 4 d. *to gain* 20 *shillings in the C. weight.*

Answer. Adde 20 s. unto 6 li. 10 s. and they

make 7 li. ten s. Then say, if $7\frac{1}{2}$ pound yield me 112 pound, what shall $1\frac{2}{3}$ pounds yield? Multiply and divide, and you shall find $24\frac{8}{9}$ li. And so many pound ought he to sell to gain 20 s. in his C. weight.

If one pound weight cost 3 s. 4 d. *and I sell the same again for* 4 s. *what is gained in a hundred pound of money laid out in that commodity?*

Answer. You may say, If $3\frac{1}{3}$ s. give 4. what will 100 pound gain? But then when you have found, you must subtract 100 pounds out of the Product, the rest is your neat gaine, or else to produce the neat gain in your work at the first, Subtract the just price out of the overprice, as I taught before in the first beginning of Losse and Gain, and your conclusion shall be all one. Multiply and divide, by which of the two wayes you thinke good, and you shall finde that he gaineth 20 pounds in the 100 pound.

Item, *If the pound weight which cost* 4 s. *be sold again for* 3 s.--4 d. *I demand what is lost in the* 100 *pounds of money.*

Answer. Say, If 4 s. lose $\frac{2}{3}$ s. what shall 100 lose? Multiply and divide, and you shall finde 16 li. 13 s. 4. and so much is lost upon the 100 of money.

Item, *If C. weight of any commoditie cost* 45 *pounds, and the buyer repenting, would lose five pounds*

pounds in the 100 *of money, I demand how the pounds may be sold, his losse to be neither more nor lesse than after the rate aforesaid of five by the hundred?*

Answer. By the Rule of three, if 100 lose 5, what shall 45 lose? Work, and you shall finde $2\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 li. 15 s. Lastly say, if 112 yieldeth but 42 li. 15 s. what one pound? Multiply and divide, and you shall find 7 s. 7 d. $\frac{17}{28}$. And so much is the pound worth after that losse.

A Grocer hath bought three pieces of Raisins, weighing $175\frac{1}{2}$ *pounds,* $182\frac{1}{4}$ *pounds:* 191 *pounds: tare for each fraile* $2\frac{2}{4}$ *pounds, as* $25\frac{1}{2}$ s. *the C weight. The question is, what they amount to in money?*

I answer 6 li.——3 s.——$4\frac{21}{28}$ d.

A *Grocer hath bought three sacks of Almonds weighing* $267\frac{1}{2}$ *pound, tare two pound,* $257\frac{1}{2}$ *pounds, tare* $2\frac{1}{2}$ *pound,* 252 *pound, tare* 3 *pound, at* 2 s. $10\frac{1}{2}$ d. *the pound, what amount they to in money?*

I answer, 110 li.——12 s.——$3\frac{1}{4}$ d.

The ninth Chapter intreateth of lengths and breadths of Arras and other Clothes, with other questions incident unto length and breadth.

I*F a piece of Arras be 7 Ells and $\frac{1}{4}$ long, and 5 Ells and $\frac{2}{3}$ broad, how many Ells square doth the same piece contain?*

Answer. Multiply the length by breadth, that is to say, $7\frac{1}{4}$ by $5\frac{2}{3}$. And thereof will come $43\frac{11}{11}$ Els: so many Els square doth the same piece containe.

Item *more, a piece of Arras doth containe 22 Els square, and if the same were in length $3\frac{1}{4}$ Els, I demand how many Els in breadth the same piece doth contain.*

Answer. Divide 22 Els by $3\frac{1}{4}$ and thereof commeth $6\frac{10}{13}$. So many Els doth the same contain in breadth.

Item *more, a Merchant hath $3\frac{1}{4}$ Els of Arras, at $1\frac{2}{3}$ Els broad, which he will change with another man for a piece of Arras, that is $\frac{7}{8}$ Els square. The question is, how many Els of that squarenesse ought the first Merchant to have?*

Ans-

Answer. Multiply the first Merchants piece, his length by the breadth, and you shall finde, it containeth $5\frac{5}{12}$ Els, which $\frac{5}{12}$ Els you shall divide by $\frac{7}{8}$ and you shall finde $6\frac{4}{21}$ Els, and so many Els of that squarenesse ought the latter Merchant to give the first.

Item, *a Student hath bought* $3\frac{1}{2}$ *yards of broad cloth, at 7 quarters broad, to make a gown, and should line the same throughout with Lambe at a foot square each skin: the question is now how many skins he ought to have?*

Answer. Seeke first the number of yards square that his cloth containeth, which to do, multiply $3\frac{1}{2}$ his length, by $1\frac{3}{4}$ his breadth, and you shall finde $6\frac{1}{8}$ yards square: then say by the *Rule of three*, if one yard square give 9 foot, what shall $6\frac{1}{8}$. Work, and you shall finde $55\frac{1}{8}$ skins.

Item *more, a Lawyer hath a rich piece of seeling come home which is 24 foot & 3 inches long, and 7 foot and* $2\frac{1}{2}$ *inches high: the Ioyner is to be paid by the yard square: the question is, how many yards this containeth?*

Answer. Multiply his length by his breadth, that is to wit, $24\frac{1}{4}$ foot by $7\frac{5}{24}$ foot, and you shall finde $174\frac{77}{96}$ foot square, which 174 you shal divide by 9 (for so many foot make a yard square) and you shall finde 19 yards 3 foot

and $\frac{22}{69}$ of a foot, and so many yards doth this piece hold.

Item, *bought a piece of Holland cloth containing 36 Els $\frac{1}{3}$ Flemmish. The question is how many Els English it makes?*

Answer. You must note, that five Els *Flemmish* doth make but three Els *English.*

Therefore say by the Rule of three, if five Ells *Flemmish* make but three Ells *English*, how many Ells *English* will 36 $\frac{1}{3}$ Ells *Flemmish* make? Multiply and divide, and you shall finde 21 $\frac{4}{5}$ and so many *English* doth 36 $\frac{1}{3}$ Els *Flemmish* containe. The like is to be done of others.

Item, *more, I have bought 342 Ells Flemmish, of Arras worke, at two Els broad Flemmish, and I would line the same with Ell broad Canvas of English measure. The question is, how many Els English will serve my turn?*

Answer. For as much as three Ells English are worth five Ells *Flemmish*, therefore put three Ells *English* into his square, in multiplying three by himselfe, which maketh nine. Likewise multiply the *English* Ell, which is five quarters, every way into himselfe squarely, and you shall finde 25. Then multiply 342 which is the length of the piece, by 2, which is the breadth, and thereof commeth 684, then say

ſay by the Rule of three, as before: if 25 Ells ſquare of *Flemmiſh* meaſure, be worth nine Els ſquare of *Engliſh* meaſure, what are 684 of *Flemmiſh* meaſure? Multiply and divide, and you ſhall finde $246\frac{6}{25}$ Els *Engliſh*.

The ſame is alſo wrought by the Backer Rule of three, in ſeeking the ſquares contained in the *Flemmiſh* Ell of two Els broad (which are 18) and alſo in ſeeking the ſquares contained in the *Engliſh* Ell (which are 25) then ſay by the Rule of three backward. If 18 quarters require 342 Els, what ſhall 25 quarters give? Multiply and divide by the Rule of three Reverſe, and you ſhall finde as before $246\frac{6}{25}$ Ells *Engliſh*?

Item, *more, at three ſhillings foure pence the Flemmiſh Ell, what is the Engliſh Ell worth after the rate?*

Anſwer. Say if three quarters give $3\frac{1}{3}$ s. what giveth five quarters? Multiply and divide, and you ſhall finde 5 s. $6\frac{2}{3}$ d.

Item, *more, at* 8 s. 4 d. *the Flemmiſh Ell ſquare, what is the Engliſh Ell worth after that rate?*

Anſwer. According to the reaſon of the laſt Queſtion, conſider that a *Flemmiſh* Ell ſquare is equal to nine quarters of a yard *Engliſh*, and an

an *English* Ell square is equall to 25 quarters of a yard. Therefore say by the Rule of three, if 9 quarters give $8\frac{1}{3}$ s. what 25 quarters? Work and finde 23 s. $1\frac{1}{9}$ pence. And so is the *English* Ell worth.

Item, *more, at* 6 s. 8 d. *the Ell square: what shall a piece of Cloth cost that is* $7\frac{1}{2}$ *Els long, and* $3\frac{1}{4}$ *Els broad?*

Answer. Multiply the breadth by the length, and you shall finde $24\frac{3}{8}$ Els square cost $6\frac{2}{3}$ s. what $24\frac{3}{8}$ Els? Multiply and divide, and you shall finde 8 pounds, 2 s. 6 pence, and so much the same piece of cloth cost.

Item *more, a Mercer sold* 3 *pieces of Silke. To wit* $24\frac{1}{4}$ $13\frac{2}{3}$ *and* 25 *yards, at* $9\frac{1}{4}$ s. *the yard, and was glad to receive in part of payment againe, a cloth containing* $34\frac{1}{2}$ *yards at* $7\frac{2}{3}$ *shillings the yard. The question is now, what the Debtor is in the Creditors Debt? Work, and you shall find he oweth the Mercer* 22 *pounds,* 3 *shillings,* $2\frac{1}{4}$ *pence.*

The

The tenth Chapter intreateth of reducing of Pawnes of Geanes into English yards.

Note, that 100 *Pawns doe make* 26 *yards, whereupon three Pawns* $\frac{11}{13}$, *do make one yard and one Pawn after the rate and proportion is* $\frac{11}{20}$ *of a a yard.*

In 4563 *Pawnes of Geanes, how many yards English?*

Answer. Say by the Rule of three, if a hundred Pawnes do make 26 yards, what will 4563 Pawnes make? Multiply and divide, and you shall finde 1186 yards $\frac{19}{50}$. So many yards doe 4563 Pawnes make.

Otherwise, take some other number at your pleasure, as ten Pawnes, which is the $\frac{1}{16}$ part of 100, then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{3}{5}$ and then say also by the Rule of three, if ten Pawns give $2\frac{3}{5}$ yards, what will 4563 Pawns give? Worke, and you shall finde 1186 $\frac{19}{50}$ yards, as before.

More, at 2 s. 6 d. *the Pawns of Geans, what will*

will the English yard be worth after the rate?

Answer. Say by the rule of three, if $\frac{11}{30}$ of a yard cost $2\frac{1}{2}$ s. what one yard? Multiply and divide, and you shall finde 9 s. $7\frac{1}{11}$ d.

More if $346\frac{1}{2}$ Pawnes cost 30 li. 13 s. 4 d. *sterling, what is that the English yard after the rate?*

Answer. Say by the Rule of three, if $346\frac{1}{2}$ Pawns cost $30\frac{2}{3}$ pounds, what are $3\frac{11}{13}$ Pawns worth (for so many Pawns make a yard?) Multiply and divide, and you shall find $\frac{2220}{8787}$ parts of a pound, which in known numbers is worth 6 s. 9 d. $\frac{6271}{9009}$.

The eleventh Chapter intreateth of rules of Loane and Interest, with certaine necessary questions and proofes incident thereunto, &c.

ITem, *lent my friend* 225 *pounds for* $5\frac{1}{2}$ *moneths simply without any Interest, upon condition, to have the like courtesie againe when I need. But when I came to borrow, he could spare me but* 149 l. 8 s. 4 d. *The question*

ſtion is now how long time I ought to have the uſe thereof, to countervaile my friendſhip before time ſhewed him?

Anſwer. Say by the backer Rule of three, If 326 pounds give 5 $\frac{1}{2}$ moneths, what time will 149 $\frac{5}{12}$ pounds give? Multiply and divide, and you ſhall finde twelve moneths, and ſo long time ought I to uſe his money.

The Proofe.

Item, *lent my friend* 149 li. 8 s. 4 d. *for twelve moneths. The queſtion is now how much money he ought to lend me again for 5 $\frac{1}{2}$ moneths to recompence my friendſhip ſhewed him?*

Anſwer. Say by the Backer or Reverſe Rule of three, If twelve moneths give 149 $\frac{5}{12}$, what ſhall 5 $\frac{1}{2}$ moneths give? Worke, and you ſhall finde 326 pounds, and ſo much ought he to lend me to requite my gentleneſſe or good turn.

Two other branches, yet more, for proofe out of the ſame queſtion.

Item, *lent my friend* 149 li. 8 s. 4 d. *for* 12 *moneths, to have the like friendſhip again when I need. And comming to borrow of him, hee very courteouſly tooke mee* 326 *pounds (for that hee could well then ſpare the ſame) The queſtion is now, how long I ought to occupie it, not uſurping friendſhip, but in his due time to reſtore it again.*

An-

Answer. Say by the Rule of three reverse, if 149 $\frac{2}{10}$ pounds give 12 moneths, what shall 326 pounds give? Multiply and divide, and you shall finde, that at 5 $\frac{1}{2}$ moneths terme, I ought to restore it again.

Proofe.

Item, *Lent my friend* 326 *pounds for* 5 $\frac{1}{2}$ *moneths. The question is now, how many pounds he ought to lend me for* 12 *moneths to recompence this pleasure again?*

Scholer. Work by the Rule of three, reverse as you have done before, and you shall finde 149 li——8 s——4 d.

Again, foure other selected questions, of Loane and Interest, all out of one branch, and each one also a necessarie question, and a particular proofe to other.

Item, *Lent my friend* 430 *pounds at Interest for three moneths, to receive after the rate of* 8 *pounds in the* 100 *pounds for* 12 *moneths. The question is, what the interest commeth to?*

You may if you please, work it at two workings by the Rule of three direct, in saying, if 12 moneths

moneths give 8 pounds , what giveth three moneth? Multiply and divide; and it giveth 2 pound.

Then for the ſecond work ſay: If a hundred pound yield 2 pounds, what yieldeth 430 li.? Multiply and divide: and you ſhall finde 8 li. 12 s. and ſo much comes the loane of 430 li. to for 3 moneths after the rate of 8 pounds in the hundred pounds of 12 moneths.

Otherwiſe wrought thus by the rule of three at twice alſo.

If 100 pound give 8 pounds, what giveth 430 pounds? Multiply and divide, and you ſhall finde 34 pounds $\frac{2}{5}$. Then again for the ſecond work ſay: If 12 moneths give 34 pounds $\frac{2}{5}$, what giveth three moneths? Work and find 8 li. 12 s. as before.

Otherwiſe yet at one working: By the firſt part of the rule of five numbers forward, in ſaying, if 100 pounds in 12 moneths, gaine 8 pounds, what ſhall 430 pounds gaine in three moneths? Multiply the firſt by the ſecond for your Diviſor, and the other three, the one into the other for the Dividend, and you ſhall finde eight pounds 12 ſhillings, as aforeſaid.

Proofe.

Item, A *friend of mine received of me 8 pounds 12 ſhillings for the Intereſt and Vſe of 430 pounds for three moneths terme: The queſtion is now*

[illegible], what hee tooke in the 100 *pound for* 12 *mo-neths after that rate?*

Anſwer. For moſt briefe, ſay by the firſt part or rule of five numbers forward: If 430 pounds in three moneths did pay 8 li. 12 s. what doth 100 pounds in 12 moneths take after the rate? Work, and you ſhal find 8 pounds, and ſo much hee took upon the 100 pounds for 12 moneths.

A third Queſtion and proofe alſo by the Backer Rule of five Numbers.

ITem, *lent my friend* 430 *pounds to receive for the intereſt thereof, after the rate of* 8 *pounds in the* 100 *for* 12 *moneths. The queſtion is now how long time my friend ought to give the uſe thereof, that it may be returned with* 8 li. 12 s. *gains.*

You may worke it, if you pleaſe, by the Rule of three direct at twice, in ſaying: If 100 li. yield 8 pounds, what yieldeth 430 pound? Multiply and divide, and finde 34 pound and $\frac{1}{2}$.

Then again for the ſecond work ſay, if 34$\frac{2}{5}$ pounds, give twelve moneths, what giveth 8$\frac{3}{5}$ pounds, Multiply and diuide, and you ſhall find three moneths, and ſo long time ought my friend to uſe it to return with 8 li.-12 s. gain.

Otherwiſe at one working by the Backer Rule

Rule of 5 numbers, in ſaying: If 100 pounds in 12 moneths doe gaine 8 pounds how much time ſhal 430 pounds be a gaining of 8 pounds 12 s.? Multiply the firſt and the ſecond into the laſt for your Dividend, and the third and fourth multiply together for your Diviſor, and then divide, and you ſhal finde three moneths, the juſt time that my friend ought to uſe it to return it with 8 li. 12 s. gain.

A fourth derived queſtion out of this Branch, which is a proofe of this laſt, and alſo of the other two going before.

ITem, *how much money ought a Merchant to deliver after* 8 *pounds in the* 100 *for twelve moneths, that in three moneths he may gain* 8 li. *twelve ſhillings?*

Anſwer. You may alſo, if you pleaſe, work it by by the Golden Rule of three at twice: firſt ſaying, If three moneths give $8\frac{3}{5}$ pound, what 12 moneths gaine? You ſhall finde $34\frac{2}{5}$. Then ſay againe, If 8 pounds be come of 100 pounds, what ſhall come of 34 li. 8 s.? Work, and you ſhall finde the anſwer to the queſtion, which is 430 pounds, and ſo much ought the Merchant to deliver.

But moſt briefly it is anſwered by the Backer Rule of 5 numbers, where I argue thus, ſaying:

If

If 100 li. be 12 moneths a gaining of 8 li. then but for three moneths terme onely to take 8 li. 12 s. must needs be a good round summe to worke it, set your numbers thus: 100 —— 12 —— 8 —— 3 —— 8 $\frac{3}{5}$ multiplying the first into the second, and also by 43 the product of the fifth, for your dividend, and the third and fourth together with 5. the Denominator of your fraction for your Divisor: then divide, and you shall finde as before 430 pounds: the true solution to your question.

The twelfth Chapter intreateth of the making of Factors, which is taken in two sorts.

The first is, when the estimation of the Factor is taken upon the sending of the Merchant, as if the estimation of his person be $\frac{1}{4}$ it it is understood that hee shall have $\frac{1}{4}$ of the gain, the Merchant the other $\frac{3}{4}$.

The other sort is, when the estimation of his making is out of the sending of the Merchant, as if the order and agreement between them were such, that the Merchant shall put in 800 li, and the Factor for his making shal have $\frac{1}{4}$: neverthelesse he shall have but $\frac{1}{5}$ of the gain or

or profit, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 pounds in all maake 1000 pounds, whereof the 200 pound, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factour, under such condition, that the said Factor shall have $\frac{1}{4}$. And after certain time they finde in profit 124 li. 6 s. 8 d. I demand how much the Merchant shall have hereof, and how much ought the Factour to have?

Answer. *When the estimation of the Factor is out of the sending of the Merchant, it maketh,*

li.	s.	d.		
99	*9*	4	*for the*	*Merchant.*
24	17	4		*Factor.*

But if that his estimation be at the sending of the Merchant, then it maketh but,

li.	s.	d.		
93	*5*	o	*for the*	*Merchant.*
31	1	8		*Factor.*

For the Merchant is then to have $\frac{1}{4}$, and the Factor $\frac{1}{3}$.

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 pounds

to have the $\frac{1}{5}$ part of the profit : I demand now for how much his person is esteemed, when the same is counted upon the sending of the Merchant.

Answer. *According to the tenour and order before prescribed in the first Rule, that is, if his estimate be $\frac{1}{4}$ hee shall have the $\frac{1}{4}$ of the gaine. Therefore say by the Rule of three direct. If $\frac{1}{4}$ taken put in 400 pound, what is the estimate, or putting in of $\frac{1}{5}$ taking? Multiply and divide, and you shall finde 320 pounds, and so much is the person of the Factor estimated.*

Otherwise.

To finde the estimation of the person of the Factour, you shall consider, that seeing it was agreed between them, that the Factour should take the $\frac{1}{5}$, then the Merchant shall have the residue, which are $\frac{4}{5}$: wherefore the gaine of the Merchant unto that of the Factour is in such proportion as 5 unto 4. Then if you will know the estimation of the person of the Factor: say if 5 give 4. what will 400 give? Multiply and divide, and you shall finde 320 pound. And so much is the person of the Factor esteemed to be worth.

Other conditions then these aforesaid, may also be between Merchants and Factors, without respect either of sending, or not sending of the Merchant, where most commonly the estimation

mation of the body of the Factour is in such proportion of the stocke which the Merchant layeth in, as the gain of the said Factor is unto the gain of the Merchant. As thus, if a Merchant do deliver into the hands of his Factor 400 pound, and he to have halfe the profit, the person of the said Factor shall be esteemed to be worth 400 pound: and if the Factor do take but $\frac{1}{3}$ of the gain, he should have but $\frac{1}{2}$ so much of the gain as the Merchant taketh, which must have $\frac{2}{3}$, wherefore the person of the Factour is esteemed but the halfe of that which the Merchant layeth in; that is to say, two hundred pound.

And if the Factor did take the $\frac{2}{5}$ of the gain, then the Merchant shall take the residue which are $\frac{3}{5}$, wherefore the gaine of the Merchants unto the Factor is then in such preportion as 3 unto 2: whereupon if you will then know the estimation of the person of the Factour, say, If three give 2, what shall 400 give? Worke, and you shall finde 266$\frac{2}{3}$ pounds. And so much is the person of the Factour esteemed to be worth.

And if the Merchant should deliver unto his Factor 400 pound, and the Factor would lay in 80, and his person, to the end hee might have the $\frac{1}{2}$ of the gaine, I demand how much shall his person be esteemed?

Answer. *Abate 80 from 400, and there will remayn 320. And at so much shall his person be esteemed.*

A Merchnt hath delivered unto his Factor 900 pounds to governe in the trade of Merchandiſe, upon condition that hee ſhall have the $\frac{1}{3}$ of the gain, if any thing be gained, and alſo to beare the $\frac{1}{3}$ of the loſſe, if any thing be loſt. Now I demand how much his perſon was eſteemed at?

Anſwer. *Seeing that the Factor taketh the $\frac{1}{3}$ of the gain, his perſon ought to be eſteemed as much as $\frac{1}{2}$ of the ſtocke, which the Merchant layeth in: that is to ſay, the $\frac{1}{2}$ of 900 pound, which is 450. The reaſon is, becauſe $\frac{1}{3}$ of the gain that the Factor taketh, is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gain that the Merchant taketh, and ſo the Factor his perſon is eſteemed to be worth 450 pounds.*

A Merchant hath delivered unto his Factor 600 pound, and the Factor layeth in 250 li. and his perſon. Now becauſe hee layeth in 250 li. and his perſon, it is agreed betweene them, that he ſhall take the $\frac{2}{5}$ of the gain. I demand for how much his perſon was eſteemed?

Anſwer. *For as much as the Factor taketh $\frac{2}{5}$ of the gain, he taketh $\frac{2}{3}$ of that which the Merchant taketh, for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factours laying in ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant layed in. Then ſubtract 250, which the Factour did lay in from 400 pound, which ſhould have bin his whole ſtocke, and there remayneth 150 pound for the eſtimation of his perſon.*

More, a Merchant hath delivered unto his Factor 800 pound, upon condition that the Factor

ctor ſhall have the gaine of 160 li. as though hee laid in ſo much ready money: I demand what portion of the gain the ſaid Factor ſhall take?

Anſwer. *See what part the 160 (which the Factor layed in) is of 960, which is the whole ſtocke of their company, and you ſhall finde* $\frac{1}{6}$. *And ſuch part of the gain ſhall the Factor take.*

But in caſe, that in making their covenants, it were ſo agreed between them, that the Factor ſhould have the gain of 160 pound of the whole ſtock which the Merchant layeth in, that is to ſay, of the 800 pound: then ſhould the Factour take $\frac{1}{5}$ *of the gain: for 160 is* $\frac{1}{5}$ *of 800 pound.*

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandize, which is diſtinct into ſeven Rules, with divers other neceſſary queſtions incident thereunto.

The firſt Rule.

TWo Merchants willing to change their Merchandize the one with the other: The one hath 24 broad clothes at 10 li.--10 s. the piece: The other hath Mace at 12 ſhillings the pound. The queſtion is, how many

pounds of Mace hee ought to give for his Cloth, to save himselfe harmlesse, and be no loser?

Answer. *Seek first by the Rule of three what the 24 Clothes cost at 10 pound 10 shillings the piece, and you shall finde 252 pound. Then to finde the quantity of Mace, say againe by the Rule of three, If 12 shillings buy one pound, what shall 252 pound buy me? Worke, and you shall finde 420 pound of Mace: and so many pound ought he to give for his Clothes.*

The Proofe.

Two barter. The one hath 420 pounds of Mace at 12 s. the pound, to barter or change broad Clothes at 10 pounds 10 shillings the piece. The question is, how many broad Clothes to he ought to give for all his Mace?

Answer. *First say, if one cost 12 s. what 420? you shall finde 5040 s. Then say again, if $10\frac{1}{2}$ pounds give 1 cloth, what shall 5040 shillings give? Worke and you shall finde 24 Clothes, your desire.*

The second Rule.

Two change Merchandise for Merchandise: The one hath Pepper at two shillings foure pence the pound, to sell for ready money. But in barter hee will have no lesse then three shillings the pound. And the other hath Holland

land at five shillings six pence the Ell readie mony. The question is now at what price hee ought to deliver the Ell in the barter to save himself harmlesse.

Answer. *Say by the Rule of three direct, If* $2\frac{1}{3}$ *ready money give* 3 s. *in barter, what shall* $5\frac{1}{2}$ *give in barter? You shall finde* $7\frac{1}{14}$ s. *and at that price ought the second Merchant to sell his Holland in barter.*

The Proofe.

Two barter. The one hath Holland at five s. 6 pence the Ell to sell for ready money. And in barter hee will have $7\frac{1}{14}$ s. The other hath Pepper at 2 s——4 d. the pound, to sell for ready money. The question is now, how hee ought to sell in barter?

Answer. *Say by the Rule of Three direct, if* $5\frac{1}{2}$ *ready money give* $7\frac{1}{14}$ s. *in barter, what ought* $2\frac{1}{3}$ *to take in barter: Multiply and divide, and you shall finde* 3 s. *your desire.*

The third Rule.

Two barter: The one hath cloth of Arras at 30 s. the Ell ready mony, but in barter he will have 35 $\frac{1}{2}$ s. And the other hath White-wines which hee delivered in barter for 16 pounds the Tun. The question is now, what his wines cost the Tun in ready money.

Answer. *Say by the Rule of three direct, i* $35\frac{1}{2}$ *s. in barter, give but* 30 *s. ready money, what did* 16 *pound in barter cost? Work, and you shall finde* 13 *li.* 10 *s.* $\frac{10}{71}$. *And so much cost his Wines for a Tun ready money.*

The Proofe.

Two barter Merchandize for Merchandize: The one hath white Wines at 13 li.-10 s. $\frac{10}{71}$ s. the Tun to sell for ready money. But in barter he delivered it for 16 pounds. The other, to make his match good and save himselfe harmlesse, delivereth Arras at $35\frac{1}{2}$ s. the Ell. The question is now, what an Ell of his Arras cost in ready money?

Answer. *Say by the Rule of three direct: If* 16 *pounds in barter give but* 13 *li.* 10 $\frac{10}{71}$ *s. in ready money, what shall* $35\frac{1}{2}$ *s. yield in barter? Work, and you shall finde* 30 *s. your desire.*

The fourth Rule.

Two barter. The one hath Kerseys at 14 *pounds the piece ready money. But in barter hee will have* 18 *pounds. And yet he will have the* $\frac{1}{3}$ *part of his over-price in ready money. And the other hath Ginger at eight groats the pound, to sell for ready money. The question is, how hee ought to deliver the Ginger by the pound in barter to save himselfe harmlesse, and make the barter equall.*

Answer.

Answer. Item, for the working of this question, and such other the like, you must understand, if the party over-selling his wares, require to have also some portion in ready mony, as $\frac{1}{2}\frac{1}{3}\frac{1}{4}$, &c. Then shall you first rebate the same demanded part, whatsoever it be, from the overprice, and also from the just price. And those two numbers that shall remayn after the subtraction is made shall be the two first numbers in the *Rule of three*, And the just price of the same Merchandize shall be the third number, which by the operation of the Rule of three direct, shall yield you a true solution, how, and at what price you shall over-sell that your Merchandize, to save your selfe harmlesse, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteen) which is the overprice of his Cloth, which $\frac{1}{3}$ of eighteen is six, which you must subtract from 18. there rest 12. And also abate it from 14, which is the just price of the Cloth, and there remayneth 8, which 8 and 12 are the two first numbers in the Rule of three. Then take eight groats, or $2\frac{2}{3}$ shillings for the third number. Then say by the Rule of three direct. If eight pounds give 12 pounds, what shall $2\frac{2}{3}$ s. give?

14———	18
6	*6*
8———	12

Mul-

Multiply and divide, and you shall finde 4 s. And for so much shall the second Merchant sel his Ginger, or his commodity in barter, to ballance the same equall.

The proofe.

Two barter, the one hath fine Kerseys at 14 pounds the piece ready money. But in barter hee will have 18 pounds. And yet he will have the $\frac{1}{3}$ part of his overprice in ready money: And the other hath Ginger, which hee having cunning enough to make the barter equall, delivered in barter for 4 s. the pound. The question is now, what his Ginger cost him ready money?

Answer. After you have made the subtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14 (as before was taught you:) then will there remayne eight and 12 for your two first numbers in the Rule of three. Then say, if twelve give eight, what shall come of 4 the over-price of the pound of Ginger, multiply and divide, and you shall finde 2 shillings 8 pence your desire.

Two Merchants barter Merchandise for Merchandise. The one hath Devonshire *Whites at* 7 li. 13 s. 4 d. *the piece ready money: but in barter hee doth them away for* 8 li.-13-s-4-d. *and yet he will have the $\frac{1}{3}$ part of his price in ready money. And the other hath Cottens at 3 pounds the piece ready money. The question is now, at what price*

Price he ought to sell or exchange his Cottens in barter to save himself harmlesse, and make the barter equall?

7	13	4	8	3	4
2	14	$5\frac{2}{3}$	2	14	$4\frac{1}{3}$
4	18	10^3	5	8	$10\frac{2}{3}$

Answer. First seeke the $\frac{1}{3}$ part of 8 li.--3--s. 4 d. which is, 2 li --14- --5 $\frac{1}{3}$ d. which rebated from 8 li---3 s---4 d. there resteth as appeareth by the example abovesaid, 5 li--8 s--10 $\frac{2}{3}$ d. which is $\frac{2}{3}$ parts of 8---3---4 d. also rebated from 7----13----4 d. there resteth, 4 li---18 s--- 10 $\frac{1}{3}$ d. the two first numbers in the Rule of three, and the 3 pounds, which is the neat price of the piece of Cotten, is the third number; Then say by the Rule of three direct, as was taught before. If 4 l--18 s--10$\frac{1}{3}$ d. give 5 l--8 s-10$\frac{1}{2}$ d. what shall 3 pounds give? multiply and divide, and you shall finde 3 li--6 s. $\frac{3}{8}$ $\frac{2}{9}$ pence, the just price that he ought to deliver his Cottens in barter.

The fifth Rule.

Two Merchants will change Merchandise for Merchandise. The one hath Kerseys at 43 s. *the piece to sell them for ready money. And in barter, he will sell them for* 56 s. 8 d. *and he will gain after ten pounds upon the* 100 *pounds. And*

yet

yet he will have the $\frac{1}{2}$ *of his over-price in ready money. The other hath flaxe at* 3 d. *the pound ready money. The question is now, how hee shall sell the pound of his flaxe in barter*

Answer. See first at 10 pound upon the 100 pounds what the 56$\frac{2}{3}$ s. commeth to, in saying (by the Rule of three direct) if 100 pounds give 110 pounds, what 56$\frac{2}{3}$ s? Multiply and divide, and you shall finde 3 li.——2 s.–4 d. of which the $\frac{1}{2}$ that hee demandeth in ready money, is 1 li.——11 s——2 d. the same 31 s——2 d. abated from 40 s. and also from 56 s——8 d. there will remain 8 s. 10 d. and 25 s——6 d. for the two first numbers in the Rule of three, and 3 pence the price of the pound of flaxe for the third number. Then multiply and divide, and you shall finde 8$\frac{2}{3}$$\frac{1}{3}$ d. And for so much shall hee sell the pound of flaxe in barter.

The sixth Rule.

Two are willing to exchange Merchandise: the one hath Norwich *Grograns at* 25 s. *the piece ready money: and in barter he will have* 30 s. *and he will have the* $\frac{1}{4}$ *part of his overprice in ready mony. The other hath* Norwich *Stockings at* 40 s. *the dozen to sell for ready mony. But in as much as the first Merchants Grogranes are no better, he would deliver them so to ballance the barter, that hee may gain* 10 *pounds in the* 100 *pounds. The question is now, how hee shall sell his*

his Hose the dozen in barter according to his request.

Answer. Say, if 100 give 110 li. what shall 40 s. give, which is the just price of the dozen of stockings? Multiply and divide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of 30 s. which is 7 s. 6 d. and subtract it from 25 s. and also from 30 s. and there will remayn 17 s—6 d. and 22 s—6 d. for the two first numbers in the Rule of three, and 44 shillings, which is the just price (with his gaine in the dozen of Stockings) for the third number. Then multiply and divide, and you shall finde 56 s. $6\frac{2}{3}$ d. and for so much he is to sell his dozen of stockins in barter.

The seventh Rule.

Two Merchants will change their Merchandise one with the other: The one hath 720 *Ells of Cambrick at* 5 s. *the Ell to sell for ready mony, but in barter he requireth* 6 s. 8 d. *And yet notwithstanding he loseth by it after* 10 *pounds upon the* 100 *pounds, whereupon he requireth one half of his over-price in ready money: and the other Merchant having skill enough to make the barter equall, delivered* English *Saffrons at* 30 s. *the pound. The question is now, what his Saffrons cost the pound in ready money?*

Answer. You must first seeke what is lost upon the 100 pound, which to doe, you may say, (if you please) if 100 pound lose 10, what

shall

shall $6\frac{2}{3}$ lose? Worke, and you shall finde $\frac{2}{3}$ s. (or 8 d.) which must be rebated from 6 s.--8 d. so resteth 6 s. still. Or you may say, if 100 pound give me but 90 pounds, what shall 6 s. 8 d. give? Worke this way either, and you shall finde also as before directly in your quotient 6 s. your desire. Then are you next to cast up what the 720 Ells of Cambricke commeth to at 6 s. 8 d. the Ell, and you shall finde 240 pounds: the $\frac{1}{2}$ whereof the Cambrick Merchant will have in ready money, (which is 120 pounds:) Nextly you must cast what the Cambricke commeth to after his losse in the 100 pound, which as you found, is but 6 s. an Ell, and you shall find 216 pounds: Now must you subtract his ready money (which is 120 pounds in all) out of 240 pound, and also out of 216 pound, and there will remayne 120 pounds, and 96 pounds for your two first numbers in the Rule of three, and 30 shillings is the over-price of your Saffron for the third number: Then multiply and divide, and you shall finde 24 shillings. And so much did his Saffrons cost in ready money.

Two *Merchants barter. The one hath* 50 *clothes to put away for ready money at* 11 *pounds the cloth, and in barter putteth them away for* 12 *pounds, taking Holland cloth at* 20 d. *the Flemmish Ell, which was worth no more but* 18 d. *The question is now, what Holland payeth for the Cloth, and what he winneth or loseth by the bargain?*

Answer.

Answer. 50 Clothes at 11 pounds the Cloth commeth to 550 pounds, and put away at 12 pounds the piece, maketh 600 pound. Then to finde what Holland payeth for the Cloth, say by the Rule of Three direct, If 20d. buy one Ell, what 600 pounds? Work and you shal find 7200 Ells. Now to finde the estate of his gain or losse, you must seeke what his 7200 Ells commeth to at 18 d. the Ell: Worke by the Rule of Proportion direct, and you shall finde 540 pounds, which is not so much as his cloths were worth in ready money by ten pounds: and so much lost the first Merchant by his Exchange.

A Venetian *hath in* London 100 *pieces of silk, to put away for ready mony at* 3 *pounds the piece. But in barter he delivered them for* 4 *pounds the piece, taking Wools of a Felmonger at* 7 li. 10 s. *the C. weight, which was worth no more but six pounds the C. ready money. The question is now; what Wools payeth for the silkes, and which of them winneth or loseth by the barter?*

Answer. 100 pieces of silke at 3 pound, is in all 300 pounds, & 4 pounds, is 400 pounds. Then to finde what wools payeth for the silkes, say by the Rule of three direct: If $7\frac{1}{2}$ buy one 100 weight, what 400 pound? Work, and finde $53\frac{1}{3}$ C. weight of wooll. Now to finde the estate of their gain, and losse, cast up his wooll at 6 li. the C. (for so much they were worth ready money) and you shall finde 320 pound, which is 20 pound more then the silks were

so

to be sold for ready money, whereby the *Venetian* gained 20 pounds by the Barter.

A Merchant hath $53\frac{1}{3}$ *weight of wooll at 6 pounds the C, to sell for ready money, but in barter he will have 7 pounds 10 s: and another doth barter with him for Silks, which are worth three pounds a piece ready money. The question is now, how hee ought to deliver his Silks the piece in barter, and how many payeth for the wools.*

Answer. Say by the Rule of proportion,(or by the Rule of three direct) if 6 pounds for C. weight ready money yield me 7 li. 10 s. what wil 3 li. yield, which is the just price of a piece of Silk in Barter, to make the Trucke equall? Work and finde 3 li. 15 s. the price of a piece of Silk in Barter. Then say, If 3 li. 15 s. require one piece of Silk, how many pieces of Silk are bought with 400 pound, which is the value of $53\frac{1}{3}$ C. weight of wooll at 7 li. 10 s? Work by the Rule of three direct, and you shall finde 106 pieces of Silk and $\frac{2}{3}$ of a piece, and so many of Silk pay for the wooll, and neither party hath advantage of other.

Two men will change Merchandise the one with the other. The one of them hath Beere at 6 s. 8 d. the Barrell to sell for ready money, but in Barter he will sell the Barrell for 8 s. *and yet he will gain moreover after* 10 *pound, upon the* 100 *pounds. And the other hath white* Spanish *wooll at* 2 s. *the Rove to sell for ready money. The question is now, how hee shall deliver the Rove of wooll in Barter to save himselfe harmlesse.*

Answer.

Answer. Say, if $6\frac{2}{3}$ s. which is the just price of the barrell of beere, be sold in barter for 8 shillings: for how much shall 20 shillings (which is the just price of the Rove of Wooll) be sold in barter? Worke by the rule of three direct, and you shall finde 24 s. Then for because the first Merchant will gain after 10 pounds upon the 100 pounds, hee maketh his 100 pounds, 110 pounds. And therefore say by the Rule of three, If the second Merchant of 110 pounds do make but 100 pounds, how much shall hee make of 24 s? Multiply and divide, and you shall finde 21 s. 9 d. $\frac{9}{11}$ of a peny. And for so much shall hee sell the Rove of Wooll to be delivered in barter, to the end the first Merchant may give 10 in the 100.

Two Merchants will change their Commodities the one with the other. The one of them hath white Paper at 4 s. *the Ream, to sell for ready money. And in barter he will do it away for* 5 s. *and yet he will gain moreover after the rate of* 10 *pounds upon the* 100 *pounds. And the other hath Mace at* 14 s. 6 d. *the pound weight to sell in barter. Now I demand what the pound did cost in ready money.*

Answer. Say, if 5 s. (which is the over price of the Paper in barter) become of 4 s. the just price of how much shall come $14\frac{1}{2}$ shillings, which is the surprize of the pound of Mace in barter? Multiply and divide, and you shall find $11\frac{1}{7}$ s. Then for because the first Merchant of Paper will gain after 10 upon the 100, Say if

100 do give 110, what shal $11\frac{2}{3}$ shillings gives Work, and you shall finde 12 s.--$9\frac{1}{2}\frac{2}{3}$ d. and so much did the pound of Mace cost in ready money.

The fourteenth Chapter intreateth of exchanging of money from one place to the other.

EXchange is no other thing, then to take or receive money in one Citie, to render or pay the value thereof in another Citie, or else to give money in one place, and receive the value thereof in another, at terme of certain dayes, moneths, or faires, according to the diversitie of the place.

But this practice chiefly consisteth in the knowledge of the money or Coines in divers places, of which for thy benefit (after a few examples given to the introduction of this worke) I will set down certain notes of the diversitie of the common and usuall coines in most places in Christendome for traffique.

And first I will begin at *Antwerpe*, where they use to make their accounts by *Deniers de grosse*, that is to say, pence *Flemmish*, whereof 12 do make 1 s. *Flemmish*, and 20 s. doe make one pound *de grosse*.

Item,

Item, *a Merchant delivered at* Antwerpe, 400 *pounds* Flemmish *to receive in* London 20 s. *sterling, for every* 23 s. —— 4 d. Flemmish: *The question is now, how much sterling money is to be received at* London *for* 400 *pounds* Flemmish? ——

Answer. Say by the Rule of three, If 23 $\frac{1}{3}$ *Flemmish* give 20 s. sterling, what 400 pounds *Flemmish?* Worke, and you shall finde 342 li — 17 s — 1 $\frac{1}{7}$ pence, and so much sterling shall I receive in *London* for the said 400 pounds *Flemmish.*

Otherwise also wrought by Rules of practice in taking the $\frac{1}{7}$ of the *Flemmish* money delivered, and abating the same from the principall, the rest is *English* money, as before.

400 li	0 s	0 d.
57	2	10 $\frac{6}{7}$.
342	17	1 $\frac{1}{7}$ sterling.

A Merchant at London *delivered* 200 li. *sterling for* Antwerpe, *at* 23 s. — 5 d. Flemmish *the pounds sterling: the question is, how much hee must receive at* Antwerpe.

Answer. Say by the Rule of three, if 1 pound sterling give 25 s. 5 d. *Flemmish*, what 200 li. sterling? Work, and thou shalt finde 234 li — 3 s. — 4 d. So many pounds *Flemmish* shall hee receive at *Antwerpe* for the said 200 pounds sterling.

Otherwise by practice.

1 ———	13 —— 5 ——	200	
3 s-4 d.		33 ——— 6 ——— 8	
1 d.		——— 16 —— 8	
maketh sterling ———		2,4 li.-3 s.-4 d.	

In London 20 *pound sterling is delivered by Exchange for* Antwerpe, *at* 23 s. 9 d. Flemmish *the pound sterling : the question is, at what rate the* Flemmish *money ought to be returned to gain* 4 *pounds upon the* 100 *pound sterling at* London.

Answer. First, say by the Rule of three direct: If 1 pound sterling give $23\frac{3}{4}$ *Flemmish*, what 200 pounds sterling? Multiply and divide, and you shall finde 237 pounds 10 shillings. The which to returne to gaine 8 pounds sterling in *London*, say by the backer Rule. If 200 pounds sterling require the exchange 23 s. 9 d. *Flemmish*, what the exchange to make 208 li. sterling? Work by the Rule, and finde 22 s. $10\frac{1}{8}$ d. *Flemmish*, the effect in the question required.

If I take up money at Antwerpe *after* 19 s. 4 d. Flemmish, *to pay for the same at* London 20 *shillings sterling, and when the day of payment is come, I am forced to return the same money again in* London, *to pay my Bill of Exchange : so that for 20 shillings which I take up here at* London, *I must pay* 19 s. 6 d. *at* Antwerpe, *I demand whe-*

ther I do win or lose, and how much in or upon the 100 *pounds of money?*

Answer. Say by the Rule of three: If $15\frac{1}{2}$ give $19\frac{1}{3}$, what will 100 pounds give? Multiply and divide, and you shall finde 99 li. $2\frac{106}{117}$ s. which being abated from 100 pounds, there will remain 17 shillings $\frac{11}{117}$, and so much I do lose upon the 100 pounds of money.

If I take up at London 20 *shillings sterling, to pay at* Antwerpe 22 s. 4 d. *and when the day of payment is come, my Factour is constrained to take up money again at* Antwerpe, *wherewith to pay the aforesaid summe, and there he doth receive* 23 s. 4 d. Flemmish, *for the which I must pay* 20 s. *at* London: *The question is now, whether I do winne or lose, and how much upon the* 100 li. *of money after that rate.*

Answer. Say by the Rule of Proportion, If $22\frac{1}{3}$ s. give $23\frac{1}{3}$ s. what will 100 pounds give? Multiply and divide, and you shall finde 104 pounds 9 shillings $\frac{41}{67}$, from the which abate 100 pounds, and there will remaine 4 pounds 9 shillings $\frac{41}{67}$, and so much is there gained upon the 100 pounds of money.

In Antwerpe *is delivered* 200 *pounds* Flemmish *by exchange for* London, *at* 20 *shillings sterling for every* 23 *shillings* 4 d. Flemmish. *The question is, at what rate the same is to be returned to gain* 10 *pounds upon the* 100 *pounds* Flemmish *in* Antwerpe.

Answer. First, say by the Rule of three, if $23\frac{1}{3}$ *Flemmish* give 20 s. what shall 200 pounds gain? Work, and you shall finde 171 pounds 8 s. $6\frac{6}{7}$ d. Then say againe by the Rule of three direct, if 171 pounds 8 s $6\frac{6}{7}$ s. sterling give me 210 pounds *Flemmish*, what shall 20 s. sterling give? Work, and you shall finde 24 s. 6 d. *Flemmish*. And at the same rate ought the same to be returned at *Antwerpe*, to gain 10 pounds upon the 100 *Flemmish*.

A Merchant of Antwerp *delivereth* 234 *pound* 3 s. 4 d. Flemmish, *to receive at* London 200 *pounds sterling: The question is now, how the exchange goeth after this rate?*

Answer. Say by the Rule of three direct, if 200 give 20. what $234\frac{1}{6}$? Multiply and divide, and you shall finde 23 s—5 d. And for so much goeth the exchange.

Item, *the exchange from* London *into* France, *is not like as it is* Flanders, *but is delivered by the* French *Crowne*, *which is worth* 50 *Soulx Turnois the piece*.

Whereupon also you must note, that in *France* they make their accounts by Franks, Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx maketh one pound Turnois, which they call a Liure or Franke. But the Merchants, to make their accounts, do use French Crowns, which is currant among them for 51 Soulx Ternois. But by exchange it is otherwise, for it is delivered but for 50 Soulx Turnois the Crown, or as the

taker

up of the money can agree with the deliverer. And note that this △ Character representeth the Crown by exchange, and is ever 50 Soulx Turnois or *French* money.

A Merchant delivereth at London, 240 *pounds sterling, after 5 shillings 6 pence the Crowne, to receive at* Paris 50 *Soulx Turnois for every Crown. I demand how much* Turnois *or* French *money payeth the Bils for the said* 240 *pounds sterling.*

Answer. Say by the Rule of three, If $5\frac{1}{2}$ s. sterling give me 50 s: Turnois, what shall 240 pounds sterling give? Reduce the pounds into shillings, then multiply and divide: and you shall finde 2181 Liures, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ Turnois; and so much payeth the Bils at *Paris*, for the 240 pounds sterling.

A Merchant delivereth at Roan, *or elsewhere in* France, 1430 *pounds or Franks, the which Frank or pound is* 20 *Soulx, or a pound* Turnois, *to receive in* London 6 s. 4 d. *sterling for every* △ *of* 50 *Soulx* Turnois. *The question is, how much sterling money I ought to receive at* London *for my* 1430 *pound Turnois.*

Answer. Say, if $2\frac{1}{2}$ pounds give me $6\frac{1}{3}$ s. what will 1430 give me? Work, and you shall finde $3622\frac{2}{3}$ shillings sterling, which maketh 181 pounds 2 s. 8 d. and so much money is to be received at *London*, for the said 1430 Liure Turnois, after 6 s. 4 d. for every △ of 50 Soulx.

In London *is delivered* 200 *pound sterling by exchange for* Paris, *at* 5 s. 9 d. *the* △ *of* 50 *soulx*

Turnois. *The question is, at what price the said* △ *is to be returned to gain 6 pounds upon the* 100 *pound sterling at* London.

Answer. First, say (by the Rule of three direct) if $5\frac{3}{4}$ s. sterling give 50 Soulx Turnois; what shall 200 pound sterling give? Worke, and you shall find 1739 Franks or Liures, $2\frac{14}{23}$, Soulx. Then the which to returne and gaine 6 pounds upon the hundred pounds in *London*, say by the Rule of three direct, if 1739 Franks $2\frac{14}{23}$ Soulx yield 212 pounds, what the △ of 50 Soulx? worke and finde 6 s. $1\frac{1}{50}$ d. the effect required in the question.

A Merchant delivered in London 160 *pounds sterling, to receive in* Biskay *for every* 5 *s*.6 d. *one Ducat of* 374 *Marvides. The question is, how many Marvides ought I to receive at* Biskay?

Answer. Say, if $5\frac{1}{2}$ s. sterling give 374 Marvides: what shall 160 pounds sterling give? Multiply and divide, and you shal find 217600 Marvides, and so many I ought to receive at *Biskay* for my 160 pounds sterling.

A Merchant delivered in Baion, 4000 *Marvides, to receive in* London 5 s.8 d. *sterling for every Ducat of* 374 *Marvides. The question is now, how much sterling money payeth the Bils of Exchange for the said* 40000 *Marvides?*

Answer. Say, if 374 Marvides make one Ducat, what 4000 Marvides? Multiply and divide, and finde $106\frac{178}{187}$.

Then say again, if 1 Ducat give $5\frac{2}{3}$ s. what giveth $106\frac{178}{187}$ Ducats? Work, and find 30 l. 6 s. $\frac{114}{161}$.

Other-

Otherwise it is wrought more briefe at one working, as in the last question before, in considering that 5 s. 8 d. containeth on Ducat, or 374 Marvides: Therefore say by the Rule of 3, if 374 Marvides give $5\frac{2}{3}$ s. what 40000 Marvides? Work, and you shall also finde in your quotient $30\frac{24}{361}$ s. And so many pounds sterling is to be received for the 40000 Marvides.

In London 200 *pounds delivered by exchange for* Vigo, 374 *Marvides the Ducat of* 5 s. 10 d. *sterling, maketh* 25645 $7\frac{1}{7}$ *Marvides: the which to return and gain* 10 li. *upon the* 100 *pounds in* London, *say by the Rule of three direct, if* 220 li. *require* 25645 $7\frac{1}{7}$ *Marvides, what* 5 s. 10 d? *Worke, and find* 340 *Marvides, the price of every Ducat in return, which is the effect in the question required.*

These may seem sufficient for instructions.

N*Otwithstanding for thy further aid and benefit, hereafter follow six speciall and most brief Rules of practice, for* English, French, *and* Flemmish *money.*

1 How to turn Flem. to Engish sterling.
2 How to turn English sterling to Flem.
3 teacheth How to turn Flemmish to French.
4 How to turn French into Flemmish.
5 How to turn sterling into French.
6 How to turn French into sterling.

The

The fifteenth Chapter intreateth of the said six Rules of brevity, and of valuation of *English*, *Flemmish*, and *French* money, and how each of them may easily be brought to others value.

How briefly to reduce pounds, shillings, and pence Flemmish *into pounds, shillings, and pence* English *sterling.*

Rule 1. IT is to be noted, that 7 pounds *Flemmish* maketh but 6 pounds sterling: 7 s. *Flemmish* maketh 6 s. sterling, and 7 d. *Flemmish* 6 d. sterling: so that 7 yieldeth but 6. Wherein is evident that then is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into *English* money: wherefore to know how much 233 l.--13 s.--4 d. *Flemmish* maketh *English*, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction, is the summe required: so that 233 li.--13 s, 4 d. *Flemmish*, maketh 200 li-5 s. $8\frac{4}{7}$ d, sterling.

Example.			Another Example.		
li.	s.	d.	li.	s.	d.
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	$7\frac{3}{7}$	$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
200	5	$8\frac{4}{7}$ ster.	266	11	$5\frac{1}{7}$

To reduce pounds, shillings, and pence sterling, into pounds, shillings, and pence Flemmish.

Note that a pound sterling maketh 1 li. 3 s. 4 d. *Flemmish*, that is, 1 $\frac{1}{3}$ li. 1 s. sterling maketh 1$\frac{1}{6}$ s. *Flemmish*, and 1 d. sterling maketh 1$\frac{1}{6}$ d. *Flemmish*. So that there is gained (if it may be so called) $\frac{1}{6}$ of the summe being thus reduced to *Flemmish*, for of $\frac{6}{6}$ is made $\frac{7}{6}$ which is one whole and $\frac{1}{6}$. Then to know how much 237 li. 7 s 6 d. sterling maketh *Flemmish*, subtract from your sterling, the $\frac{1}{6}$ of the whole summe, and adde it to the same summe, and it maketh 276 li. 18 s. 4 d. which is the summe required.

Example.			Another Example.		
li.	s.	d.	li.	s.	d.
237	7	6 ster.	337		
$\frac{1}{6}$ 39	11	3	$\frac{1}{6}$ 56	3	4
276	18	9 *Flem*.	393	3	4

To

Rule 3: *To reduce pounds, shillings, and pence,* Flemmish, *into pounds, shillings, and pence* French.

Ye shall note, that the equality of Flemmish and French money is this, that is to say, the pound Flemmish, maketh 7 pound $\frac{1}{3}$ French, or Turnois. 1 s. Flemmish maketh 7 $\frac{1}{3}$ s. French, and a groat Flemmish, maketh $7\frac{2}{5}$ d. French.

Wherefore to know how much 143 li. 4 s. 9 d. Flemmish maketh French, ye must multiply the whole number twice by 6, beginning at pence, and so forward, and the product of your second multiplication divide by 5, so the work is finished. Or multiply the said summe by 7, and take out of it $\frac{1}{5}$, adding it to the product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 li--4 s--9 d, Flemmish, maketh 1031 li--6 s--2 $\frac{2}{3}$ d, French or Turnois.

Example.

li.	s.	d.	
143	4	9	*Flem.*
		6	
859	8	6	
		6	
5156	11	0	*Fren.*
$\frac{1}{5}$ 1031	6	$2\frac{2}{5}$	*Fren.*

The same otherwise.

li.	s.	d.
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	$11\frac{2}{5}$
1031	6	$2\frac{2}{3}$

Ano-

Another example.	Or thus:
143 l. *Flem.*	143
6	7
858	1001
6	$\frac{1}{5}$ 28—12
$\frac{1}{5}$ 5148 *French*	1029 li—12
1029 li. $\frac{1}{2}$ or 12 s. *French.*	

To reduce pounds, shillings, and pence, French, *into pounds, shillings, and pence,* Flemmish. Rule 4.

Multiply 233 li—8 s—4 d. French by 5, and divide the Product twice by 6, that is, the said number by 6, and the product or quotient again by 6, and the quotient of this second Division is the thing required. So that 233 li—8 s.-4 d. French, maketh 32 li-8 s-4$\frac{1}{2}$ d. Flemmish.

Example.		Another example.	
li. s. d.		li. s. d.	
238---8---4	*Fren.*	758	*French.*
5		5	
1167---1---8		3765	
$\frac{1}{6}$ 194—10—3$\frac{2}{8}$		$\frac{1}{6}$ 627--10	
$\frac{1}{6}$ 32—8--4$\frac{1}{9}$	*Flem.*	$\frac{1}{6}$ 104--11—8	*Flem.*

To

Rule 5. *To reduce pounds, shillings, and pence, sterling, into pounds, shillings, and pencs,* French, *or Turnois.*

The pound sterling maketh 8 li. 8 s. *French*, that is to say, $8\frac{2}{5}$ pounds: the shillings, maketh $8\frac{2}{5}$ shillings, and the peny $8\frac{2}{5}$ d. *French.* Wherefore to know what 231 li. 13 s. 4 d. sterling maketh *French*, ye must multiply your whole summe by 42, that is, by 7, and the product of it by 6, and divide this second product by 5, and that is the summe required.

Otherwise, multiply the summe sterling by 8, and adde twice to the Product $\frac{1}{5}$, and it shall produce the sum required. So that both wayes 231 li-13 s-4 d. sterling, maketh 1946 pound *French*, as here under followeth.

Sterling. **Example.** **The same otherwise.**

li.	s.	d.	Ster. li.	s.	d
231 —	13 —	4	231 —	13 —	4
		6			8
1390 —	0 —	0	1853 —	6 —	8
		7	46 —	6 —	8
			46 —	6 —	8
$\frac{1}{5}$ 9730 —	0 —	0			
			1946 —	0 —	0
$\frac{1}{5}$ 1946 —	0 —	0 *Fren.*			

French.

Ano-

Another example.		The same.	
753	Ster.	753	Sterling.
6		8	
4518		6024	
7		$\frac{1}{2}$ 150——12	
31626		$\frac{1}{2}$ 150——12	
$\frac{1}{2}$ 6325———4 *Fren.*		6325———4	French.

To reduce pounds, shillings, and pence, French, *into pounds, shillings, and pence, sterling.*

To know how much 1256 li. 12 s. 6 d. *French* maketh in sterling money: multiply the sum by 5, and divide the product by 7 and 6 at twice, and the last quotient shall be the thing required, that is to say, 1256 li. 12 s. 6 d. maketh 149 pounds, 11 s. 11$\frac{2}{7}$ d. sterling.

Example.		Another example.	
li. s. d.	*French.*	li. s. d.	
1256--12--6		2531 0 0	*French*
5		5	
6283---2--6		12655	
$\frac{1}{6}$ 1047-3--9		$\frac{1}{6}$ 2109--3--4	
$\frac{1}{7}$ 149--11-11$\frac{2}{7}$	Ster.	$\frac{1}{7}$ 301--6--2$\frac{2}{7}$	Ster.

Note,

Note, that when any money is given by exchange at *London* for *Roan*, at 71 $\frac{1}{2}$ d. or rather 71 $\frac{1}{7}$ for the Crown of 50 s. *French*, there is neither gain nor losse: for it is one mony for another, accounting 8 li. 8 s. *French*, for one pound sterling. So the giver loseth the time of payment, which is about 15 days, and he that taketh it, hath the gain of the same.

They of *Roan*, that put forth or take money by exchange for *London*, ought to have like consideration.

Item, when any man giveth at *London* 64 pence $\frac{1}{3}$, or rather 64 $\frac{2}{7}$ d. to have at one of the Faires of *Lions* a Crown *de Marc*, hee that so giveth the money, loseth the time, and he that taketh it, gaineth the same: for 62 pence $\frac{1}{2}$ is equall in value to 45 s. *French*. He that putteth or taketh money at *Lions* for *London*, ought to consider the same.

Item, when any deliver in *Antwerpe* 75 pence, to receive at *Lions* a Crown *de Marc*, hee that putteth it forth, loseth the time, and he that taketh it gaineth the same. For 75 groats *Flemmish*, is equall in value to 45 s *French*.

Thus for this time I make an end of the practice of Exchanges, and the instructions thereunto belonging, and according to my promise: yet further to gratifie such as are desirous to know the common Coynes used for trafficke among Merchants in these Cities following, a briefe declaration of their Monies, and the reckonings, and account of them.

The

The sixteenth Chapter containeth a declaration of the valuation and diversity of Coynes of most places of Christendome for traffick; And the manner of Exchange in those places from one Citie or Town to another: which known, is right necessary for Merchants, by means whereof they do finde the gain or losse upon the Exchange.

Item, for as much as the greatest diversity of money of Exchange is at *Lions*, therefore I will begin duely of the money of that place.

At *Lions* they use Franks, Soulx, and Deniers Tournois. A Franke maketh 20 Soulx, and one Soulx 12 Deniers: but the Merchants to keep their books of Accounts, doe use French crowns of the Mark at 45 Soulx the piece, and doe divide it into 20 Soulx, 1 Soulx is 12 Deniers.

Item, a Marke of Gold maketh 65 △ of the Mark, which serveth for exchange, and divide it into 8 ounces. The ounce into 24 pence or Deniers, the Denier into 24 grains, and so the summe or whole by imagination or gesse. △ This marke standeth for a Crown:

Also at *Lions*: there are four Fairs in a yeer, at the which they doe commonly Exchange, which are from three moneths to 3 moneths.

At *Geant* they use the Soulx: one Ducat maketh 3 pound.

At *Naples* they use Ducats, Taries, and Grains, the Ducat maketh five Taries, and one Tary 20 Grains: but they take 6 Ducats (which maketh 30 Taries) for the ounce.

A Ducat maketh ten Carlins, and a Carlin ten Grains, so that 2 Carlins make a Tary, and 100 Grains make a Ducat.

At *Rome* they use the Ducats of the Chamber: one Ducat is worth 12 Guilis, and one Guilis ten Soulx.

At *Venice* they use Ducats currant at 124 Soulx a piece, or 24 Deniers, and one Denier maketh 32 Picolis.

At *Palerme* and *Messine* they write, after ounce, tary, and grains, and one ounce is worth 6 Ducats of 30 Taries, and 1 tary is 20 grains, and 1 graine 6 Picolis, 1 Ducat is also worth 24 Carlins.

At *Millan* they use li. s. d. of Ducat Imperials, and △ of exchange is worth 4 li.

At *Lucques*, *Florence*, and *Ancone*, they use the △ of Gold: in Gold the *French* Crown is worth 7 li. but at *Boloigne* 3 li. 10 s.

At *Barcelone* they use the Soulx: the Ducat of Exchange is worth 22 Soulx.

At *Valence* and *Saragosse* they use the Liver, Soulx, and Denier: the *French* Crowne of Exchange is worth 20 Soulx, and one Soulx is 12 Deniers.

At the Faires of *Castile* they use the Marveides,

veides, the Ducat is worth 375 Marveides.

At *Lisbone* they use the Rayes, one Ducat of Exchange is worth 400 Rayes.

At *Noremburge*, *Frankeford*, and *Augusta* in *Germany*, they use the *Krentzars*, whereof 60 make a Floren.

At *Antwerpe* they use li.--s.- and d. de Gros, and they exchange into the Denier de Gros, to wit, our English peny.

At *London* they use the li. the s. and d. sterling, and they exchange in pence sterling.

The Exchange of Lions *at sundry places.*

Item, at *Lions* there is exchange in three sorts, at the Cities and Towns following.

First, they deliver at *Lions* one Mark to have or receive at *Naples* almost 41 ½ Ducats at *Venice* 70 Ducats currant: at *Rome* 63 Ducats of the Chamber: at *Lucques* and *Florence* 65 △ of Gold, at *Millan* 82 △.

And contrariwise, at the said Cities aforesaid, they do give so much of money, to have a Mark at *Lions*.

Secondly, they give at *Lisbone* one △ of Marke of 45 Soulx Turnois a piece, to have at *Geans* almost 68 Soulx, at *Palerme* and *Messine* almost 24 Carlins, at *Barselone* 22 Soulx, at *Valence* or *Saragosse* 20 Soulx, at the Faire at *Castile*, 350 Marveides, at *Lisbone* 360 *Rayes*, in *Antwerpe* 57 Deniers *de Gros*, and at *London* 70 d. sterling.

And contrariwise, they give in the said Cities almost as much of their money to have a French Crown of the Mark at *Lions*.

Thirdly, they do give at *Lions* a △ of the Sun, to have almost 93 Krentzars at *Frankeford*, *Augsburge*, *Noremberge*, or other Cities in *Almaine*.

Also at *Lions* onely they do pay, they change the $\frac{2}{3}$ in Gold, and $\frac{1}{3}$ in money, or else all in money, in giving $1\frac{1}{2}$ for the hundreth.

Changes at Naples *and other Townes.*

Item, at *Naples* they give or deliver almost 112 Ducats, to receive at *Rome* 100 Ducats of the Chamber at the old value.

Through *Lucques* and *Florence* they deliver 100 Ducats Carlins, to receive there almost 86 △ of Gold.

Through *Palerm* and *Messine*, one Ducat of 5 Tary, to receive there almost 164 grains.

Through *Millain*, one Ducat to receive there almost 90 Soulx.

Through *Geans* one Ducat to receive there almost 65 Soulx; The whole summe to be payed within ten dayes after the sight of the Bill of Exchange.

Also at *Naples* they deliver one Ducat to receive in *Antwerpe* almost 67 d. or Deniers *de Gros*, within two moneths. At *London* almost 60 d. sterling in three moneths. At *Barselone* almost 20 Soulx within two moneths.

At

At *Valence*, almost 18 Soulx within two Moneths, At *Lisbone* 333. Rayes within three moneths: and at the Faire at *Castile*, almost 340 Marvides at the same Faire.

Change of Venice *to other places.*

At *Venice* they deliver 100 Ducats currant to receive in *Almain* almost 140 Florens at 60 Krentzers the piece.

At *Lucques* and *Florence* almost 108 △ of Gold in ten days.

Likewise at *Venice* they deliver a Ducat currant to receive at *Palerme* and *Messine* almost 21. Carlins, at *Millan* almost 93. Soulx: at *Geanes* almost 62. Soulx, the whole at ten dayes end.

Of the Pair *of* Pari.

As touching the Exchange, it is necessary to understand or know the *Pair*, which the *Italians* call *Pari*, which is no other thing then to make the money of the change of one City or Town, to or with the mony of another, by meanes wherof they doe finde the gain or losse upon the Exchange.

Example.

Item, having received Letters of credit of one of *Antwerpe*, that the △ of the Sun is there worth 7 Soulx: The question is, what the same is worth at *London*, when the *Pair* of Exchange goeth for 23 shillings?

Answer, *Say, if 23 give but 20, what giveth 7? Worke, and finde* 81. 1, $\frac{4}{3}$ d. *and so much is the* △ *of the Sun worth at* London.

The seventeenth Chapter containeth also a Declaration of the diversitie of the VVeights and Measures of most places of Christendome for traffique. At the end of which Discourse are two Tables, the one for weight, and the other for measure, proportionate and reduced to an equality of our English measure and weight, by the aide whereof, the ingenious may easily by the *Rule of three*, convert the one into the other at pleasure, &c.

AT *London*, and so all *England* thorow, are used two kindes of Weights and Measures, as the *Troy* weight, and the *Haberdepoise*. From the *Troy* weight is derived the proportion and quantity of all kinde of dry and liquid Measures, as Pecks, Bushels, Quarters, &c. wherewith is bought and sold all kinde of graine and other Commodities met by the Bush. And in liquid, Ale, Beere, Wine, Oile, Butter, Honey, &c. upon these grounds and Statutes is bread made, and sold

ſold by the *Troy* weight: and ſo is gold, ſilver, pearle, precious ſtones, and Jewels. The leaſt quantity of this *Troy* weight is a grain: 24 of theſe grains make a peny weight, twenty peny weights an ounce, and 12 ounces a pound, two pounds or pints of this weight maketh a quart. And ſo aſcending into bigger quantities, is produced the Meaſures whereby are ſold our other naturall ſuſtenance: *viz.* Ale or Beere, with all other neceſſary commodities, as Butter, Hony, Herrings, Eeles, Sope, &c All which laſt before rehearſed, though their Meaſures (wherein they are contained) be framed and derived from the *Troy* weight, yet are they in traffique with divers commodities, as Lead, Tinne, Flax, Wax, with all other commodities both of this Realme, and of other forraigne Countries whatſoever, bought and ſold by the *Haberdepoiſe* weight after ſixteen ounces to the pound, and 112 pound to the C weight. And to every C is allowed but 12 pound weight at the common beame. From hence is alſo derived the weight of *Suffolke* Cheeſe, which containeth 32 Cloves, 8 pound to a Clove, and weigheth in all 256 pounds. And alſo the Barrell of *Suffolke* Butter is, or ſhould be of like weight with the weight of Cheeſe, *viz.* 256 pounds. More 14 of theſe pounds make a Stone, and 26 Stone containeth a Sacke of *Engliſh* Wooll, Forraigne Wools; to wit, *French*, *Spaniſh*, and *Eſtrich*, is alſo ſold by the pound, or C. weight, but moſt com-

See further of theſe Weigh's and Meaſures in Reduction, beginning, pag. 133.

commonly by the Rove, 25 pound to a Rove: other commodities of Tale, are bought and sold by the C. five score to the C. Except headed ware, to wit, Cattell, Nails, and Fish, which are sold after sixscore to the C. There are also two other sorts of Measures, to wit, the Ell and the Yard. By the Ell is usually met ; Linnen cloth, as Canvas, &c. And by the yard, Silks, woollen clothes, &c.

Antwerpe.

At *Antwerpe* are also two sorts of weights, their gold and silver weight, and their commen weight. Gold and silver is weighed by the *Marke*, the *Marke* is 8 ounces, the ounce 20 Esterlings, and the Esterling 32 as our grains. The Goldsmiths divide that into smaller, but not the Merchants: The proof of Gold is made by Karects, whereof 24 maketh a Mark of fine Gold; the Karect is 24 grains: the proofe of the money is made by Deniers: 12 Deniers is one s. fine, that is, a *Mark* of fine silver : the Denier also is divided into 24 graines; and the grain into four quarters.

Item, 100 *Marks in* Antwerpe. *Troy weight, maketh at* Lions 103 *Marks*, 2 ½ *ounces, and* 20 *grains*, 23 *p. At* Noremberg 103 *Marks* 2½ *ounces*, 2 *Quints*, 3 *Deniers*; *at* Frankford, 105 *Marks*; *at* Augsburg 104 *Marks*, 3 *ounces*, 1 *Quint*. *At* Venice 103 *Marks*, 1 *ounce*, 7 *Deniers*, 18 *grains*. *At* London 66 *pounds*.

The

The Marke of gold or silver at Antwerpe, *Troy weight which is 8 ounces, maketh 7 ½ ounces common weight, with which all other Merchandise is weighed. So that the Troy weight is greater than the common weight by 6¼ in the C. By this weight of* Troy, *they also weigh* Muske, Amber, Pearle, &c.

All silks are bought at Antwerpe *by the* Burges Ell, *which is greater than the common measure, by which they retaile by two in the hundred. Their common Ell is ¾ of our yard, and ⅗ of our Ell.*

Lions.

At *Lions* is used 3 sorts of weight, whereof the first is the common Towne weight, with which they weigh all kinde of Spicery, and divers other Merchandise. The second is called *Geneva* weight, which is 8 in the C greater than the common weight, with which they weigh Silks, &c. The third is *French* weight, called commonly the *Marke weight*, and 100 pounds thereof maketh 106¼ li. *Geneva*, and 114¾ of their common weight: with which *French* weight, is weighed all things that payed custome or toll.

At *Lions* is also used two sorts of Ells or Aulnes. The one wherewith they measure grosse clothes, as canvasse, and such like. The other is called the French Ell or Aulne, with which they measure all other kinde of Merchandize, whereof seven common Town Ells maketh 11 ordinary French Ells.

Roan.

Roan.

At *Roan* $6\frac{1}{2}$ Muides of Salt, being the measure of the place, make a C. at *Armuiden* in *Zeland*, and the C of *Bronage* measure of *Armviden*, maketh at *Roan* 11 Muides, 30 Muides make a last of Corn, and 16 a last of Oats, 100 pound weight there, maketh at *London* $114\frac{1}{2}$, and $190\frac{1}{4}$ at *Antwerpe*. And 100 Ells make at *London*, $115\frac{1}{4}$.

Noremburge.

A 100 pound weight at *Noremburge* maketh at *London* $111\frac{1}{2}$, at At *Antwerpe* $107\frac{1}{7}$, and 100 Ells at *Noremburge* make at *London* $75\frac{2}{5}$ at *Antwerpe* $95\frac{1}{5}$, &c.

Lisbone.

The C. weight at *Lisbone* maketh 4 Roves, every Rove 32 pounds, so that their C. weight is 128 pounds, and their pound containeth 14 ounces, and 100 pounds of their weight maketh at *London* $113\frac{1}{8}$.

Their Silke, cloth of gold, and woollen is measured with a measure which they call a cubit, containing about $\frac{3}{4}$ of a Varre of *Castile*. Howbeit their common Measure is called a Varre, which maketh five Palmes, and containeth $1\frac{1}{4}$ of a Varre of *Castile*, our Ell of *London* is equall with the Varre of *Lisbone*.

All

All kinde of Merchandize brought from *Flanders*, *Roan*, or *Brittain*, payeth at *Lisbone*, as a duty or custome to the King, 20 in the C. which they call the tenth in Merchandise, and the other tenth in money.

Note also, that all kind of Merchandise comming to *Lisbone* by land, payeth lesse in custome, then that that commeth by water.

Civill.

The Rove of *Civill* is 30 pound, 4 Roves make their C. weight, which is 120 pounds. The 100 pounds of *Civill* maketh at *London* 102 pounds. Their other common Measure is a Varre, whereof 100 maketh at *London* 74 Els, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* be two sorts of weight, the one called *La Grosse*, the other *La Suttle*, with the grosse is weighed all kind of great wares; and with the smal all kind of spicery, and such like; 96 pounds of grosse weight there, maketh at *London* 100 pound, and 100 pounds of spicery there without any tare or allowance, make at *London* 94, and with tare 65.

Their owne common measure are Braces, whereof 100 make at *London* 55 $\frac{1}{2}$ Ells at *Antwerpe* 92 $\frac{2}{11}$, &c.

Florence.

At *Florence* the 100 li. weight maketh at *Aquila*, for Saffron 110, and 145 pounds of *Florence*, make at *Roan* but 100 pounds, the weight of *Florence*, and that of *Luke* is all one.

Their other measures are Braces, whereof 100 maketh at *Antwerpe*, *Burges* measure, 81 $\frac{2}{3}$ Els, 100 Braces there, make at *London* 49 Els, &c.

Lucque.

The *Lucque* Sattens commonly sold at *Lions* by weight, and 133 $\frac{1}{3}$ pounds maketh at *Lions* 100 pound, so that 1 pound $\frac{1}{3}$ maketh at *Lions* but one pound.

Their other measures are Braces, whereof 100 of them make at *London* 50 Els, at *Antwerpe* 83 $\frac{1}{3}$ Els, &c.

Aquila.

At *Aquila*, their 100 pounds maketh at *London* 71 $\frac{1}{4}$, their 136 $\frac{2}{3}$ pounds of Saffron maketh at *Geneva* but 100, and 11 li. of *Geneva* maketh 15 li. at *Aquila*.

Valentia.

At *Valentia* be two sorts of weights, a great and a small. The C. weight or great weight con-

containeth foure Roves, the Rove 36 li. so the C. great weight is 144 li. and the C. weight small containeth but 120 pounds, and is also parted into foure Roves, which is 30 pounds to a Rove. By the smal is sold the Scarlet grain, with all other kinde of Spicery, and by the great is sold Wooll, with all such like grosse wares. The $1\frac{1}{3}$ pounds of Silke at *Valentia*, maketh at *Lions* one pound *Geneva* weight. The charge of great Merchandize at *Valentia* containeth 432 pound, and in small wares 360 pounds.

The weight here and at *Barsellone* is all one.

There 100 pound weight maketh at *London* 78 pound, at *Antwerpe* 75.

Dansick

At *Dansick* or *Spruceland* the rule is, that whosoever buyeth any Merchandize there, buyeth it by the ship-pound, which is 320 li. 20 Lispounds make a ship-pound, & the Lispound containeth 16 pound, which Ship-pound of *Dansick* maketh at *Antwerpe* $266\frac{2}{3}$ li. Their 100 li. weight, maketh at *London* 86 &c.

Their other common measures are Ells, whereof 100 maketh at *London* $72\frac{1}{4}$ and at *Antwerpe* $820\frac{1}{2}$ Els.

Toulhouse.

At *Toulhouse* 6 Cabes of Woad maketh a Charge, two Cisterns of Corn-measure, and all

kinde

kinde of graine maketh a Charge, the Cesterne weigheth 160 l. weight of that place. Their 100 in weight, maketh at *London* but $91\frac{1}{4}$ pound.

Geans.

At *Genua* or *Geans*, 100 li. of their weight maketh at *London* $71\frac{1}{4}$, and at *Antwerpe* $68\frac{1}{2}$ 100 li. weight at *Genua*, maketh at *Venice*, to wit, Suttle 106 li.

Their other common Measures are *Palmes*, whereof 100 make at *London* $20\frac{1}{5}$ Ells: and at *Antwerpe* $34\frac{1}{5}$.

The rest are supplyed in two Tables, which hereafter followeth: whereby the ingenious may gather his desire.

The Table of the agreement of the Weights of divers Countries, the one with the other, being reduced to an equality, as followeth.

112 pounds weight at London, make at	
Antwerpe	$107\frac{1}{1}$
Frankeford	099
Colen and *Ausburgh*	$102\frac{1}{4}$
Noremburg.	$100\frac{1}{3}$
Roan	098
Paris	$102\frac{1}{4}$
Lions	$118\frac{1}{2}$
Deep	$100\frac{1}{4}$
Geneva	$090\frac{1}{3}$
Towlouse	$122\frac{1}{4}$
Rochell	$124\frac{1}{4}$
Marcellis	$124\frac{1}{4}$
Civill, &c.	$109\frac{1}{4}$

112 pounds weight at London, make at	
Venice grosse weight.	$105\frac{1}{8}$
Venice subtle weight.	$166\frac{2}{7}$
Aquina	$157\frac{1}{7}$
Vienna	$089\frac{1}{8}$
Preslaw	$134\frac{1}{6}$
Leipsig	$101\frac{1}{4}$
Dansick	$129\frac{1}{4}$
Lubeck	$097\frac{1}{8}$
Barcellon	$144\frac{1}{2}$
Lisbone	099
Geans	$157\frac{1}{5}$

The

The other Table of agreement of Measures of divers Countries reduced unto an equality, by the aide whereof you may with the use of the *Rule of Three*, convert either more or lesse of any one Measure unto the other.

60 Els, or 7 Yards at London make at

Antwerpe	100	*Els.*
Noremburg	$104\frac{1}{2}$	*Els.*
Frankeford	125	*Els.*
Leipsig	125	*Els.*
Preslaw	125	*Els.*
Dansick	183	*Els.*
Vienna in *Austria*	87	*Aulnes.*
Lions in *France*	$60\frac{1}{6}$	*Aulnes.*
Paris in *France*	57	*Aulnes.*
Roan in *Normandy*	52	*Aulnes.*
Lisbone	60	*Varres.*
Sivil in *Spain*	81	*Varres.*
Castile in *Spain*	81	*Varres.*
Methera Isles	62	*Varres.*
Venice	108	*Braces.*
Lucques	120	*Braces.*
Florence	$122\frac{1}{2}$	*Braces.*
Millan	138	*Braces.*
Rome	90	*Canes.*
Geans	$288\frac{1}{2}$	*Palms.*

The

The eighteenth Chapter treateth of Sports, and Pastimes, done by Number.

IF you would know the number that any man doth think or imagine in his minde, as though you could divine, bid them triple it, or put twice so much more to it as it is, which done, aske him whether it be even or odde: if he say odde, bid him take one to it, to make it even, and for that one keep one in your minde. Now after hee hath taken one to it, to make it even, bid him give away halfe, and keepe the other halfe for himselfe which when hee hath done, bid him triple that halfe, and again, after he hath tripled it, aske him whether it be even or odde: if he say odde: then bid him take one to make it even again, and for that last one, keep two in your minde: now after he hath made his number even, bid him cast away the one halfe, and keep the other half, from which half that he keepeth cause him subtilly to put away or give you nine out of his number, as oft as he can, and for each 9 that he giveth you, keep 4 in your minde, and thereunto joyn the 3 which I bade you keep, and you shall have your desire.

Example

Example.

Imagine he thought 7, the triple whereof is 21, *and becauſe it is odde, hee is to take* 1 *to make it even, which firſt* 1 *given, is for you to keepe in minde. Then the half of his* 22 *being caſt away, he reſerveth ſtill* 11, *which after you have bid him triple, it maketh* 33 *: then in giving of him one againe to make it even, upon that laſt* 1 *reſerve* 2 *in your minde, then his halfe of* 34 *maketh* 17*; from whence he can give you* 9 *but once. Therefore that yielding to you* 4, *and the* 3 *that you keepe, make* 7 *your deſire.*

Another kinde of Divination, to tell your friend how many pence or ſingle pieces, reckoning them one with another, he hath in his purſe, or ſhould think in his minde.

Which to do, firſt bid him double the pieces hee hath in his purſe, or the number he thinketh (if hee participate his number or ſecrecie unto ſome one friend that ſitteth by him that can but multiply, and adde never ſo little: if their number be great, then ſhall they worke as you bid them ſo much the ſurer.)

Now after he hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 alſo; which done, bid him tell you the juſt ſumme of his laſt multiplication, which ſumme the giver thinketh it nothing available, becauſe it is ſo great above his pretended imagination: yet thereby ſhall you preſently with the helpe of Subtraction tell his propoſed number.

The Rule is this.

Imagine he thought 17, Double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplyed by 5, as here is practised, it yieldeth 195, which 195 is the summe delivered you in the worke: then for a generall Rule you shall evermore cut off the last figure toward your right hand, with a dash of your pen, as here is performed, as a figure nothing available unto your worke, and then rebate 2 from your first figure, after 5 is cut off, and the rest shall evermore be your desire, as by this example doth appeare.

17
2

34
5

39
5

195
2

17

Another of a Ring.

If in any company you are disposed to make them merry by manner of divining, in delivering a Ring unto any one of them, which after you have delivered it unto them, that you will absent your selfe from them, and they to devise after you are gone, which of them shall have the keeping thereof, and that you at your returne will tell them what person hath it, upon what hand, upon what finger, and what joynt. Which to doe, cause the persons to sit down all in a row, and to keep likewise an order of their fingers: now, after yee are gone

out from them to some other place, say unto one of the lookers on, that he double the numbers of him that hath the Ring, and unto the double bid him adde 5, and then cause him to multiply the Addition by 5, and unto the product, bid him adde the number of the finger of the person that hath the Ring. And lastly, to end the worke, beyond that number towards his right hand, let him set down a figure signifying upon which of the joynts hee hath the Ring, as if it be upon the second joynt, let him put down 2. Then demand of him what number he keepeth, from the which you shall abate 250, and you shall have three figures remayning at the least. The first toward your left hand, shall signifie the number of the person which hath the Ring, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shall betoken the number of the joynt.

Example.

Imagine the seventh person is determined to keep the Ring upon the fifth finger, and the third joynt: first double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplyed by 5, yieldeth 95, unto which 95, adde the number of the finger, and it maketh 100: and beyond 100 toward the right hand, I set downe 3 the number of the joynt, all maketh 1003, which is the number

that is to be delivered you, from which abating 205, there resteth 753, which prefigureth unto you the seventh person, the fifth figure, and the third joynt.

But note, that when you have made your subtraction, if there doe remayne 0, in the place of tens, that is to say, in the second place, you must then abate 1, from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth finger, as if there should remayne 803, you must say, that the seventh person upon his tenth finger, and upon his third joynt, hath the Ring.

Another of three Dice.

If a man do cast 3 Dice, you may know the points of one of every of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same summe to multiply by 5, and unto the product adde the points of one of the other Dice; and behinde the number towards the right hand, to put the figure which signifieth the points of the last Die, and then to aske what number he keepeth, from which abate 250, and there will remayne 3 figures, which doe note unto you the points of every Die.

Another of things hidden.

If three divers things are to be hidden of

three divers perſons, and you to divine, which of the three perſons hath the three divers things, do thus: imagine the three things to be repreſented, *A*, *B*, *C*. Then ſecondly, keep well in your minde which of the perſons you meane to be the firſt, ſecond, and third. Then take 24 Counters or Stones, and your three things, and give *A* to the party whom you imagine to be your firſt man, and therewithall give him one of your 24 Counters in his hand, and *B*, unto your ſecond man, and therewithall 2 Counters. And *C*, unto your third man, and therewithall 3 Counters: and leave the reſt, which are 18, ſtill among them: which done ſeparate your ſelfe from them, and afterwards bid them change the things among them as they ſhall think good: which done, after they are agreed, bid him that hath ſuch a thing, as before you have repreſented by *A*, for every Counter that he hath in his hand, to take up as many more. And for him that hath *B*. For every one in his hand to take up two. And for him that hath *C*, for every one in his hand to take up 4, and the reſt of them to leave ſtill upon the board. Theſe 3 things, and the three perſons being fully printed in your minde, come to the Table, and you ſhall evermore finde one of theſe 6 numbers, 1, 2, 3, 4, 5, 6, or 7. If therefore one remayne ſtill upon the board, then have they made no exchange, but keepe them ſtill as they were delivered unto them. So that the firſt man hath *A*, the ſecond

B, and the third man *C*. But if 4 remayn, then the first man hath *B*, your second man *A*, and your third man *C*. The rest of the worke and the order thereof are here apparant by the Table following.

	1	*A*		1	*B*
1	2	*B*	5	2	*C*
	3	*C*		3	*A*
	1	*A*		1	*C*
2	2	*B*	6	2	*A*
	3	*C*		3	*B*
	1	*A*		1	*C*
2	2	*B*	7	2	*B*
	3	*C*		3	*A*

Another divination of a number upon the casting of two Dice.

First let the Caster cast both the Dice, and mark well the number: then let him take up one of them, it maketh no matter which, and look what number it hath in the bottom, and adde all together: then cast the Die againe, and keepe in his minde what altogether maketh: then let the Dice stand, and bring seven with you, and thereunto adde the rest of the pits that you see upon the upper side of the Dice, and so many did the caster cast in all.

FINIS.

An Appendix concerning the Resolution of the *Square* and *Cube* in Numbers, to the finding of their side, by *Ro. Hartwel.*

A Figurate Number is a number made by the multiplication of one number or more by another. A figurate number what.

The sides of a figurative number, are the numbers by whose multiplication it is made. The sides of a figurate number what.

A Figurate number is two-fold, as { *Plain.* / *Solid.* }

And it is { *Of one multiplication,* / *Or consequently of many,* } *as a* { *Plain.* / *Solid.* }

And in each { *Both Æquilater.* / *And Inequilater.* }

A figurate number made of one multiplication, by two sides or numbers multiplyed together, is called a plain figurate number.

For every number made by the mutuall multiplication of two numbers, may be called a *Plaine*, because it bringeth forth a right-angled parallelogramme, according to his unities disposed in *length* and *breadth*, the *sides* whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5 is called a *Plaine*, and the A plain figurate number.

the *ſides* thereof are 4 and 5 as here.

* * * * *
* * * * *
* * * * *
* * * * *

Becauſe the unities thereof diſpoſed in *length* and *breadth*, as the *ſides* do expreſſe, do bring forth an *inequaliter Parallelogram*, for that the numbers, or *ſides* are inequall.

By like reaſon 36 made by multiplication of 6 by 6, is called an *Æqualiter plain*, for the *ſides* thereof 6 and 6 are equall.

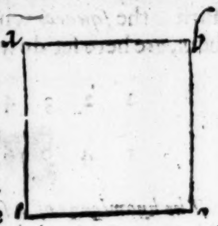
Moreover one and the ſame *plaine number* may have many *ſides*, as the *plaine number* 24, hath ſides 4 and ſixe: 3 and eight: 2 and 12. For it is produced from the mutuall multiplication of theſe numbers: whereupon for the invention of the ſides, to wit, in *inequilater Plaines*, it is needfull to give one of the *ſides*, by which the *plain* it ſelfe divided, the other *ſide* is made known. As the *plain* 48 being divided by the *ſide* 8, the *quotient* 6 is the remayning *ſide*. Notwithſtanding another reſolution and inquiſition doth happen in the *ſides* of the *Æquilater plains*.

An Equilater Plain or quadrat what.

An Æquilater plain is a number made by two equall ſides, or by any number multiplyed by it ſelfe. It is vulgarly called a ſquare or quadrat; by the Arabians Zenſus, *it is commonly expreſſed by this note z, by us q.*

A quadrat or *ſquare* in *Geometrie* is called a right lined *plain figure*, made by foure equall right lines, and ſo many right Angles, & every one of the lines is called the *ſide* of the *quadrat*,

drat, as this *figure a b c d* whose *side* is *a b*, or *b c*, as also *c d*, and *a d*.



To the similitude hereof, that number is called a *Quadrate*, which is made by the multiplication of two equall numbers, or of one in it selfe of which manner 36 is made, by 6 multiply in it selfe, or by the mutuall multiplication of 6 and 6. For if 36 unites be placed in plain forme, it bringeth forth a perfect *Geometricall Quadrat*, having in every side sixe unities as here.

The number whereof the Quadrat is produced by multiplication in it selfe, is called the side or root of the Quadrat. The side or root of a number what.

Concerning the extraction of the Quadrat or square Root.

THerefore to find the *Quadrat root*, or the *side* of any *Quadrat number*, is to search a number, which brought or multiplied in it self, maketh the number propounded: concerning the finding whereof, as it is requisite that the *sides* (being lesser then 10) of the *squares* under an hundred should be gathered by the Table of Multiplication: so the *sides* of the *grea-*

greater squares are to be sought out by *Art*. First, the *squares* whose *sides* are simple numbers, are here set down as you see.

The roots.	1	2	3	4	5	6	7	8	9
The squares.	1	4	9	16	25	36	49	64	81

The knowledge of a square is by finding out his side expressed by a whole number.

Although the finding out of the *side* of a *square* be applyed to each number given, as to a *square*, yet *square numbers* onely have a *side* to be expressed by a certain number of unites, or by rationall numbers, the other are to be expressed but onely in power. The *sides* are commonly called *Roots* by a *Metaphoricall phrase*.

The Root or side of a square is to be found by the Theorem following.

The side or root of a number.

4 If the odde degrees of a *square number* being marked from the right toward the left hand with points, you subduct from the number given, the particular *square* of the last *period*, setting the *side* thereof alone by it selfe.

2 Then going on, if you divide the remaynder (if ther be any) with the figure going before it, by the double of the side set alone by it selfe.

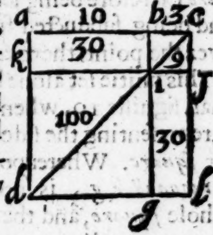
3 And multiply the *quotient*, found out (being placed by the side, which was first set alone by it selfe, and also before the doubled number

on

n the right hand)by both the numbers(name-y by the double number, and the Figure set by t selfe)being counted as one *divisor*, subducting he products from the given number, and then enue this last work of division so many times s there are pricks remayning, the *side* of the *square* shall be found out.

This artificiall device is taken out of the 4 *P.* 2 *of* Euclide. *Where by demonstration it is proved, that if a right line be cut into two* segments, *howsoever the* square *of the whole line is equall to the* squares *of the* segments, *and the two* right-angled figures *made of the* segments *as in the* figure *annexed, the two* Diagonals, *kg*, and *bf*, *are the* squares *of the* segments, *ab*, and *bc*. *Also the* complements *bk*, and *fg*, *are the* right-angled figures *made by multiplying the line ab*, by *bc*.

To extract the square root.



The first example.

The self-same parts are to be found in any *square number*. As for *example*, let the *number* be 169, whose *side* is 13. This *side* being divided into two pieces, 10 and 3, multiplying each piece by it selfe once, namely, 10 by 10, and 3 by 3: then multiply one by another, as 10 by 3, and 3 by 10, so shall you have foure *plain numbers*, whereof two are *square*, as here you see.

There-

Therefore as the *ſquare* 169 is made by adding together of theſe foure *plain numbers*, ſo by ſubducting them ſeverally it is reſolved.

10	3
10	3
100	
30	
30	
9	
169	

Firſt, therefore I marke each odde place with points, becauſe the *particular ſquares* are to be found in the odde places. Then for ſo much as the unity ſtanding under the firſt point next the left hand, and repreſenting the laſt *period*, is both a *ſquare* and the *ſide* of a *ſquare*: that *figure* therefore being ſet alone in the *quotient*, and being ſubducted from the unity ſtanding over the point, there remayneth nothing.

This unitie ſet alone by it ſelf in the *quotient*, ſhall ſignifie 10, when another *figure* is ſet by it, repreſenting the ſide of ſome other particular *ſquare*. Whereupon I ſay, that the greater *Diagonall k g*, is now ſubducted from the whole *ſquare*, and the ſide of it *k i*, or *a b*, (for they are equall one to another) and alſo the *ſide* of the *complement* is found out.

This is the firſt ſtep to this reſolution.

Moreover, I double the *figure* found out, becauſe being doubled, it is the ſide of both the *complements* taken joyntly together, namely, *k i*, and *g i*. Then ſetting 2 the doubled number under 6, I divide 6 (which in this place is as much

much as 60, and representeth both the *comple-ments*) by 2, the *quotient* is 3, representing the other *side* remayning of the *complement*, namely, *i f* or *b c*, which number I set in the *quotient*, and count it for the *segment remayning* of the right line given. Wherefore because this number 3 is the side of the remayning *Diagonall*, that is to say, of the lesser *square b f*, therefore being set by the *divisor* on the right hand, and multiplyed by it selfe, and also by the *divisor*, it bringeth forth three *plain numbers*, namely, the *square b f*, and the two *complements a i*, and *i l*, which being subducted from the numbers standing over them, there remayneth nothing.

The example is thus.

169 (13. *Which is all one, as if you had put downe the numbers found out in this manner.*
123
3
69

169
100 *The greater Diagonall.*
60 *The comple. twofold.*
9 *The lesser Diagonall.*
169

The subtilty of this invention is illustrated by many examples. The second example.

Let the *square* given be 1764. This number being marked with two points, telleth us, that the side thereof is to be written with two Figures.

First,

First therefore beginning at the point on the left hand, I seeke the side of the last period namely, 17. But for so much as it is no square number, I take 4 the side of the next lesser square, which I set alone by it selfe in the quotient, and then multiply it by it self, the *product* is 16, which being subducted from 17, there resteth 1. Moreover I double the side found out, the product is 8, I place this doubled number under 6, and by it divide 16 standing above it, the *quotient* is 2, which must be set by 4. This *quotient* 2 must be set before the *Divisor* 8 on the right hand under the point, and then must it be multiplyed both by it selfe, and into 8 the product is 164, which being subducted from the figures standing over them, there remaineth nothing: whereby I gather that the number given is a just *square*.

The Example standeth thus.

$$\begin{array}{l} \cancel{1} \\ \cancel{1}\cancel{7}\cancel{6}4 \;(42 \\ \hline 168\cancel{1} \\ \cancel{2} \\ \hline 164 \\ \hline 1764 \end{array}$$ *The Collection.*

The same manner of working is to be followed in greater square numbers given, saving that the former part of the worke is to be used but once,

once, but the later part is to be followed so many times as there are points remayning excepting the last.

As in 54756, I say, that the *side* of the *square* next unto 5 is 2: therefore 2 being set in the *quotient*, and multiplyed by it self, make 4, and taken from 5, the remaynder is 1. Moreover I double the *quotient*, the product is 4, which I set under the next *figure* toward the right hand, and thereby divide 14, the *quotient* is 3, which three being set both in the *quotient*, and also before the *Divisor* toward the right hand, I multiply both the numbers by it, the *product* is 129: this being subducted from 147 standing above it, the remaynder is 18. But because there is yet one point remayning, with which I have not medled; I therefore againe double all the whole *quotient*, for in this case I must take 23 for the side of one former *square*, and generally in great nunbers, when I light upon more particular *sqnares* then two, I must esteeme them but as two, and take the sides which are first found out, but as the *sides* of one onely *square*. Therefore twice 23 is 46: by this I divide 185, the number to be set in the *quotient* is 4, which number also must be set before the *Divisor* on the right hand: then must 464 be multiplyed by 4: the *product* is 1856, this *product* being subtracted from the numbers standing over it, there remayneth nothing. The example standeth thus.

The third Example.

Note.

The

$$
\begin{array}{l}
118 \\
54756(234 \\
44364 \\
129 \\
\hline
4 \\
1836 \\
54756
\end{array}
$$

The Collection.

See also the Example following.

1094 2864(3308.

Therefore out of this invention is this *consectarie.*

4 Example of a surd number. *The number whose side cannot be expressed by whole numbers, is not a square number.*

Such are all *prime numbers*, and the *squares* themselves excepted) all other *compound numbers*. For if in them you desire to finde out the *square side*, you shall labour in vain; because they are not *squares*, for to the whole numbers arising in the *quotient*, there will be some *fraction* adjoyned, whereby it commeth to passe, that the number of the *side* is not to be expressed by a true number, and it is commonly called a *surd number*.

Notwithstanding, if you adjoyne to the *side* out, the number remayning, taking his found ation from the double of the *side augmented*

denomin

mented by an unity, you shall finde the next side that may be like to the *side of a square*.

As if from 40 you take the nearest *square*, to wit, 36, the remaynder is 4. Here therefore the side sought for of the *square* exceedeth not the *side* found out by an unity, but either by one, or more parts of some whole number: wherefore I double 6, the side found out, and adde an unity to it being doubled, the totall is 13, this number I set under 4 the remaynder, and say that the side of 40 demanded as neere as may be, is $6\frac{4}{13}$: the *Denominator* of the *Fraction* being added to the greatest *square* in the number given namely unto 36, maketh the next greatest *square* above it, namely, 49, whose side is 7. But this surd side, to wit, $6\frac{4}{13}$, multiplyed by it selfe, maketh $39\frac{121}{169}$, which are not just equall to 40, the given number.

Judge the like concerning the rest which are not *squares*.

Thus much concerning plain figurate numbers, but especially such as are square numbers.

Concerning solid figurate Numbers.

A *Solid figurate Number is made of two multiplications by three numbers, or sides, multiplyed together, admitting length, breadth, and thicknesse.*

A solid figurare number.

Therefore every *number* made by the mutuall multiplication of three numbers, may be called a *solid*, because it bringeth forth *a right angled Parallelipipedon*, disposed according to his unities in *length*, *breadth*, and *thicknesse*, the *sides* whereof are the three multiplying *numbers*. As the *number* 30 made by the mutuall multiplication of 2, 3, and 5, is called an *Inequilater solid number*, and the sides thereof are 2, 3, and 5; because the unites thereof disposed by a certain distance one from another, in *length*, *breadth*, and *depth*: as the sides doe expresse, do bring forth resemblance an *Inequilater Parallelipipedon*, for that the *numbers* or sides are inequall.

By like reason 216 made by multiplication of 6 by 6, and the product thereof by 6, is called an *Equaliter solid*, for the sides thereof 66, and 6 are equall.

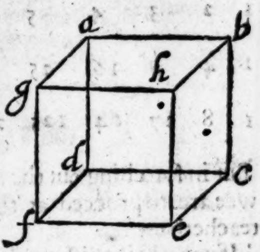
An Equilater Solid or Cube.

An Equilater, is a number made by three equall sides, or by any number multiplyed by it selfe, and that product againe by the foresaid number. And it is called an Equilater and Equiangled Parallelipidedon or Cube, and is commonly represented by us thus C.

A Cube in Geometry is a right-angled Parallelipipedon having six equall *surfaces*, and 8

solid

ſolid angle, and 12 ſides, as this figure *a. b. c. d. e. f. g. h.* whoſe ſide is *a b*, or *a d*, alſo *b c*, or *c d*, either *c e*, or *e f*, likewiſe *e h*, or *h g*, alſo *g f*, or *d f*, or *d a*, and *g a*.



The number whereof the Cube is produced by Multiplication in it ſelfe twice, is called the ſide or root of the Cube, which being found out in whole numbers, the Cube is known. The ſide or root of Cube.

Concerning the extraction of the Cubick Root.

THerefore every *Cube* in numbers hath ſuch a ſide as may be expreſſed in whole numbers, but in magnitudes it is not alwayes ſo, as indeed in magnitudes there are many things not to be expreſſed in *whole number*. Now for as much as the ſide of any *Cube* under 1000, is a ſimple figure, it is neceſſary, before we undertake to finde out the ſide of any great number, to know what *Cube* is made of each ſimple figure, and what is the ſide of any *Cube* leſſer then 1000, as I have here ſet them down.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	226	343	512	729

But in ſearching out the *ſides* of greater *Cubes*, wee are to proceed as the *theorem* following teacheth us.

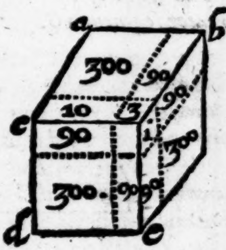
If you diſtinguiſh with *points* as it were into periods, the given *Cube* beginning at the firſt figure on the right hand, and omitting each two figures continually ; and firſt of all ſubduct the *particular Cube* of the laſt *period* from the given number, ſetting the *ſide* thereof in the *quotient:* and then ſet triple of the *quotient* under the *figure* next following the former *point*, on the right hand, and the *ſquare* of the *quotient* being tripled beneath it one *degree* more toward the left hand : and afterward divide the number above written by the triple of the *ſquare*, ſetting the *quotient* by it ſelf, and then multiply the *diviſor* by the *quotient* found out, and the tripled *ſquare* by the *ſquare* of the *quotient*, and the *quotient cubically*, ſubducting the *products* (ſo orderly added together, that each *figure* may anſwer the *numbers* whereof it was multiplied) from the *number* given, and renue this laſt manner of diviſion ſo many times as there are *points* remayning, the *ſide* of the *Cube* ſhall be found out.

This

This artificiall device is drawne out of that *theorem*, which *Ramus* made, imitating that of *Euclide* concerning *square numbers* in this this manner.

If a right line be cut into two segments, the Cube of the whole line shall be equall to the Cubes of the segments, and the two solid figures comprehended three times under the square of his segment, and the segment remayning. The extraction of the Cubick side or root.

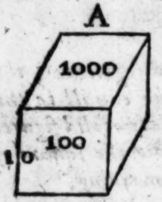
As the line *ci*, which is 13, is cut into two *segments*, 10 *and* 3 therefore the *Cube* of the whole line, namely 2197, is equall unto the *Cubes* of the *Segments*, namely unto 1000, and 27 also to *the two-fold Solids* or *Parallelipipedons thrice* taken,



whereof three have like *soliditie*, the *soliditie* of each of the three lesser is 90, being made of the *Square of the Segment* 3, that is to say of 9, multiplyed by the other *Segment* 10. These three *Parallelipipedons* joyntly taken together, make 270. But of the three greater *Parallelipipedons* each containeth 300, being made of 100, the *Square* of the greater *Segment* 10, multiplyed by the *lesser Segment* 3,

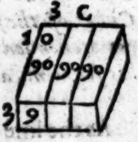
 and

and they being taken joyntly together, make 900.

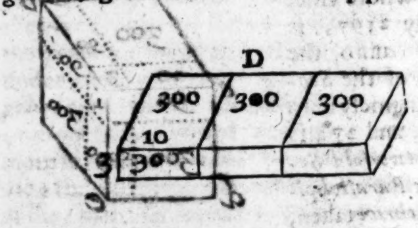


The Cube of the greater segment 10.

The Cube of the lesser segment 3.



The 3 *lesser Parallelipipedons.*



The three greater Parallelipipedons.

The *Cube* therefore hath eight *particular solids in number*, which are made of the parts of the number given, namely of 10 and 3 in this manner. First, let there be foure *plain numbers* made, each part being multiplied by it selfe; and one by another.

19	3
19	3
	9
	30
	30
	100

If againe I multiply the *Plaines* by the same parts, there will arise 8 *solids*, as you see here.

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	*All these being added*
90	300	*together, are equall to*
90	300	*the* Cube *of the whole,*
300	1000	*to wit,* 2197.

Therfore the same way that is kept in making the Cube, *it also to be followed in resolving the* Cube.

The first example to extract the Cubick root.

As for example, I marke the *Cube* given points in this manner, 2197

Then I subduct the particular *Cube* of the number set under the last point 2 but for so much as that number is no *Cube*, I take the neerest to it, namely, an unity, which also I set in the *quotient*. This unitie in the number given, is 100, but in the *quotient* it is but 20, the unite subducted from 2, the remaynder is 1, which must be written over the number given

given. So that the greater *Cube A*, is to be supposed to be subducted from the number given.

This is the first step of this work.

1
2197 (1
~~1~~

After I triple the *quotient* found out (that is to say, I multiply it by 3) this triple representeth the three *sides* (joyntly taken together) of the three lesser *solids* marked with *C*, I place the tripled number under 9. Again, I multiply the *quotient* square wise, and triple the *product*, which maketh likewise 3. This *product* resembleth the three *square sides* (taken joyntly together) of the three *greater solids*, marked with *D*, I place the *product* on a degree lower toward the left hand underneath 1. With it I divide 11, which written above it, the quotient is 3. This *segment* or *quotient* 3, being multiplyed by 3, the *Divisor* maketh 9, which in respect of the place wherein it standeth, is 900, and representeth the three greater *solids* marked with *D*, taken joyntly together. Furthermore the same *quotient* being multiplyed square wise, maketh 9, and multiplyed afterward by the *tripled number* standing under 9, it maketh 27, which in respect of the place wherein it standeth, is 270, and representeth the 3 *lesser solids* marked with *C*. Last of all, the

same

ſame *quotient* multiplyed *cubically*, breedeth the leſſer *Cube B*. Theſe 3 *products* therefore being added together, and the totall ſubducted from the *numbers* ſtanding over it, there remayneth nothing, which importeth the given number is a *Cube*.

The example is as you ſee.

1	2197 (13
2197 (13	1000 *The greater Cube.*
1 3	3
3	3
9 Or thus:	900 *The 3 greater Parallelipipe.*
27	270 *The leſſer Parallelipipedons.*
27	27 *The leſſer Cube.*
2197	2197

The matter may be explained by many examples.

Let the ſide of the given *Cube* 16387064, be ſought out, contrive it therefore (as it were) into certain periods with points. Then firſt of all, ſearch out the *ſide* of the *Cube* next to the left hand: But for as much as 16 is no *Cube*, take 2 the *ſide* of the next *Cube* under it, that is to ſay, of 8, and ſet in the *quotient*, and ſubduct 8 the *Cube* thereof from 16, there remayneth 8. The firſt work is not to be renued throughout the whole *number*, but the *rules* following muſt be repeated as often as there are *points* remayning. *The*

The ſecond example of the Cubick root.

The firſt ſtep to find out the root, is in this manner.

8
16387064 (2
8

Moreover, triple the *quotient* now found out, and the *product* is 6, which is to be placed under 8, namely, under the figure following the next prick toward the right hand. Then multiply the *quotient* by this tripled *number* (or which is all to one purpoſe, ſquare the *quotient*, and then triple the *product*) it maketh 12, ſet that number in a lower place one degree neerer the left hand, & make it the *diviſor*: divide 83 by 12, obſerving this rule in chooſing your *quotient*, that it be no greater, then that the numbers afterward produced by multiplication may not exceed the numbers ſtanding over it. So that here you ſhal take 1 in 8, but 5 times. Afterward by this number 5, multiply the *diviſor* 12, and by the ſquare of 5, multiply the tripled number 6, and laſt of all multiply 5 cubically: ſo ſhall you produce three numbers, namely, 60, 150, 025, to be deſcribed in ſuch ſort as you ſee. Theſe numbers added together, and ſubducted from 8387, the remayner is 762.

The ſecond ſtep to finde out the root, in this manner.

8762
16387064 (25

6

12

60
150
125

7625

And because there is yet one point remayning, this last manner of Division must be wrought again.

First, therefore I triple the *quotient*, the product is 75. which must be so placed, that the first figure thereof, namely 5, may stand under 6, the second under the 0. Again, multiply the *quotient* by this *tripled number* (or which is all one, square the quotient, & triple the product) it maketh 1875, which must be the *Divisor*, whose first figure namely 5, must be placed under 7, the *last figure* of the *tripled number*. Then see that 1 may be contained in 7, many times, but I can take it but 4 times, I set 4 in the quotient, and multiply the *Divisor*, by 4, the product is 7500, afterward I square 4, it maketh 16 which I multiply by the tripled number 75, the product is 1200. Last of all I multiply 4 *cubically*, it maketh 64, these products added all

toge-

together, make 762064, which number being subducted from the *Cube* given, there remayneth nothing, whereby I gather that the number given it exactly *cubicall.*

The third step to finde out the side is in this manner.

762
16387064 (254

75
1875

7500
1200
64

762064

The third example of the Cubick root.

Behold also the example following.

614125000 (850

Another manner of working.

HItherto the Princely high-way to finde out the side of the Cube hath been declared.

But there are moreover certain other wayes also bending thereto, and leaning to the same principles, whereof this is one.

The second form

1 Having found out in the Table of simple *Cubes*, the first figure representing the side of the *cube* contained in the number standing under the first *point* on the left hand, set it in the *quotient,*

quotient, & ſubduct the particular *Cube* of that figure as you did before: then *ſquare* that *figure*, and triple that *ſquare*, the *product* ſhall be the *Diviſor*, the firſt *figure* whereof ſhall be ſet under that *figure* which is on the right hand next of all to the point (now examined) before going.

See how many times the *Diviſor* is contained in the number written over it, and multiply the *Diviſor* in the *quotient*, and ſubduct the *product* from the *dividend*: yet here you muſt take heed, that you take not a greater *quotient*, then that the *products* made afterward thereby may be ſubducted from the *number* given. 2

The ſubduction being done, triple the firſt *figure* which was ſet in the *quotient*, and adde to the triple the laſt *number* which was ſet in the *quotient* on the right hand of the *product*. 3

This totall multiplyed by the ſquare of the *figure* laſt found out, and ſet down the *product* ſo, that the firſt *figure* thereof toward the right hand may ſtand under the *point* next before going on the ſame hand, and finally, ſubduct the ſame from the *number* given. 4

As in 804357. The particular *Cube*, namely, 729 being taken from the number ſtanding under the laſt *period* upon the left hand, there remayneth 75357, the ſide of that particular *Cube* being 9, I ſet in the *quotient*. Then I *ſquare* that *ſide*, it maketh 81, and triple the *ſquare*, the *product* 243 is my *Diviſor*, which I ſet under the given number, ſo that 3 may ſtand under 3 with

The fourth example of the Cubick root.

with this *Divisor*, divide the number standi
over it, you shall finde 2 to be contained i
three times. Therefore I set 3 in the quotie
and multiply the *Divisor* by it, the *product*
729, which being subducted from 753, the r
maynder 24.

The Induction is thus:

```
        2
      754
   804357 (93
      243
      729
```

Moreover I triple 9, the product is 27, b
which on the right hand I set 3 the quotie
last found out, the totall is 273.

This *number* I multiply by 9 the square o
the quotient last found out, the product shall b
2457, which being subducted from the sup
riour number, there remayneth nothing.

The Induction is thus:

```
      24
   804357 (93
      732
        9
     2457
```

Another

Another manner.

THe self-same work may be dispatched another way, a little differing from the former in this manner.

The figure in the *quotient*, being found out by subducting the particular *Cube*, and also the second figure in the quotient being found by division, let the totall quotient be tripled, and let the tripled number be multiplyed by the former figure in the quotient. Then let the product be multiplyed againe, by the latter figure found out, and let a *cipher* be set on the right hand of that *product*. Last of all, let the *Cube*, of the latter figure found out, be added to this *product*, and let the totall summe be subducted from the number given. As in 373248. The third forme.

The first induction is in this manner.

30
373 248 (7

343

Moreover I *square* the side found out, it maketh 49, and triple the square, the *product* is 147 which shall be the *divisor*, by this I divide 302, the number written over it, the *quotient* is 2. Now I triple the *totall quotient* 72, it maketh 216, and multiply this triple by 7, the former *figure* in the *quotient*, the *product* is 1512. I multiply this *product* also by 2, the latter *figure* of the *quotient*, and set a *cipher* on the right hand of it, so as it maketh 30240, unto this number last of all I adde 8, the *Cube* The fifth example.

of

of the latter *figure* found out, the total is 3024
which being subducted from the *figure* abov
it, there remayneth nothing.

The second Induction is thus.

30
~~373~~248 (72

147

30248

All the *points* of the *number* given being ex-
amined, if any thing remayne, it signifieth th
number given is no *Cube:* wherfore the tru
side of it cannot be exactly given in *numbers*
Yet if it please you to sift out the neerest *sid*
that may be, by the first kinde of reduction o
mixt numbers, you shall reduce the *number* gi-
ven unto a *cubicall* fraction of a greater *deno-
mination*, and afterward seeke out the *cubica*
side of that *fraction*.

To finde the neerest Cubick root in a surd number.

For example sake, because 120 is no *Cube*
therfore let it be reduced into sixty *cubical*
parts, after this manner. Multiply 60 *cubically*
in it self, it maketh 216000, by this being ta-
ken for the *denominator* of the *fraction*, multi-
ply 120 the *number* given, the *product* is
25920000 whose *cubicall side* is $\frac{295}{60}$ that is $4\frac{11}{12}$
the neerest to the true *side* that can be.

*For the extraction of all sorts of roots, the table
of Logarithmes set forth by* Mr. Briggs *are most
excellent, and ready.*

FINIS.

A Table of Board and Timber measure, more perfect then ever hath beene made; shewing also the Squares betweene 4 and 37 from quarter to quarter, calculated by Robert Hartwell.

Board measure.	Inches & quarters.	Squares.	Timber measure	Board measure	Inches & quarters.	Squares.	Timber measure
36.0.0	4	16	08.0.0	16.0.0	9	81	21.3.3
33.8.8	1	18	06.0.0	15.5.6	1	85	20.3.3
32.0.0	2	2	86.4.0	15.1.6	2	90	19.2.0
30.3.1	3	2	78.5.4	14.7.7	3	95	18.1.8
28.8.0	5	20	69.1.2	14.4.0	10	100	17.2.8
27.4.3	1	27	64.0.0	14.0.5	1	10	16.4.6
25.1.8	2	5	57 6.0	13.7.5	2	11	15.7.1
25.0.4	3	38	52.3.6	13.3 9	3	115	15.0.2
24.0.0	6	36	48.0.0	13.0.9	11	121	14.2.8
23.0.4	1	39	44.3.0	12.8.0	1	126	13.7.1
22.1.5	2	42	41.1.4	12.5.3	2	132	13.0.9
21.3.3	3	45	38.4.1	12.2.7	3	138	12.5.2
20.5.7	7	49	35.2.6	12.0.0	12	144	12.0.0
19.8 6	1	52	33.2.3	11.7.5	1	150	11.5.1
19.2.0	2	56	30.8.6	11.5.2	2	156	11.0.7
18.5.8	3	60	28.8.0	11.2.9	3	162	10.6.6
18.0.0	8	64	27.0.0	11.0.7	13	169	10 2.2
17 4.6	1	68	25.4 1	10.8.7	1	175	9.8.7
16.9.4	2	73	24.0.0	10.6.7	2	182	9.4 9
16.4.6	3	76	22.7.5	10.4.7	3	189	9.1.4

Board measure	Inches & quarters.	Squares.	Timber measure	Board measure.	Inches & quarters.	Squares.	Timber measure
10.28.	14	169	8.8.1	6.8.6	21	441	3:2:2
10.1.1	1	203	8.5.1	6.7.7	1	451	3:8:2
9.9.3	2	210	8.2.3	6.6.6	2	462	3:7:3
9.7.6	3	217	7.9.6	6.6.1	3	473	3:6:5
9.6.0	15	225	7.6.8	6.5.4	22	484	3:5:7
9.4.5	1	232	7.4.3	6.4.7	1	495	3:4:9
9.2.9	2	240	7.2.0	6.4.0	2	506	3:4:1
9.1.4	3	248	6.9.7	6.3.3	3	517	3:3:3
9.0.0	16	256	6.7.5	6.2.6	23	529	3:2:7
8.8.6	1	267	6.5.4	6.1.9	1	540	3:2:0
8.7.3	2	272	6.3.5	6.1.2	2	552	3:1:3
8.6.0	3	280	6.1.6	6.0.6	3	564	3:0:6
8.4.7	17	289	5.9.8	6.0.0	24	576	3:0:0
8.3.5	1	297	5.8.1	5.9.4	1	588	2:9:4
8.2.3	2	306	5.6.4	5.8.8	2	600	2:8:8
8.1.1	3	315	5.4.8	5.8.2	3	612	2:8:2
8.0.0	18	324	5.3.3	5.7.6	25	625	2:7:6
7.8.9	1	333	5.1.8	5.7.0	1	637	2:7:1
7.7.8	2	342	5.0.5	5.0.5	2	650	2:6:5
7.6.8	3	351	4.9.2	5.5.8	3	662	2:6:0
7.5.8	19	361	4.7.8	5.5.4	26	676	2:5:5
7.4.8	1	370	4.6.7	5.4.8	1	689	2:7:0
7.3.9	2	380	4.5.5	5.4.3	2	702	2:4:7
7.2.9	3	390	4.4.3	5.3.8	3	715	2:4:1
7.2.0	20	400	4.3.2	5.3.3	27	729	2:3:8
7.1.1	1	410	4.2.1	5.2.8	1	742	2:3:2
6.0.2	2	420	4.1.1	5.2.3	2	756	2:2:7
6.9.3	3	431	4.0.1	5.1.8	3	767	1:2:4

Board measure.	Inches & quarters.	Squares.	Timber measure	Board measure	Inches & quarters.	Squares.	Timber measure
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.9	4.3.0	2	1120	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1136	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.4.9
4.9.2	1	855	2.0.2	4.2.0	1	1174	1.4.7
4.8.8	2	871	1.9.8	4.1.7	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1210	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1237	1.4.0
4.7.2	2	930	1.8.6	4.0.5	2	1247	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1280	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	974	1.7.7	3.9.7	1	1313	1.3.1
4.5.7	2	987	1.7.5	3.9.4	2	1331	1.2.9
4.5.3	3	1000	1.7.2	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	37	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.2	2	1056	1.6.3	3.8.4	2	1416	1.2.2
4.4.0	3	1072	1.6.1	3.8.2	3	1425	1.2.0

The use of this former Table.

IF upon a *Scale or Ruler* you divide one *inch* into ten *equall parts or primes*, and againe by *diagonals* and *parallell lines*, you subdivide each of them into ten *equall parts or seconds*, with your *compasses*, you may take of more exact running measure for *board and timber*, then then by any other meanes whatsoever, and so place the same, or this *Table* if you will, upon any *Ruler*.

Also by meanes of the *columnes of Squares*, you may readily finde a *square* equall to any *Parallelipipedon* or piece of *timber*, which is thicker then it is broad. As for example, suppose a piece of *timber* to be ten *inches thicke*, and 9 *inches broad*: if I multiply those *sides* one by another, they will produce 290. then seeking the *columne of squares* for 290, which I finde not; but I finde 289 the neerest *number* to 290, to stand against 17: therefore I say 17 *inches fere*, will make a square equall to such an unlike *squared piece*: then looking in the *columne of timber measure* against 17, you shall finde that 5 *inches*, 9 *prime*, or $\frac{9}{10}$, and 8 *seconds*, or

$\frac{18}{100}$ of an *inch in length* of that piece will make a *foot of timber*.

Likewise for *board measure*, you may finde how much in *length* of *breadth* of *board* must be in one *foot*.

By the like meanes suppose *for example* that a *board*, appointed to be measured, is 15 *inches broad*, if I desire to know how much in *length* thereof will make a *foot*, I seek in the *columnes* that stand under *unites* and *quarters*, for 15 $\frac{1}{4}$, and also against the same in the *columne* under the title of *board measure*, where I finde 9 *inches* 1 *prime* or *tenth* of an *inch*, and 4 *seconds* or *hundreds* of *an inch* will make a *foot at* that *breadth*: the like may be practised for any other *breadth of board* whatsoever.

Certain *Tables* shewing the Interest of an *summe of money* whatsoever unto 40 yeeres how much *Annuities* respited or forborn commeth unto. And for *buying* or *selling* of *Annuities* for the said time; and also the same i *reversion* after any *number of yeeres* unto 3 What they may be worth in present ready money, by *R. C.* and now diligently correcte and amended by *Robert Hartwell*.

Definition of Interest.

PRincipall, *is the summe from which the* Interest *is reckoned.*

2 Interest *is the summe reckoned for the lending or forbearance of the* Principall *for an termes or time.*

3 Interest simple *is that which is counted fro the* Principall onely.

4 Interest compound *is that which is counte for the* Principall, *together with the* Arrerage.

5 Interest profitable *is that which is adde to the* Principall.

6 Interest Damageable *is that which is to b subtracted from the* Principall.

The use *per annum* of	1 li.	is	2 s.
	10 s.		12 d.
	5 s.		6 d.
	2 s. 6 d.		d.
	12		1 d. $\frac{1}{3}$ of a peny.

A Table shewing what 1 li. *with interest, and nterest upon interest after* 10 *in the* 100 *comes to very yeer under* 41 *yeers. As followeth.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	1	2	0	7	8	0	21
2	1	4	2	8	2	9	22
3	1	6	7	8	19	1	23
4	1	9	3	9	16	11	24
5	1	12	2	10	16	8	25
6	1	15	5	11	18	4	26
7	1	18	11	13	2	2	27
8	2	2	10	14	8	5	28
9	2	7	1	15	17	3	29
10	2	11	10	17	8	11	30
11	2	17	1	19	3	10	31
12	3	2	9	21	2	3	32
13	3	9	0	23	4	6	33
14	3	15	11	25	10	11	34
15	4	3	6	28	2	0	35
16	4	11	10	30	18	0	36
17	5	1	1	34	0	0	37
18	5	11	2	39	8	1	38
19	6	2	3	41	2	10	39
20	6	14	6	45	5	2	40

By the former Table, if you deſire to know what 1 li. commeth to with *intereſt*, and *intereſt* upon *intereſt* after 10 in the 100 for any *number* of yeeres unto 40. Look in the *row* or *margent* (over which is written *yeeres*) and againſt it on the right hand cloſe unto it in the *row or margent of pounds, ſhillings, and pence*, (which is titled thus, li.s.d. you ſhall finde your deſire.

Example.

I would know what 1 li. *with intereſt, and intereſt upon intereſt commeth to in* 7 *yeers?*

I look in the row of yeers for the *number* 7. And againſt it on the right hand I finde 1 li. 18 s. 11 d. Alſo what it commeth unto in 13 yeers. I ſeeke among the yeers for 13, and againſt it I finde 3 li. 9 s.

Again, for 21 yeers. I looke for 21 among the yeers, and I finde 7 li. 8 s. 0 d. But if you would know for a greater ſum then 1 li. Then multiply your ſumme by that ſumme of 1 li. in the Table for any of thoſe yeers, and you ſhall eaſily finde it. As thus, I would know what 10 li. commeth to for 7 yeers with intereſt, &c. I ſee that 1 li. commeth to 1 li. 18 s. 11 d. in that time. Then ſay I that 10 li. muſt be 10 times as much in that ſpace, which is 19 li. 9 s. 2 d. Alſo of 10 li. in 13 yeers. I ſee that 1 li. in that time commeth unto 3 li. 9 s. Then muſt 10 li. be 10 times as much in that ſpace, which

is

is 34 li. 10 s. Alſo what 10 li. commeth to in 21 yeers. I finde firſt that 1 li. in that ſpace commeth to is 7 li. 8 s. Then I ſay 10 muſt be 10 times as much, which is 74 li. Laſtly, I would know what 100 li. commeth to in 7 yeers, I ſee it muſt be 100 times as much as 1 li. commeth to in that ſpace, which is 194 li. 11 s. 8 d. Hereby you ſee the common ſaying is not true, that 100 li. doth double it ſelfe in 7 yeers, for it wants thereof 5 li. 8 s. 4 d. But in 8 yeeres 100 li. commeth to 210 li. 8 s. 4 d. Which you ſee is more then double it ſelfe by 10 li. 8 s. 4 d. And in this ſort may any that can but caſt with Counters, or indeed by memory finde the increaſe of any ſumme whatſoever for any of the number of yeers in the foreſaid Table, after they have found what 1 li. commeth unto for that time, as before is ſpecified.

A Table ſhewing if 1 li. *annuitie to endure for any number of yeers, under* 41, *be all reſpited or forborn, untill the laſt payment grow due, and then all be received together: with intereſt, and intereſt upon intereſt after* 10 *in the* 100 per annum,

annum, *what they will amount unto by any of the said number of yeers. As followeth.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	1	0	0	64	0	0	21
2	2	2	0	71	8	0	22
3	3	6	2	79	10	10	23
4	4	12	10	88	9	11	24
5	6	2	1	98	6	11	25
6	7	14	3	109	3	7	26
7	9	9	8	121	1	11	27
8	11	8	8	134	4	2	28
9	13	11	7	148	12	7	29
10	15	18	8	164	9	10	30
11	18	10	7	181	18	10	31
12	21	7	8	201	2	9	32
13	24	10	5	221	5	0	33
14	27	19	5	245	9	6	34
15	31	15	9	271	0	5	35
16	35	18	11	299	2	6	36
17	40	10	10	330	0	9	37
18	45	11	11	364	0	10	38
19	51	3	2	401	8	11	39
20	57	0	6	442	11	10	40

By this Table you may know what any Annuitie being respited or forborn for any number of yeers unto 41. with interest upon interest, after ten in the 100 will come unto: first seeking in the *Table* what 1 li. will come unto: in that time, and that being found to multiply it by the summe you desire to know.

Example.

First, I would know what 1 li. *Annuitie being forborn or respited for* 14 *yeers commeth unto.*

I looke in this last Table (which is for purpose, and I finde 27 li. 19 s. 5 d.

Againe, what 1 li. *Annuitie* respited for 21 *yeeres* cummeth to, I looke in the said Table for 21 *yeeres*, and I finde 64 li. Also the like for 1 li. for 30 *yeeres* respited. I look, and I finde it to be 164 li. 9 s. 10 d. as by the said Table may appeare. Now for greater *Annuites*, as 30 li. *per annum*, respited or forborn, what it amounteth to in 16 *yeers*. I seek first for 1 li. in this last *Table* before for 16 *yeers*, and against it I finde 35 li. 18 s. 11 d. Then say I, that 30 li. *per annum* being respited for that time, will come to 30 times as much, which is 1078 li. 7 s. 6 d. Also if there be an *Annuity* of 45 li. due and unpayed for 12 *yeers*, I looke in the said Table what 1 li. commeth to, 12 yeeres being respited, and I finde it is 21 li. 7 s. 8 d. Then I conclude that

5 li.

5 li. must be 45 times as much, which is 962 li. 5 s.

Lastly, I have an Annuity of 50 li. per annum, *which hath been behinde for* 16 *yeers, and must be answered unto me with interest, and interest upon interest, all at one payment, what shall or ought I to receive in all at the* 16 *yeers end?*

I seek what 1 li. comes unto in that time (as before taught) and I finde 35 li. 18 s. 11 d. Then must my 50 li. *per annum* forborne for that time, come to 50 times as much, which is 1797 li. 6 s. 10 d. And thus may you finde any other *summe* great or small, for any *number* of *yeeres* contained in the foresaid *Table*, without the helpe of *Arithmeticke*, if you can but use your *Counters*, or by memory count well.

A Table shewing if 1 li. *Annuity (to indure for any number of yeeres unto* 41) *be to be sold for present ready money, how much ought that ready money to be, reckoning* 10 per 100 per annum

annum *abating interest, and interest upon interest. As followeth.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	0	18	2	8	12	11	21
2	1	14	8	8	15	5	22
3	2	9	8	8	17	7	23
4	3	3	4	8	19	8	24
5	3	15	9	9	1	6	25
6	4	7	1	9	3	0	26
7	4	18	4	9	4	8	27
8	5	6	8	9	6	1	28
9	5	15	2	9	7	4	29
10	6	2	10	9	8	6	30
11	6	9	9	9	9	7	31
12	6	16	3	9	10	6	32
13	7	2	0	9	11	4	33
14	7	7	4	9	12	2	34
15	7	12	1	9	12	10	35
16	7	16	5	9	13	6	36
17	8	0	5	9	14	1	37
18	8	4	0	9	14	7	38
19	8	7	3	9	15	1	39
20	8	10	3	9	15	6	40

This Table before last specified is very necessary and commodious for all *Gentlemen* or others, that shall have cause to buy or sell *Annuities* or such like, for by this they shall know what they do, whether they demand, or take too little or too much, after the rate of ten in the 100, by which proportion all these *Tables* are ruled.

As for example, I am to buy an Annuitie of 16 li. per annum, *for* 12 *yeers, and am demanded for it ready money* 120 li. *I would know, if I give this rate, whether I give too much or too little, according to the proportion of* 10 *in the* 100 per annum, *&c.*

I looke in the *Table* last before what 1 li. is worth for 12 *yeers*, and I finde against 12 this *summe* 6 li 16 s. 3 d. Now I say that 16 li. *Annuitie* for that time, and after that proportion commeth to 16 times as much, which is 100 li. So that I see the party demanded of mee 11 li. too much after the rate of 10 in the 100 *per annum*, and therefore I must draw him to a lower price, or leave it.

Again, I am offered an Annuity of 20 li. per annum *of* 14 *yeers for* 130 li. *I would know if I give it, whether I give too much or too little, according to the proportion aforesaid.*

I seek first what 1 li. *Annuitie* is worth for 14 *yeers*, and I finde in the said last *Table* 7 li. 7 s. 4 d. Then say that the *Annuitie* of 20 li. *per annum*, will come to 20 times as much, and will be worth 147 li. 6 s. 8 d. according to

the

the proportion before mentioned:and is more then his demand by 17 li. 6 s. 8 d. So that I see, if I accept of it, I shall have a good bargaine. And thus may you know readily by looking in your Table, and finding what 1 li. is worth for any time therein contained, how much any greater summe will come unto, if you multiply it by that summe of 1 li. as before is sufficiently shewed.

But suppose this I have 300 li. *ready money, and would bestow the same for a valuable Annuitie answerable thereunto according to the proportion aforesaid. I would know what Annuitie to endure* 21 *yeers this* 300 li. *will buy?*

I looke in the former Table what 1 li. Annuity will cost for that time, and I finde 8 li. 12 s. 11 d. Then I say by the *Rule of Proportion.* If 8 li. 12 s. 11 d. will buy 1 li. Annuity for 21 yeeres: what Annuity shall 300 li. buy or be worth for that time, I reduced the *summes* to least denomination (which is pence) and I finde 34 li. 10 s. 9 d. And after this manner (by the helpe of this *rule*) may you finde all other summes for any time contained in the foresaid last *Table*.

An-

A Table shewing what 1 li. *in reversion for any number of yeers under* 31 *is worth in ready money, the buyer staying untill the thing be falne in hand.*

yeers.	li.	s.	d.	li.	s.	d.	yeers.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	3	29
15	0	4	9	0	1	1	30

This

This last *Table* differeth, and is contrary to the other three before mentioned. For whereas the others increased more and more according to the *number of yeers* specified. This doth grow and diminish lesse and lesse, as the *number of yeers* increaseth. As for example.

There is a Tenement, the fee simple whereof after 7 yeeres will be worth 40 li. *what am I to give for it in ready money, now staying untill it fall in hand.*

To know this I looke in this last Table for 7 yeers, and against it I finde 10 s. 3 d. So that a thing that after 7 yeers will be worth 1 li. is worth now in ready money but 10 s. 3 d. Then say I that the foresaid Tenement (which after 7 yeers will be worth 40 li.) is now worth 40 times 10 s. 3 d. which is 20 li. 10 s.

Againe, there is Farme which after 9 yeeres will be worth the Fee-simple 420 li. *what is it now worth in ready money, staying untill it fall in hand.*

I looke in the said Table what 1 li. is worth in Reversion after 9 yeers, and I finde 8 s. 5 d. Then say I, that the *Farme* of 420 li. so long in Reversion, will be now worth in ready mony, 420 times as much, which is 176 li. 15 s.

Lastly, there is a Lordship to be sold, the Fee-simple whereof after 14 *yeers will be worth* 7500

li. *I would know what the same is now worth in ready money for the Reversion.*

I looke in this last *Table* for 14 yeeres, and against it I finde 5 s. 3 d. so much 1 li. is worth in reversion after 14 yeeres. Then say I, that 7500 li. is worth no more in reversion for that time then 7500 times 5 s. 3 d. which is 1968 li. 15 s. And after this manner may you finde out any other other summe whatsoever. And though some men of their owne experience can ayme (as they think) neer enough the mark to serve their owne turns: yet I dare undertake they shall never so exactly doe it, nor justifie what they doe, as if they did it by Art.

FINIS.

New *Tables* of *Interest* at 8 *per centum per annum*, exactly calculated for 30 yeeres by *Robert Hartwell*, with necessary *questions* for the use of them.

The first Table expressing the increase of one pound principall, put out and forborn for any number of yeers under 31, at 8 per centum, per annum.

yeers.	li.	s.	d.	q.	li.	s.	d.	q.	yeers.
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	0	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	2	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	0	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	1	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The deſcriptiou and uſe of the Tables *of Intereſt at* 8 per annum, *being profitable.*

The firſt of them.

THeſe Tables conſiſt of foure *Columnes*, in the firſt and fourth whereof is written over the head, yeeres, and under the firſt number of yeeres deſcending from 1 to 15, likewiſe in the fourth the number of yeers deſcending from 16 to 30. And againſt every yeere in the ſecond Columne toward the right hand the pounds, ſhillings, pence and farthings, which one pound, or 20 s. principall will amount unto, being put forth and forborn for the number of yeeres ſet againſt it; (but the pounds, ſhillings, pence, &c. in the third Columne belongeth to the yeers ſet in the laſt Columne.

1 Example.

Let it be required what one pound or 20 *ſhillings, being put forth and forborn for* 12 *yeers ariſeth to at* 8 per 100, per annum, *intereſt upon intereſt.*

Seek in the firſt Columne under the title of yeers, for 12 the number of yeers propoſed in the queſtion, and right againſt it toward the right hand in the ſecond Columne, you ſhall find 2 li—10 s—4 d—1 q. which is the principall and increaſe thereof due for the time required.

2 Example.

If 100 li. *be put forth for* 17 *yeers according to*

to the same interest, I demand what it will amount to in that time?

Look in the Columne under the title of yeers for 17, and right against it towards the left hand in the Table is found 3 li.—14 s—0 d.—0 q, which is the increase of 1 li. by which you may thus gather the increase of 100 li. or any other *summe*; a hnndred times 3 li. is 300 li. then 100 times 14 *shillings* is 70 li. both which added together do make 370 li.—0 s.—0 d. which is the *increase* of 100 li. put forth and forborn 17 *yeers* the solution to the question.

li.	s.	d.	q
300	0	0	0
70	0	0	0
370	0	0	0

3 Example.

Suppose 60 li. *be put forth for 19 yeers according to that rate, what will it increase to in that time.*

Seek 19 under the *title* of yeers, and against it toward the left hand is found 4 li—6 s—3 d—3 q now say 60 times 4 li. is 240, and 60 times 6 *shillings* is 360 *shillings*, or 18 li. and 60 times 3 d. is 180 d. or 15 *shillings*, and 60 times 3 *farthings* is 3 *shillings* 9 d. all which added together make 258 li. 18 s. 9 d. the increase thereof demanded.

li.	s	d	q
240	0	0	0
18	0	0	0
0	15	0	0
	3	9	0
258	18	9	0

to the same interest, I demand what it will amount to in that time?

The second Table *shewing what one pound annuity, or yeerly rent is worth at the end of any number of yeers under 31, being forborne, at* 8 per centum, per annum.

yeers	li.	s.	d	q.	li.	s.	d.	q.	yeers
1	1	0	0	0	30	6	5	3	16
2	2	1	7	0	33	15	0	0	17
3	3	4	1	0	37	9	0	0	18
4	4	10	11	2	41	8	11	0	19
5	5	17	3	3	45	15	2	3	20
6	7	6	8	0	50	8	5	2	21
7	8	18	5	1	54	9	1	2	22
8	10	12	8	3	60	17	10	1	23
9	12	9	9	0	66	15	3	2	24
10	14	9	8	3	73	2	1	1	25
11	16	12	10	3	79	19	1	0	26
12	18	19	0	2	87	7	0	0	27
13	21	19	10	3	95	6	1	2	28
14	24	4	3	2	113	19	3	3	29
15	27	3	0	2	[illegible]	5	7	3	30

The use of the second Table, *(whose disposition is altogether like the former) according to the title thereof, being profitable.*

1 Example.

There is a Lease worth 28 li. per annum, *to endure* 14 *yeers, I demand what it will rise unto at the end of those yeers, I demand what it will rise unto at the end of those yeeres, being all forborn with the interest upon interest at the rate prescribed in this* Table.

Look in the third Table for 14 yeers, against which toward the right hand, you shall finde 24 li—4 s—3 d—2 q. Now multiply 28 li. by 24 there ariseth 672 li. then 28 li. by 4 s. yieldeth 112 s. or 5 li. 2 s. Again 28 li. by 3 d. produceth 84 d. or 7 s. finally, 28 by 2 farthings yieldeth 56 farthings or 1 s. 2 d. All which added together make

li.	s	d	q
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	10	2	0

678 li-0 s. 2 d. to be received at the end of 14 yeers, the same rent or annuity being respited.

2 Example.

If 60 li. *yeerly rent or annuity be forborn* 20 *yeers: I demand how much it will increase at the end of the said terme?*

In

In the Table I find that 1 pound in 20 years will arise to 45 li.--15 s--2 d--3 q. therefore 60 li. in the like terme will yield 60 times as much; which I will reckon thus: 60 times 45 li. is 2700 li. 60 times 15 s. is 900 s. or 45 l. 60 times 2 d. is 120 d. or 10 s. last of all, 60 times 3 q. is 180 farthings, or 3 s. --2 d. all which together amount unto 2745 li——13 s.——9 d. the value thereof to be received at the end of the terme.

li	s	d	q
2700	0	0	0
45	0	0	0
10		0	0
3		9	0
2745	13	9	0

3 Example.

The yearly rent of 1 li--13--4 d. *being behind and unpaid the space of 7 yeeres at the end of which terme the Tenant is compelled to pay the same with the interest thereof according to the above named rate. I demand what the payment ought to be?*

The increase of 1 li. *yeerely rent* answering to 7 *yeers*, is 8 li-18 s-5 d-1 q. which for 6 li. *rent* is to be taken 6 times, which ariseth to 53 li-10 s-7 d-1 q, now because 13 s--4 d. is two third parts of 1 li. therefore I take $\frac{2}{3}$ of 8 li. 18 s-5 d-1 q, which is the increase of 1 li. forborn for 7 *yeers*, that is 5 li-18 s-11 d-2 q, which together make 59 li-9 s-6 d-3 q, the *summe* to be received, as was required.

li	s	d	q
35	10	7	q
5	18	11	2
59	9	6	3

The third Table declaring what one pound due at the end of any number of yeers under 31 is worth ready money at 8 per centum, per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	0	18	7	0	0	0	10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	0	0	3	11	2	21
7	0	11	8	1	0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	3	0	2	8	1	26
12	0	7	11	1	0	2	6	0	27
13	0	7	4	0	0	2	3	3	28
14	0	6	9	2	0	2	1	3	29
15	0	6	3	3	0	1	11	3	30

This

This third Table *is diſpoſed as the firſt, the uſe according to the Title thereof, being damageable.*

1 Example.

Suppoſe there is 750 li. *due to be payed at the end of 9 yeers, the Creditour would ſell this debt for preſent money, what ought that money to be at the rate deſcribed in the* Table.

Seeke in this third Table for 9 yeeres at the left ſide of the Table, and right againſt it toward the right hand, you ſhall finde 10 ſhillings, which multiplyed or taken 750 times, yieldeth 7500 ſhillings, which is 375 li. the value of that debt in preſent money.

2 Example.

There is a Leaſe worth 500 li. after the end of 7 yeers; what is it worth preſent money, according to the rate deſcribed in the table ſtaying till it fall.

I ſeek in the Table for the 7 yeers, and right againſt it I finde 11 s—8 d; now I multiply 5500 by 11, it yieldeth 5500 ſhillings, or 275 li. then 500 times 8 d. maketh 4000 d, which is 16 li-13 s-4 d, which added together is 291 li-13 s-4 d. the value of the Leaſe to be paid before it fall in hand.

li	ſ	d	q
275	0	0	0
16	13	4	0
221	13	4	0

The

The fourth Table *expressing what one pound yeerly rent or annuity for any number of yeers not exceeding* 30 *is worth ready mony at* 8 per centum, per annum.

yeers.	li.	s.	d.	q.	li.	s.	d.	q.	yeers.
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	10	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	6	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	1	10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	19	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The

The fourth Table is disposed altogether as the form ... and the use thereof in like sort being damageable.

1 Example.

There is an annuity or rent of 20 s. per annum *to endure* 25 *yeers, it is required what it is worth ready money?*

1 Look in the Table for 25 *yeeres*, and right against it you shal find 10 li-13 s-5 d-3 q, which is the solution.

2 Example.

What is the Lease of certaine Land valued at 140 li. per annum, *to begin presently and endure* 18 *yeers worth ready money?*

Search in the *Table* for 18 *yeers*, the *terme* named in the *question*, and right against it toward the left hand you shall finde 9 li-7 s-5 d-1 q. which expresseth that *one pound rent* to be bought for that *terme* is worth so much; therfore that summe 140 times is the value required. Now 140 times 9 li. is 1260, and 140 times 7 s. is 980 s. or 49 li, likewise 140 times 5 d. is 700 d. or 2 li-18 s-4 d. and 140 *farthings* is 2 s-11 d. all which added together make 1312 li-1 s. 3 d. for the value of the said *Lease*, paying no *rent*.

li	s	d
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3 Ex-

3. Example.

A Lease taken for 21 *yeers at* 13 li-6 s. 8 d. per annum, *which after* 5 *yeers expired, the Tenant is desirous to give a fine, and bring the rent down to* 8 li. per annum, *for the rest of the term, the demand is what fine is to be payed?*

Subtract 5 *yeers* from 21, the remayne 16, is the time unexpired; likewise from the present *rent* abate 8 li. the rest will be 5 li-6 s-8 d. now the drift of the *question* is, what 5 li-6 s-8 d. *yeerly rent* or *annuity* to indure 16 yeeres is worth present money?

The value of 1 li. *rent* or *annuity* answering to 16 *yeers* is, 8 li-17 s-0 d. 1 q. Now 5 times 8 li. is 40 li. and 5 times 17 s. 4 li-5 s. and 5 times one farthing, is 1 d-1 q. and because 6 s-8 d. is $\frac{1}{3}$ of 1 li. I take $\frac{1}{3}$ of 8 li-17 s-0 d 1 q, which is 2 li-19 s 0 d. all which added together, make 47 li-4 s-1 d-q. which is the *fine* that ought to be paid to bring the *rent* to 8 li. *per annum*.

li.	s	d	q
40	0	0	0
4	5	0	0
2	19	1	1
47	4	1	1

The fifth Table declaring what yeerly rent or annuitiy of one pound ready money will purchase for any number of yeers under 31, *at* 8 per centum, per annum.

yeers	li.	s.	d.	q.	li.	s.	d.	q.	yeers
1	0	1	7	0	0	2	9	3	16
2	0	14	0	2	0	2	8	3	17
3	0	9	8	2	0	2	8	0	18
4	0	7	6	6	0	2	7	0	19
5	0	6	3	0	0	2	6	2	20
6	0	5	4	3	0	2	5	3	21
7	0	4	9	2	0	2	5	1	22
8	0	4	4	0	0	2	4	3	23
9	0	4	0	0	0	2	4	1	24
10	0	3	8	2	0	2	4	0	25
11	0	3	6	0	0	2	0	3	26
12	0	3	3	3	0	2	3	1	27
13	0	3	1	3	0	2	3	0	28
14	0	3	0	1	0	2	2	3	29
15	0	2	11	0	0	2	2	2	30

In

In the fifth Table *the* Numbers *and* Columnes *are all disposed as the former* Tables, *and needeth no further explanation but onely Examples.*

1 Example.

The Table *declareth at first sight what yeerly rent or annuitie one pound ready money will purchase for any terme in the* Table *expressed.*

But if the *ready money* be above one pound, then if any *value* or *rent* set down in this Table, be multiplyed by the *number* belonging to the *yeers in question*, the *product* will shew what *yeerly rent* or *annuitie* that *ready money* will purchase for the *time proposed.*

2 Example.

A certain man hath 750 li, *to purchase an Annuitie to endure* 27 *yeers, so as it may yield him the like profit, as if it were put out according to the rate in the* Table *expressed, it is required what that annuitie ought to be?*

Because the *annuitie* is to endure 27 yeeres, seek out the value or rent set against 27 yeers, in this fift Table, which is 2 s--3 d--1 q now this

This being the *Annuity* of 20 s. ready *mon*[ey] will purchase for that *termes*, it must be multi-plyed by 750 li. as followeth, because 2 s. is the tenth part of 20 s. therefore take the tenth part of 750 li. which is 75 li. which set first down, then 750 times 3 d. is 9 l-7 s-6 d. whic[h] set under the former, last of all 750 *farthing*[s] is 15 s-7 d-ob. All which added together, pro[-]duce 85 li.-3 s.-1 d-ob. the *yeerly Annuiti*[e] required.

li	s	d
75	0	0
9	7	6
	15	7
85	3	1

Deo soli laus, omnis honor & gloria tribuatur.

AMEN.

FINIS.